Hold-up in multi-sized particulate solid-liquid flow through horizontal pipes

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Hold-up is an important design parameter for pipelines transporting solid liquid mixture as it correctly determines the in situ volume fraction of different phases. Extensive experiments have been reported to establish the effect of solid concentration, flow velocity and particle size on hold-up. Based on the data base, a simple correlation has been proposed for determination of hold-up for multi-sized solid-liquid flow through pipes. The proposed correlation \[ HR = 1 + 0.64 \left( \frac{W_o}{U_o} \right)^{0.50} \] incorporates the effect of all the factors affecting hold-up such as solid concentration, flow velocity, particle size, pipe diameter, etc. The predictions based on the proposed correlation match reasonably well with the experimental data.

Transportation of solids using pipes has been established to be more techno-economically attractive compared to conventional modes of transportation for complex terrain. These pipelines offer high degree of efficiency, reliability and round the year availability, besides being environmental and eco-friendly. Over the years, many studies have been conducted to formulate a methodology for the design of these pipelines by judicious selection of parameters, like flow velocity, concentration of solids, particle size distribution, pressure drop, pipeline wear, etc. One of the important parameters in the design of such pipelines is hold-up. Hold-up for a phase is defined as the ratio of in situ concentration of that phase to the delivered/input concentration of the same. The importance of hold-up can be understood from the fact that design of slurry pipelines is based on correlations developed on the basis of in situ concentration and not on the delivered concentration.

Hold-up effect is clearly evident at low velocities in horizontal solid-liquid flows through pipes because of the flow characteristics, viz.; (a) velocity profile across the pipe cross-section in the vertical plane; (b) solid concentration profile across the pipe cross-section in the vertical plane; and (c) the local relative velocity between the two phases.

Phenomenon of hold-up in solid-liquid flows in pipes has attracted attention of scientists for the last four decades. Worster¹ has shown the link between the type of the flow and the input volume fraction with the hold-up way back in fifties. He observed that increasing input volume fraction of solids results in reduced hold-up. He also observed that hold-up ratio is greater in a heterogeneous regime of flow than a homogeneous regime of flow. Bonnigton² investigated the flow of water and sand particles with mean size of particles as 1220 microns in a 38.1 mm (1.5" pipe). He has shown that the hold-up increases with decreasing liquid velocity. Newitt³ et al. studied the flow of light perspex, zircon and fine sand particles and observed that phenomena of hold-up exists for coarser particles only. Gandhi⁴ has pointed out that the differences observed in the predictions from various correlations could be due to the difference between the in situ and the delivered solid concentrations. He also proposed a method to predict the hold-up for a horizontal pipeline for solid-liquid flows. Spedding and Nguyen⁵ have developed a tool for prediction of hold-up in solid-liquid flows based on the applications of conservation of mass to field theory of heterogeneous flow. Vishwanathan and Mani⁶ have also proposed an expression using the method of least square relating hold-up with various dimensionless parameters. Spedding and Chen⁷ have also shown that the phenomena of hold-up in solid-liquid system has a direct relationship with the volumetric input ratio. They have tested several models of gas-liquid hold-up for solid-liquid flows. The phenomena of hold-up has been well documented by Govier and Aziz⁸, Roco and Shook⁹, and others.¹⁰ ¹¹. Govier and Aziz⁸ have proposed a relation to predict the hold-up in solid-liquid mixtures for equisized particles. The methods proposed by Roco and Shook⁹ and Wilson et al.¹⁰ are applicable to
commercial slurries but are too complex to be used. Coulson and Richardson have described the concept of hold-up for liquid and gas flow.

Review of the literature reveals that very little work has been carried out to establish the dependence of hold-up phenomena on various parameters in solid-liquid flows. The methodologies proposed are from the database on equi-sized particulate slurries and hence their applicability is severely restricted for multi-sized particulate slurry flows. As most of the commercial slurries are multi-sized with particle size varying over three order of magnitudes, an attempt has been made to quantify the effect of various parameters affecting the phenomena of hold-up in multi-sized particulate slurry flow through pipelines and propose a relation for hold-up which is easy to use and applicable to commercial slurries in the present study. The emphasis of the work is towards correlating the hold-up with a non-dimensional parameter.

Hold-up in multi-phase systems has been quantified in different ways by various investigators and are well documented. Generally, for solid-liquid mixtures, hold-up ratio is defined as:

\[ HR = \frac{C_{\text{in-situ}}}{C_{\text{eff}}} \]  

where \( C_{\text{in-situ}} = \int c \, dA / A \) and \( C_{\text{eff}} = \int c \, v dA / Q \). Here, \( c \) is the local in situ concentration of solids measured over a small area, \( v \) is the local velocity of the solids over the same area, \( A \) is the total area of the flow and \( Q \) is the flow rate. The hold-up ratio \( HR \) has been assumed to represent the hold-up for solids in solid-liquid flows through horizontal pipe line.

**Parameters Affecting Hold-up**

Hold-up is primarily affected by the interplay of concentration and velocity profiles. It is the parameter, which represents in situ concentration of the solid phase and hence it is expected that hold-up will be affected by different process parameters in the same way as in situ concentration profile. The parameters directly affecting concentration profiles are mean particle size, shape and specific gravity of particles, pipe diameter and turbulence level. Some of these parameters, namely efflux concentration, flow velocity and turbulence level affect concentration profile in a way that it becomes more uniform. Remaining parameters, namely pipe diameter, particle size and specific gravity increase the skewness in the concentration profile. To bring out the cumulative effect of all these parameters on hold-up, the best way is to define a dimensionless parameter. Based on this hypothesis, HR can be proposed as:

\[ HR = f \left( \frac{W_o}{U_s} \right) \]  

Here, \( W_o \) is the hindered settling velocity and \( U_s \) is the frictional velocity. In this relation, \( W_o \) incorporates the effect of particle size, shape, specific gravity and efflux concentration, where as \( U_s \) represents the effect of pipe diameter, velocity and turbulence intensity. The behaviour of this function can be speculated. Smaller value of \( (W_o/U_s) \) either corresponds to a smaller value of \( W_o \) or a higher value of \( U_s \) or both. This suggests that particle size, specific gravity and pipe diameter are small where as velocity and efflux concentration are higher. All these combinations show that concentration profile will be uniform and hence a lower value of hold-up. Higher value of \( (W_o/U_s) \) corresponds to a higher value of \( W_o \) and lower value of \( U_s \), i.e. particle diameter are larger and velocity and concentration are smaller. This implies a skewed concentration profile resulting in a higher hold-up value.

**Computation of hold-up and \( (W_o/U_s) \)**

The philosophy of the proposed hypothesis requires evaluation of the hold-up, hindered settling velocity and the frictional velocity. The hold-up is defined as the ratio of in situ concentration and efflux concentration. The in situ concentration is evaluated from the relation:

\[ C_{\text{in-situ}} = (1/A) \int c \, dA, \]  

Here, \( c \) is the measured local concentration which is integrated over the pipe cross sectional area to get the in situ concentration. Hold-up, thus, can be calculated if in situ concentration profile is known for a given efflux concentration. The hindered settling velocity is evaluated from the relation suggested by Richardson and Zaki, as:

\[ W_o = V_o (1-C_{\text{eff}})^2, \]  

where, \( Z \) is evaluated from the following relations depending upon the range of \( R_e \):

\[ Z = 4.65 + 1.95 \left( \frac{d}{D} \right) \] for \( 0.002 < R_e \leq 0.2 \)  

\[ Z = (4.35 + 17.5 \left( \frac{d}{D} \right)) R_e^{0.03} \] for \( 0.2 < R_e \leq 1.0 \)  

\[ Z = (4.45 + 18.0 \left( \frac{d}{D} \right)) R_e^{0.1} \] for \( 1 < R_e \)

and \( V_o \), the unhindered settling velocity is calculated from the expressions given in the literature:
\[ V_o = \frac{g}{18\mu} \left( \rho_s - \rho \right) d_j^2 \] for laminar motion 
\( R_e < 1; \) Stokes law \hspace{1cm} \ldots \hspace{0.5cm} (8)

\[ V_o = 0.2 \ d_j^{1.18} \left( \frac{g(\rho_s - \rho)}{\rho} \right)^{0.72} / \left( \mu/\rho \right)^{0.45} \] for transition region \hspace{1cm} \ldots \hspace{0.5cm} (9)

\[ (1 < R_e < 1000; \) Allen's equation \]

\[ V_o = 1.74 \ d_j^{0.5} \left( \frac{g(\rho_s - \rho)}{\rho} \right)^{0.5} \] for turbulent motion 
\( R_e > 800; \) Newton's law \hspace{1cm} \ldots \hspace{0.5cm} (10)

In the above expressions \( d_j \) is the particle size, \( \rho_s \) is the density of solid particles, \( \rho \) is the density of liquid, \( D \) is the pipe diameter and \( \mu \) is the viscosity of the liquid.

The value of shear velocity \( U_* \) is calculated from relation (11); 
\[ U_* = \sqrt{\left( \frac{g}{r} \Delta p \right)} \] 
\hspace{1cm} \ldots \hspace{0.5cm} (11)

where, \( \Delta p \) is the pressure drop/length (metre of slurry), \( r \) is the hydraulic radius (for pipe flow \( r = D/4 \))

For extending the analysis for multi-sized particulate slurries, the weighted mean diameter has been taken as the representative diameter as suggested by Mishra\(^1\). \(^3\).

Data used in the analysis
Data collected from the experiments conducted on the pilot plant test loop at IIT Delhi, has been used in the present analysis. The experimental data analysed are from the experiments of Kaushal\(^1\), Mukhtar\(^3\), Seshadri et al.\(^6\)-\(^8\), Mishra\(^1\) and Seshadri & Singh\(^19\),\(^20\). The parameters used from these studies were specific gravity, pipe diameter, particle size distribution, efflux concentration, flow velocity, pressure drop and mean particle diameter. The range of these parameters is given in Table 1. It is important to see that the range of parameters covers the heterogeneous as well as the homogeneous flow regime. It is also important to see that solid materials used give a reasonably wide range of specific gravity and mean diameter of particles.

Results and Discussion
Table 1 presents the details of data collected from the literature along with the computed values of hold-up and \( W_s / U_* \). To establish the effect of velocity, efflux concentration and particle size on hold-up the systematic data of Mishra\(^1\) have been used and shown in pictorial form in Figs 1-3.

![Figure 1](image1.png)

**Fig. 1** — Variation of hold-up ratio with flow velocity at different efflux concentrations

![Figure 2](image2.png)

**Fig. 2** — Variation of hold-up ratio with efflux concentration at different flow velocities

**Effect of velocity**
Fig. 1 depicts the variation of hold-up ratio as a function of velocity. For a given efflux concentration hold-up ratio decreases with increase in velocity. The rate of decrease of hold-up ratio is maximum for the lowest efflux concentration and is least for the maximum efflux concentration. This indicates that the effect of the flow velocity is more pronounced on the hold-up at low concentrations of solids in comparison to higher concentration of solids.

**Effect of concentration**

![Figure 3](image3.png)

**Fig. 3** — Variation of hold-up ratio with concentration at different flow velocities

Literature has revealed that at high efflux concentration, the concentration profile is uniform and hence leads to a smaller value of hold-up ratio. Fig. 2 clearly shows this phenomenon as at a given flow velocity hold-up ratio decreases with increase in efflux concentration. The effect is more pronounced at the lower flow velocity as compared to higher flow velocities.
Table 1 — Data used in the analysis

<table>
<thead>
<tr>
<th>Sl No.</th>
<th>Investigator</th>
<th>Pipe diameter (mm)</th>
<th>Material and Sp. Gravity</th>
<th>Efflux concentration (% by Vol.)</th>
<th>Velocity (m/s)</th>
<th>Pressure drop (water column m/100 m of pipe length)</th>
<th>Weighted mean diameter ( \times 10^{-4} ) (m)</th>
<th>( W/J, U_2 ) ( \times 10^{-2} )</th>
<th>HR</th>
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<td>1</td>
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</table>

velocities. The rate of decrease in hold-up with concentration is maximum for lower flow velocity and minimum for highest flow velocity used in experiments.

**Effect of particle size**

To study the effect of particle size hold-up ratio has been calculated from the Data of Mishra\(^{13}\), which gives the in-situ particle size distribution of different
Holdup Ratio

Fig. 3 — Variation of hold-up ratio with particle diameter at an efflux concentration of 9.3% (by volume) for different flow velocities

Fig 4 — Comparison between experimental and predicted values of hold-up ratio

particle sizes along with the overall particle size distribution. Gravitational force is main cause to affect the hold-up phenomenon for the different size of particles as depicted in Fig. 3. It is seen that bigger particle sizes give high hold-up ratio and smaller sizes lead to a lower hold-up ratio. This is because bigger particles have higher settling velocities, which results in highly skewed concentration profiles resulting in higher hold-up. For all particle sizes, hold-up ratio decreases with increase in slurry flow velocity.

Correlation between HR and (W/U) for multi-sized slurries

Figs 1-3 clearly establish the effect of parameters namely efflux concentration, flow velocity and particle size on hold-up ratio. Based on the preceding discussion it is feasible to propose a relationship for the dependence of HR on the fraction \((W/U)\) as:

\[ HR = 1 + A (W/U)^B \]  \hspace{1cm} (12)

where \(A\) and \(B\) are constants. Expression of the form shown above is best suited for prediction of hold-up for multi sized slurry flows because in the homogeneous regime \(W_0/U_*\) will be very small and thus hold-up will be very close to 1. For heterogeneous regime, \(W_0/U_*\) is expected to be higher resulting in value of hold-up in excess of one. Values of constants \((A \text{ and } B)\) have been calculated using the method of least squares as, 0.64 and 1.50 respectively. Prediction of hold-up ratio based on the above correlation gives a reasonably good fit for the value of dimensionless parameter \((W/U)\) below 0.17 and almost 80% of the data points are with in an error band of 1% (Fig. 4).

Conclusions

The present study clearly establishes the effect of velocity, efflux concentration and particle size on the hold-up ratio and clearly brings out the difference between the in-situ concentration and efflux concentration. The in situ concentration does not have a constant value and is affected by different parameters. The present investigation has identified the dependence of these parameters, namely particle diameter, input velocity and efflux concentration, on in situ concentration and the hold-up ratio. Furthermore, a simple correlation has been proposed between the hold-up ratio and the dimensionless parameter \((W/U)\). The prediction of hold-up ratio based on the proposed correlation shows good agreement with experimental results for a value of less than 0.17 of the dimensionless parameter \((W/U)\). It is also seen that the weighted mean diameter gives a reasonable accurate representation of the particle size effect for multi-sized particulate slurries.

References

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2. Bonnington S T. Experiments on the hydraulic transport of mixed sizes solids (British Hydro mechanical Research Association), 1959, 637.