Effects of non-uniform temperature gradient and magnetic field on the onset of convection in fluids with suspended particles under microgravity conditions

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Received 9 March 1999; revised received 13 December 2000

The effects of a non-uniform temperature gradient and magnetic field on the onset of convection driven by surface tension in a horizontal layer of Boussinesq fluid with suspended particles confined between an upper free / adiabatic boundary and a lower rigid / isothermal boundary have been considered. A linear stability analysis is performed. The microrotation is assumed to vanish at the boundaries. The Galerkin technique is used to obtain the eigenvalues. The influence of various parameters on the onset of convection has been analysed. Six different non-uniform temperature profiles are considered and their comparative influence on onset is discussed. It is observed that the electrically conducting fluid layer with suspended particles heated from below is more stable compared to the classical electrically conducting fluid without suspended particles. The critical wave number is found to be insensitive to the changes in the parameters but sensitive to the changes in the Chandrasekhar number. The problem has possible applications in microgravity space situations.

Recently there has been great interest in the theory and modelling of materials processing in the microgravity environment. The development of convection and corresponding heat transfer are examples of the physical phenomena to be encountered in these types of problems. Among the effects to be considered here are those of surface tension, crystalline anisotropy, non-equilibrium solidification and convection in the melt. These are relevant to the growth of large single crystals, the manufacture of semi-conductor devices and metallurgical processing. The results of space exploration, particularly the mechanism of prevention of buoyancy driven convection, are useful in understanding the physical processes involved in manufacturing these materials. Even though the microgravity environment in space is known to reduce the convection driven by buoyancy force, Marangoni convection will be generated due to the variation of surface tension with temperature. Simulating the microgravity environment in the laboratory to prevent altogether the buoyancy driven convection is difficult.

The Rayleigh-Bénard situation in Eringen's4-6 micropolar fluids has been investigated by many authors7-14. The main results from all these studies is that for heating from below stationary convection is the preferred mode. But it is a well known fact that the onset of convection in Benard's experiments is produced not simply by buoyancy force but primarily by variation of surface tension with the temperature. The latter effect is generally referred to as Marangoni instability. Pearson15 was the first person to make an analytical study of this effect. According to Pearson's15 theory for a critical value of the Marangoni number, the layer displays a short-wave pattern of stationary cellular convection. The effect of a uniform vertical magnetic field on the thermoacapillary instability of a Newtonian layer of electrically conducting fluid (Marangoni magneto-convection) was first considered by Nield16 and later by Rudraiah et al.17,18. Subsequently, Maekawa and Tanasawa19 considered the same problem with inclined magnetic field. All the above works are for a non-deformable surface. Sarma20, Kaddame and Lebon21 and Wilson22,23 studied Marangoni magneto-convection considering a deformable free surface. Recently, Milaninova and Slatchev24 made a weak non-linear analysis of Marangoni magneto-convection.

The objective of this paper is to suggest additional mechanisms of controlling convection using suspended particles, applied magnetic field and non-uniform basic temperature gradients. The micropolar fluid description is used in the paper for the fluid with suspended particles. The single term Galerkin expansion technique has been utilized to obtain the critical Marangoni number.

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Mathematical Formulation and Solution

Consider an infinite horizontal layer of a Boussinesquian electrically conducting micropolar fluid layer of depth 'h' permeated by an externally applied magnetic field $H_0$ normal to the fluid. A cartesian coordinate system is taken with the origin in the lower boundary and z-axis vertically upwards. The x-axis is along the lower plate. Let $\Delta T$ be the temperature difference between lower and upper boundaries of the fluid. The interface at the upper boundary has a temperature dependent surface tension $\sigma(T)$. Expanding $\sigma(T)$ by Taylor series about $T_0$, we get

$$\sigma(T) = \sigma(T_0) + \frac{(T - T_0)}{1!} \left( \frac{d\sigma}{dT} \right)_{T_0} + \frac{(T - T_0)^2}{2!} \left( \frac{d^2\sigma}{dT^2} \right)_{T_0} + \cdots.$$  

Since $T - T_0$ is quite small in our analysis, we may write

$$\sigma(T) = \sigma_0 - \sigma_1(T - T_0)$$  \hspace{1cm} (1)

where $\sigma_1 = - \left( \frac{d\sigma}{dT} \right)_{T_0}$ and $\sigma_0 = \sigma(T_0)$.  

The form of $\sigma(T)$ in Eq. (1) was used by Pearson$^{15}$. The governing equations for the problem are

$$\nabla \cdot \mathbf{q} = 0$$  \hspace{1cm} (2)

$$\rho \left[ \frac{\partial \mathbf{q}}{\partial t} + (\mathbf{q} \cdot \nabla) \mathbf{q} \right] = -\nabla p + (2\zeta + \eta) \nabla^2 \mathbf{q} + \xi \mathbf{v} \times \mathbf{w} + \mu_m \mathbf{H} \cdot \nabla \mathbf{H}$$

$$+ \mu_m \mathbf{H} \cdot \nabla \mathbf{H}$$

$$\frac{\partial \mathbf{q}}{\partial t} = \frac{\partial \mathbf{\omega}}{\partial t} \left[ \mathbf{q} + \gamma M_1 \mathbf{V} \times \mathbf{q} + \gamma M_2 \mathbf{V} \right]$$

$$\frac{\partial \mathbf{\omega}}{\partial t} = \left( \lambda + \lambda M_1 \mathbf{V} \times \mathbf{\omega} + \lambda M_2 \mathbf{V} \right)$$

$$\frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = \chi \nabla^2 T$$

$$\frac{\partial \mathbf{H}}{\partial t} + \mathbf{q} \cdot \nabla \mathbf{H} = (\mathbf{H} \cdot \nabla) \mathbf{q} + \gamma_m \nabla^2 \mathbf{H}$$

$$\nabla \cdot \mathbf{H} = 0,$$  \hspace{1cm} (3)  

(4)  

(5)  

(6)  

(7)

where $\mathbf{q}$ is the velocity, $\mathbf{\omega}$ is the spin, $T$ is the temperature, $\mathbf{H}$ is the magnetic field, $P = \rho + \frac{\mu_m}{2} H^2_0$ is the hydromagnetic pressure, $\rho_0$ is the density of the fluid at a reference temperature $T_r = T_0$, $\xi$ is the coupling viscosity coefficient or vortex viscosity, $\eta$ is the shear kinematic viscosity coefficient, $I$ is the moment of inertia, $\lambda'$ and $\lambda''$ are the bulk and shear spin viscosity coefficients, $\beta$ is the micropolar heat conduction coefficient, $C_i$ is the specific heat, $\zeta$ is the thermal conductivity, $\alpha$ is the coefficient of thermal expansion and $\gamma_m = 1/\mu_m \sigma_m$ is the magnetic viscosity ($\sigma_m$ : electrical conductivity and $\mu_m$ : magnetic permeability).

The Eqs. (2) – (7) are solved subject to containment conditions appropriate for a rigid and thermally perfect conducting wall on the underside and a free surface on the upper side. This free surface is adjacent to a non-conducting medium and subject to a constant heat flux (i.e. adiabatic). Further, the no-spin boundary condition is assumed for micro-rotation. Since the shear stress for a non-classical fluid with suspended particles is no different from that of classical fluids, the boundary conditions for flat free boundaries used by Nield$^{16}$ in respect of Newtonian fluids are appropriate for micropolar fluids also.

In the quiescent state the velocity $\mathbf{q}$, the spin $\mathbf{\omega}$, the temperature $T$ and the magnetic field $\mathbf{H}$ have the following solution:

$$\mathbf{q} = 0, \quad \mathbf{\omega} = 0, \quad \mathbf{H} = H_0 \mathbf{k}, \quad \frac{-h}{\Delta T} \frac{dT}{dz} = f(z)$$  \hspace{1cm} (8)

where $f(z)$ is a non-dimensional basic temperature gradient satisfying the condition

$$\int_0^1 f(z) \, dz = 1.$$  \hspace{1cm} (9)

The various non-uniform basic temperature gradients considered in this paper are presented in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Non-uniform temperature gradient</th>
<th>$f(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Linear</td>
<td>1</td>
<td>$1$</td>
</tr>
<tr>
<td>2 Heating from below</td>
<td>$\left{ \begin{array}{ll} \varepsilon^{-1} &amp; 0 \leq z &lt; \varepsilon \ 0 &amp; \varepsilon &lt; z \leq 1 \end{array} \right.$</td>
<td>$1$</td>
</tr>
<tr>
<td>3 Cooling from above</td>
<td>$\left{ \begin{array}{ll} 0 &amp; 0 \leq z &lt; 1 - \varepsilon \ \varepsilon^{-1} &amp; 1 - \varepsilon &lt; z \leq 1 \end{array} \right.$</td>
<td>$1$</td>
</tr>
<tr>
<td>4 Step function</td>
<td>$\delta(z - \varepsilon)$</td>
<td>$1$</td>
</tr>
<tr>
<td>5 Inverted parabolic</td>
<td>$2(i - z)$</td>
<td>$1$</td>
</tr>
<tr>
<td>6 Parabolic</td>
<td>$2z$</td>
<td>$1$</td>
</tr>
</tbody>
</table>
This type of basic temperature gradients arise due to sudden heating or cooling, radiation, through flow, etc. (see Lebon and Cloot).

We now suppose that the initial state is slightly disturbed. The linearized equations of motion allow the solution of a disturbance in the form

\[ \begin{bmatrix} W, \Omega_z, T, H_z \end{bmatrix} = \begin{bmatrix} \exp[(lx + my)] \end{bmatrix}, \]

where \( l \) and \( m \) are the horizontal component of the wave number \( \bar{a} \). We use this expression in the linearized version of the basic equations and non-dimensionalise the resulting equations using the following definitions:

\begin{align*}
\left( x^*, y^*, z^* \right) &= \left( x, y, z \right) / h, \quad \bar{q}' = \frac{q'}{\chi/h}, \quad \bar{\omega}' = \frac{\omega'}{\chi/h^2}, \\
T^* &= \frac{T'}{\Delta T}, \quad H^* = \frac{H'}{H_0}
\end{align*}

We assume the principle of exchange of stability to be valid and hence deal with only stationary convection which is governed by the following equations after using Eq. (11):

\begin{align*}
(1 + N_1) \left( D^2 - a^2 \right) W + N_1 \left( D^2 - a^2 \right) G + Q Pr (D^2 - a^2) DH_z &= 0, \quad \ldots \ (12) \\
N_1 \left( D^2 - a^2 \right) W - N_3 \left( D^2 - a^2 \right) G + 2N_1 G &= 0, \quad \ldots \ (13) \\
(D^2 - a^2) T + f(z) (W - N_5 G) &= 0, \quad \ldots \ (14) \\
(D^2 - a^2) H_z + \frac{Pm}{Pr} DW &= 0, \quad \ldots \ (15)
\end{align*}

where \( D = \frac{d}{dz} \), \( a^2 = l^2 + m^2 \).

\begin{align*}
N_1 &= \frac{\zeta}{\xi + \eta} \text{ (Coupling Parameter)}, \\
N_3 &= -\frac{\eta}{(\xi + \eta)h^2} \text{ (Couple Stress Parameter)}, \\
N_5 &= \frac{\beta}{\rho_0 C_v h^2} \text{ (Micropolar Heat Conduction Parameter)}, \\
Pr &= \frac{\zeta + \eta}{\chi} \text{ (Prandtl number)}, \\
Pm &= \frac{\zeta + \eta}{\gamma_m} \text{ (Magnetic Prandtl number)}, \\
R &= \frac{\alpha g DT h^3 \rho_0}{(\xi + \eta)\chi} \text{ (Rayleigh number)} \\
Q &= \frac{\mu_m H_0^2 h^2}{(\xi + \eta)\gamma_m} \text{ (Chandrasekhar number)}.
\end{align*}

In writing Eqs. (12)-(15) the asterisks have been omitted.

Eliminating \( H_z \) between Eqs. (12) and (15), we get

\[ (1 + N_1) \left( D^2 - a^2 \right) W + N_1 \left( D^2 - a^2 \right) G - QD^2 W = 0. \]

Eqs. (13), (14) and (16) are solved subject to the following boundary conditions (see Nield and Rudraiah and Siddheshwar):

\begin{align*}
W &= DW = T = G = 0 \quad \text{at } z = 0, \\
W &= D^2 W + a^2 MT = DT = G = 0 \quad \text{at } z = 1
\end{align*}

(17)

where \( M = \frac{\sigma \Delta T h}{\mu \chi} \) is the Marangoni number. Eq. (17) indicates the use of rigid, isothermal lower boundary and upper, free, thermally insulating boundary (with respect to the perturbation). The condition on \( G \) is the spin-vanishing boundary condition.

We now use the single-term Galerkin expansion technique to find the critical eigenvalue. Multiplying Eq.(16) by \( W \), Eq.(13) by \( G \) and Eq.(14) by \( T \), integrating the resulting equations by parts with respect to \( z \) from 0 to 1, using the boundary conditions (17) and using \( \frac{\partial W}{\partial z} = A W_1, \frac{\partial G}{\partial z} = B G_1, \frac{\partial T}{\partial z} = C T_1 \) in which A, B and C are constants and \( W_1, G_1 \) and \( T_1 \) are trial functions, yield the following eigenvalue equation:
\[ M = \frac{\left( DT_1^2 \right) + a^2 \left( T_1^2 \right) \left( C_1 C_2 + N_2 C_3^2 \right)}{(1 + N_1) a^2 DW_i(l) T(l) C_4}, \]  
(18)

where

\[ C_1 = N_3 \left( DG_1 \right) + \left( N_3 a^2 + 2 N_1 \right) \left( G_1^2 \right), \]

\[ C_2 = -\left[ (1 + N_1) \left( \left( D^2 W_1 \right) + 2 a^2 \left( DW_1 \right) + a^4 \left( W_1 \right) \right) \right], \]

\[ -Q \left( DW_i \right) \]

\[ C_3 = \left( DG_i \right) \left( DW_i \right) + a^2 \left( W_i G_i \right) \]

\[ C_4 = \left( f(z) W_i T_i \right) C_1 - N_3 N_1 \left( f(z) T_i G_i \right) C_3 \]

In Eq. (18), \(\cdots\) denotes integration with respect to \(z\) between \(z=0\) and \(z=1\).

\( M (z, W_i, G_i, T_i) \) in Eq. (18) is a functional and Euler–Lagrange equations for the extremisation of \(M\) are Eqs (12)-(15).

We select the trial functions

\[ W_i = z^2 (1 - z^2), \quad G_i = z (1 - z), \quad T_i = z (2 - z) \]  
(19)

such that they satisfy all the boundary conditions (17) except the one given by \(D^2 W + a^2 M T = 0\) at \(z = 1\), but the residual from this is included in the residual from the differential equations. Substituting Eqs (19) in (18) and performing the integration, we can calculate the critical Marangoni number \(M_c\), which attains its minimum at \(a_c^2\).

**Results and Discussion**

The effects of non-uniform basic temperature gradient and magnetic field on the onset of Marangoni convection in an electrically conducting micropolar fluid have been studied. Six non-uniform temperature profiles are chosen for study. It is observed that for the critical Marangoni number, \(M_c\), the following inequality holds for the six models under question

\[ M_{c1} < M_{c3} < M_{c6} < M_{c2} < M_{c3} < M_{c5}. \]

i.e., the step function is the most destabilising basic temperature distribution and inverted parabolic is the most stabilising basic temperature distribution. In the case of piecewise linear and step function profiles, the critical Marangoni number \(M_c\) depends on the thermal depth, \(\varepsilon\), in addition to depending on the parameters of the problem. In the case of piecewise linear profile heating from below, cooling from above and step function profiles the minimum value of \(M_c\) is attained at \(\varepsilon = 0.93\), \(\varepsilon = 0.43\) and \(\varepsilon = 0.74\) respectively.

Before embarking on a discussion of the results let us make some comments on the parameters \(N_1\), \(N_3\) and \(N_5\) arising due to the suspended particles. Assuming the Clausius-Duhem inequality Eringen\(^4\) presented certain thermodynamic restrictions, which lead to non-negativity of \(N_1\), \(N_3\) and \(N_5\). For \(\zeta = 0\) \((N_i = 0)\) it is clear that equation (12) for \(W\) becomes independent of \(G\), i.e. it is uncoupled. As \(\zeta \to \infty\), we see that \(N_1 \to 1\) and \(N_3 \to 0\). This is the Stokesian description of suspension. Thus, it is obvious that couple stress comes into play only at small values of \(N_3\). This supports the contention that \(N_1 \in [0,1]\) and that \(N_3\) is small positive real number. Coming to \(N_5\) it has to be finite because the increasing of concentration has to practically stop somewhere and hence \(N_5\) has to be a positive, finite real number.

The typical order of magnitudes of \(N_1\), \(N_3\) and \(N_5\) mentioned above apply to fluid systems encountered in materials processing under microgravity in space.

With the above background and with the motive

![Fig. 1—Plot of critical Marangoni number \(M_c\) versus coupling parameter \(N_i\) for different non-uniform temperature gradients.](image-url)
specified in the introduction we now discuss the results presented by the Figs 1-6.

Fig. 1 is the plot of $M_c$ versus the coupling parameter $N_1$ for different non-uniform temperature gradients. Clearly $M_c$ increases with $N_1$. Increase in $N_1$ indicates the increase in the concentration of micropolar elements. These elements consume the greater part of the energy of the system in developing the gyrational velocities of the fluid and as a result the onset of convection is delayed. Therefore, the increase in $N_1$ is to stabilise the system.

Fig. 2 is the plot of $M_c$ versus the couple stress parameter $N_3$ for different non-uniform temperature gradients. Clearly $M_c$ decreases with $N_3$, and ultimately levels off to the Newtonian value. Increase in $N_3$ decreases the couple stress of the fluid which causes a decrease in microrotation and hence makes the system more unstable.

Fig. 3 is the plot of $M_c$ versus the micropolar heat conduction parameter $N_5$ for different non-uniform temperature profiles. When $N_5$ increases, the heat induced into the fluid due to these microelements also increases, thus reducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of instability. This result can also be anticipated because Eq. (5) clearly shows that the effect of the suspended particles is to deduct from the velocity. Thus, increase in $N_5$ is to stabilise the system.
Fig. 4 is the plot of $M_c$ versus Chandrasekhar number $Q$ for different non-uniform temperature gradients and two values of $N_1$. It is observed that as $Q$ increases $M_c$ also increases. It is also observed that as $N_1$ increases, $M_c$ also increases for small values of $Q$. However, for very large values of $Q$, the critical $M_c$ is less than the Newtonian value. This result may possibly suggest a value of $Q$ up to which the present theoretical study applies. Thus, the increase in the concentration of suspended particles is to stabilise the system along with the magnetic field.

Fig. 5 is the plot of $M_c$ versus $Q$ for different non-uniform temperature gradients and two values of $N_3$. The increase in $Q$ increases $M_c$ thus reiterating an earlier observation. From the figure, we see that the
A strong magnetic field succeeds in inducing micropolar heat conduction leads to delayed convection, a result which as we noted earlier is only to be anticipated.

Fig. 6 is the plot of $M_c$ versus $Q$ for different non-uniform temperature gradients and $N_5$.

These are shown in Table 2.

The authors are grateful to the referees for useful comments on the paper.

References