Vertical cracks of structural concrete beams under short-term load with unloading

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Characteristics of the process of cracking have been studied and their evaluation based on the experimental investigation of reinforced and prestressed concrete beams subjected to short term load with unloading has been carried out. The relations between strains and the characteristics of cracking are explained in the paper. The relationship between the reversible part of crack opening density and the crack opening density immediately before unloading is also presented.

As it is known, the cracks in concrete structures may originate in part as a result of insufficient compliance of construction practices (selection of diameter of reinforcing bars and concrete cover, unsuitable arrangement of stirrups, considerations of shrinkage of concrete, etc.), which may result in exceeding the tensile stress of concrete. In some cases cracks may be prevented by construction rules\(^1\), while in other cases cracks cannot be prevented as a rule, but they are admissible with requirement of crack width limits. These limits depend on the type of a structure and the interaction with the environment\(^2,3\).

In many structures, especially those with predominant bending stress, the cracks normal to the central line are decisive, that is vertical cracks in the case of horizontal structures such as beams or slabs. In most practical cases, the fulfillment of cracking requirements may be achieved by means of appropriate implicit practical rules, without direct calculation of crack widths. These practical rules that provide for control of cracking are available in most present codes, for example, the CEB-FIP Model Code\(^4\). But in special cases, particularly if this is needed for functional conditions a direct crack width calculation and check for acceptability of crack width limits is required. Also, the crack width formulae are useful to calibrate practical rules and analytical models\(^5\). Recently, research and experience have indicated that the use of thicker covers increase durability. With the use of thicker covers, however, the currently used crack control methods become unworkable and new crack width equations should be developed\(^6\). Design of a structure has to be based on the correlation between the bending moment and resulting stresses, strains, and crack widths. To achieve such correlation, the quantities as crack density and crack opening density\(^7\) may be used.

For correct design of a structure according to serviceability limit states, the behaviour of structural elements on the unloading branches is of considerable importance. During the lifetime, structures are under sustained loads, but very often the incidental overloading occurs. The estimate of the deformations and crack width after decreasing of load is necessary also for the statically indeterminate structures where the redistribution of stresses occurs\(^8,9\).

The relationships between the characteristics of cracking and the bending moment were derived for one-way reinforced concrete slabs on the basis of the results of an extensive experimental research on the deformation properties of flexural elements subjected to different type of loading\(^7,10\). Tests of reinforced and partially prestressed concrete beams subjected to different type of load, namely, short term-, moving- and long term load were carried out. These tests were conducted to know the behaviour of elements with flanged cross-sections and to investigate the influence of shear forces on the deformation behaviour of concrete elements. The short term load according to the Slovak Standard\(^11\) is defined as a load acting on a structure for a time period that is negligible with regard to the durability of the structure. Tests under gradually increasing load up to the failure are considered as short-term tests, the rheological properties of concrete have no effects on deformations and these properties are not taking into account by theoretical calculations.

The tests of the reinforced and partially prestressed beams under gradually increasing load with unloading branches were mainly concerned with the study of the
relation between the elastic deformations and the deformations immediately before unloading. Cracking was also observed during the tests and its results are presented in this paper.

**Experimental Procedure**

As a part of wider experimental programme three structural concrete (SC) beams with I and T cross sections for different span/depth ratio were tested. The span/depth ratio \( h/\ell \) of the I-beams was 7.5, and that of the T-beam was 11.4. The dimensions of the tested I-beams and their theoretical spans were chosen to follow the previous experimental research\(^\text{12}\), wherein different types of reinforcement were used. The experimental work also covered tests under long term load\(^\text{13}\). One of the concrete I-beams (I-PCB) was partially prestressed. The degree of prestressing was 0.614\(^\text{14}\). Details of the cross-sections of the tested beams, reinforcement and side-views are shown in Fig.1.

Average material characteristics of the deformed bars of the non-prestressing reinforcement employed in the beams were: yield strength \( f_{yy} = 454 \text{ MPa} \), tensile strength \( f_{tu} = 649 \text{ MPa} \), and modulus of elasticity \( E_s = 213 \text{ GPa} \). The characteristics of prestressing tendons with the nominal diameter \( \phi = 12.5 \text{ mm} \) were \( f_{ps} = 1700 \text{ MPa}, \quad E_p = 202 \text{ GPa} \). The average mechanical characteristics of the concrete are given in Table 1 for all the beams.

The simply supported beams were tested under a concentrated force \( F \) in the middle of the span. The hydraulic jack was placed under the tested beam. The force was induced by means of the two tie rods and steel rolled I-beam. The stiff loading regime was applied on both branches – on the loading branch, as well as on the unloading branch. For the control value, step-like increase of the displacement of the order of 4 mm of the loading hydraulic jack was taken. The measurements (deformations of concrete and cracks) for all loading and unloading branches of a beam were performed at the loading steps with the same deflection. The loading procedure is shown in Fig. 2 where the relationship between the loading force and the midspan deflection is plotted. The symbols (filled circles to represent the loading branches, empty circles for the unloading branches) represent the measured loading steps. In the case of the T-beam, the last three unloading cycles were made after the yield stress was reached.

The test of the T-beam was completed after the yield stress of the tensile reinforcement was reached which corresponded to the value of the loading force \( F = 245 \text{ kN} \) and to the control displacement of the hydraulic jack \( a_{hc} = 44 \text{ mm} \). The tests of I-beams were

![Table 1—Mean values of strengths of concrete and moduli of elasticity](image)

Table 1—Mean values of strengths of concrete and moduli of elasticity

<table>
<thead>
<tr>
<th>Designation of beams</th>
<th>Compressive strength ( f_{cm} ) on cubes [MPa]</th>
<th>Tensile strength ( f_{tu} ) on prisms [MPa]</th>
<th>Direct tensile strength ( f_{pt} ) [MPa]</th>
<th>Modulus of elasticity ( E_p ) [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>55.32</td>
<td>44.80</td>
<td>2.42</td>
<td>40.83</td>
</tr>
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<td>I-PCB</td>
<td>52.34</td>
<td>38.25</td>
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<tr>
<td>T-RCB</td>
<td>41.43</td>
<td>33.11</td>
<td>2.09</td>
<td>39.74</td>
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</tbody>
</table>

![Fig. 1—Cross-section, reinforcement of tested elements and position of measuring bases](image)

![Fig. 2—Loading force versus deflection relationship](image)
limited by the capacity of loading equipment, which was, $F_{\text{max}} = 400$ kN. The tests on I-beams were therefore interrupted at the loading levels $\lambda = 0.74$ and $\lambda = 0.68$ for prestressed concrete and reinforced concrete specimens, (I-PCB and I-RCB), respectively.

At every control displacement level, the strains of continuously linked-up base-lines at the compressed and tensioned edges as well as at the crossing diagonals were registered. The metal plugs embedded into the surface layer of the beam served to measurements of the distance changes between two adjacent ones by means of the mechanical strain gauges. The base lines at the top and bottom edges were of the nominal length $s = 360$ mm and their nominal vertical distance was $v = 460$ mm. The base lines shown in Fig. 1 form a "truss" consisting of "struts" and "ties" enabling the calculation not only of the strains but also of the deflections, by using a method based on Williot-Mohr translocation polygons. The network of base lines (Fig. 1) was decomposed into two truss systems. The metal plugs were located in the vertices of trusses. The measurements of elongations between two adjacent metal plugs were then evaluated by using the above-mentioned method of translocation polygons (in numerical form)\(^{15}\). The input data consisted of coordinates of vertices of a truss, elongations of struts, and of an incident matrix formed by the number of vertices adjacent to a given vertex. From the input data, the vertical and horizontal displacement of the vertices were calculated. This method offered the possibility of separating the deflections due to shear (calculating the vertical displacement from the deformations in diagonals) from those due to bending (the vertical displacements were calculated from the longitudinal deformations at the top and bottom edges). The reliability and checking of test results were improved because of the possibility to compare the deflections computed from the elongations measured on truss with those measured directly by the dial gauges (1 - 5 in Fig. 1).

In addition, after cracking, the development of cracks was observed. The widths of cracks were registered at the two (or four) loading steps before unloading and after unloading to the zero level at each loading cycle. The widths of the vertical cracks were measured at the position of tensile reinforcement and the widths of diagonal cracks were measured in the middle of the web.

The following characteristics of cracking were evaluated, namely, the crack density, the crack opening density, and the crack widths for both, vertical as well as diagonal cracks. In this paper, the results of measuring only the vertical cracks are presented.

**Results and Discussion**

As it is obvious, the deformations and cracking characteristics are dependent on the bending moment, and therefore there is a mutual dependence between them. The measured strains on the bottom edge of the beam contained also the widths of the cracks. Therefore, the ratio of crack opening density $\alpha_{t,v}$ to

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Designation} & \text{Parameters of polynomial regression function} & \text{Correlation} \\
& y = ax^2 + bx + c & \text{coefficent} \\
\hline
\text{I-RCB} & 9.00E-05 & 0.6640 & -42.583 & 0.9695 \\
\text{I-PCB} & 2.00E-04 & 0.3641 & -85.030 & 0.9822 \\
\text{T-RCB} & 1.00E-05 & 0.9580 & -20.450 & 0.9970 \\
\hline
\end{array}
\]

**Fig. 3**—Crack opening density versus strains at the bottom edge

**Fig. 4**—Ratio of the crack opening density and strain versus the bending moment in the middle of base lines
the strain $\varepsilon_{eb}$ at the given bending moment $M$ was investigated.

To find out the relationship between the width of vertical cracks and the strains of the bottom edge of the beam, the crack opening density $\alpha_{r,v}$ was defined as the sum of crack widths found on a baseline in a unit length as given below.

$$\alpha_{r,v} = \left(\sum w_{r, i} \right) / l_{v,i} \quad i = 1, \ldots, n_{r,v} \quad ... \ (1)$$

Where $n_{r,v}$ is the number of cracks found in the baseline, $l_{v,i}$ is the length of the base-lines on the bottom edge of the beam, and $w_{r, i}$ is the width of the $i$-th crack. To mitigate the effect of the choice of the base lines, the moving average method of second degree was applied by the evaluation. If $k$ is the number of a base-line ($k = 1, \ldots, 11$), the moving average of the crack opening density $\bar{\alpha}_{r,v,k}$ in this base-line will be:

$$\bar{\alpha}_{r,v,k} = (\alpha_{r,v,k-1} + 2\alpha_{r,v,k} + \alpha_{r,v,k+1}) / 4 \quad ... \ (2)$$

The crack opening densities of reinforced concrete I-beam obtained by the above method are plotted in Fig. 3 with respect to the strains of the bottom edge. Similar relationships were obtained for the reinforced concrete and prestressed concrete T- and I-beams. (in the case of reinforced concrete T-beam only values up to the yield stress were taken into account). The parameters of the regression polynomial are given in Table 2 together with the correlation coefficients for the three beams.

The relationship between the ratio of the

<table>
<thead>
<tr>
<th>Designation of beams</th>
<th>Parameter of regression straight lines</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>$a$</td>
<td>$r$</td>
</tr>
<tr>
<td>I-PCB</td>
<td>0.1002</td>
<td>0.9144</td>
</tr>
<tr>
<td>T-RCB</td>
<td>0.1602</td>
<td>0.9792</td>
</tr>
</tbody>
</table>

Fig. 5—Maximum crack widths versus crack opening density

Fig. 6—Loading force versus sum of crack widths relationship of the beam I-RCB

Fig. 7—Loading force versus sum of crack widths relationship of the beam I-PCB

Fig. 8—Loading force versus sum of crack widths relationship of the beam T-RCB
approximated crack opening densities $\alpha_{cr,v}$ to strains $\varepsilon_{sb}$ on the bottom edge $\alpha_{cr,v}/\varepsilon_{sb}$ and the bending moment $M$ in the middle of the base lines is illustrated in Fig. 4 for one of the beams. It can be seen in Fig. 4 that the cracking moment is approximately $M_{cr} = 25$ kNm, which is in good agreement with the observed cracking moment $M_{cr,exp} = 36$ kNm. The ratio $\alpha_{cr,v}/\varepsilon_{sb}$ grows at faster rate until approximately $1.7 M_{cr}$ that corresponds to the increase of crack density. Then, after the stabilisation of crack density, the increase of the ratio $\alpha_{cr,v}/\varepsilon_{sb}$ is milder up to the end of measuring, which in this case was $M = 0.64 M_{cr}$. It follows from Fig. 4 that the relation between the ratio $\alpha_{cr,v}/\varepsilon_{sb}$ and the bending moment, in the region of service loads, can be replaced by say, a bilinear relationship such as,

$$\alpha_{cr,v}/\varepsilon_{sb} = 0.9 (M/M_{cr} - 1); M/M_{cr} \in < 1; 1.7> \ldots (3)$$

$$\alpha_{cr,v}/\varepsilon_{sb} = 0.63 + 0.02 (M/M_{cr} - 1.7); M/M_{cr} > 1.7 \ldots (4)$$

Where the upper limit of the Eq. (4) is at $M = 0.64 M_{cr}$. The Eqs. (3) and (4) enable to determine the crack opening density $\alpha_{cr,v}$ when moment $M$ and corresponding strain is known.

It should be mentioned that the crack opening density corresponds with the average crack width. However, for the crack width limitation the maximum width of cracks should be known. Therefore, the dependence of the maximum crack width $w_{cr,v,max}$ on the crack opening density $\alpha_{cr,v}$ is important (Fig. 5). It can be seen from Fig. 5 that the regression straight line approximates the experimental values quite satisfactorily. It follows from the values in Fig. 5 that

$$w_{cr,v,max} = 0.1 \alpha_{cr,v} \ldots (5)$$

The parameters of regression straight lines and correlation coefficients for the three beams are given in Table 3.

As mentioned in the above, the beams were also tested for unloading. During the tests the widths of cracks were measured also after unloading to the zero level. In Figs. 6 ,7 and 8 relationship between the loading force and the sum of crack width is plotted for I-RCB, I-PCB, and T-RCB, respectively. The above figures, also show the closeness of cracks after unloading and the favourable effect of prestressing (Fig. 7). It is interesting to note the relationship between the reversible (closing) part of the crack width and the crack width before unloading. As the crack opening density was known for each base line, it provides sufficient amount of data to determine such a relationship. In Fig. 9, the reversible part of crack opening density is plotted in dependence on the crack opening density immediately before unloading for I-RCB. This relationship shows relatively small scatter and can be approximated by a straight line with sufficient accuracy. Similar results were obtained for the other two beams (Figs. 10 and 11). The parameters of regression straight lines are given in Table 4. The linear relationship between the reversible crack opening density and the crack opening density before unloading confirms the results being published in another paper about the relationship between short-term, total and elastic deflections.

<table>
<thead>
<tr>
<th>Designation of beams</th>
<th>Parameter of regression straight lines</th>
<th>Correlation coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-RCB</td>
<td>0.8419</td>
<td>0.9880</td>
</tr>
<tr>
<td>I-PCB</td>
<td>0.9877</td>
<td>0.9996</td>
</tr>
<tr>
<td>T-RCB</td>
<td>0.8229</td>
<td>0.9903</td>
</tr>
</tbody>
</table>
Fig. 11—Reversible part of the crack opening density versus crack opening density before unloading of the beam T-RCB

**Conclusions**

On the basis of experimental results, it may be concluded that Eqs. (3)-(5) enable to establish relationship between the bending moments, global strains, and the maximum width of cracks. However, it would be desirable to conduct more experiments to generalise the parameters of relationships between the above-mentioned quantities.

**Acknowledgement**

Authors are grateful to the Slovak Grant Agency VEGA (Grant No. 2/7034/01) for partial support of this work.

**Nomenclature**

- $a_{il}$ = parameter of regression functions
- $a_{0i}$ = control displacement of the hydraulic jack
- $a_{ail}$ = total deflection in the middle of the span
- $k$ = parameter of regression functions
- $E_c$ = modulus of elasticity of concrete
- $E_{p}$ = modulus of elasticity of prestressing reinforcement
- $E_n$ = modulus of elasticity of non-prestressing reinforcement
- $F$ = loading force
- $F_{\text{max}}$ = capacity of loading equipment
- $f$ = tensile strength of concrete
- $f_p$ = tensile strength of prestressing reinforcement
- $f_y$ = yield stress of non-prestressing reinforcement
- $f_n$ = tensile strength of non-prestressing reinforcement
- $h$ = depth of beams
- $l$ = number of a base line
- $l_i$ = span of beams
- $l_{\text{bl}}$ = length of the $i$-th base line
- $M$ = bending moment
- $M_c$ = cracking moment
- $M_{cr,exp}$ = observed cracking moment
- $n_{cr,i}$ = number of cracks in the base line
- $r$ = correlation coefficient
- $w_{x,ci}$ = width of the $i$-th vertical crack
- $w_{x,ci,\text{max}}$ = maximum crack width in a base line
- $a_{x,i}$ = vertical crack opening density
- $\bar{w}_{x,ci}$ = moving average of the crack opening density
- $e_{0,i}$ = strain at the bottom edge of the beam
- $\lambda$ = loading level

**References**