Ferrofluid lubrication equation for porous bearings considering anisotropic permeability and slip velocity

Rajesh C Shah & M V Bhat

Department of Mathematics, Nirma Institute of Technology, Sarkhej-Gandhinagar Highway, Ahmedabad 382 481, India

E-202, Riddhi Complex, Near Jodhpur Village, Ahmedabad 380 015, India

Received 12 September 2002; accepted 10 April 2003

A Reynolds type equation was derived for the lubrication of porous bearings considering slip velocity at the porous-film interface and anisotropic permeability in the porous matrix. A ferrofluid lubricant flowing as per Jenkins model was used with an oblique magnetic field. Various special cases were deduced from the lubrication equation.

Reynolds equation for porous bearings was derived on the following assumptions: (i) there is no variation of pressure across the film, (ii) velocity gradients across the film predominate, (iii) the porous region is isotropic, (iv) the flow in the porous region is governed by Darcy's law, (v) pressures and normal velocity components are continuous at the interfaces and (vi) there is no slip at the porous interface.

It was used by many investigators in their analyses of porous bearings with conventional lubricants. Some simplified their analysis by using Morgan-Cameron approximation to avoid series solutions of the bearing characteristics.

A ferrofluid is a suspension of solid magnetic particles of subdomain size in a liquid carrier. For example, ferric oxide particles coated with a surfactant, antimony, suspended in a base fluid, diester. Such a fluid can be made to move with the help of field gradient. This property makes a ferrofluid useful in space ships, which often go in zero gravity regions. The side not facing the sun can be made warmer and the side facing the sun cooler by using a ferrofluid.

Studies were made with ferrofluid-based porous bearings using Neuringer-Rosensweig model for the lubricant flow. Ram and Verma used Jenkins model for the lubricant flow in their study of an inclined slider bearing. Jenkins model could be considered as an extension of Neuringer-Rosensweig model.

Beavers and Joseph Beavers et al. and Sparrow et al. proved analytically and experimentally that the assumption of no slip at the porous interface could not hold when the porous matrix was formed with naturally permeable material like foam. Investigations were made using the boundary conditions proposed by Beavers and Joseph and Sparrow et al.

Kulkarni and Kumar derived a lubrication equation for porous bearings considering anisotropic permeability in the porous region and slip velocity at the interface of film and porous matrix. However, they did not derive the Reynolds-type equation for bearing study.

In this paper, we derive the Reynolds-type equation for studying porous bearings considering anisotropic permeability, slip velocity at the film-porous interface and a ferrofluid lubricant flowing as per Jenkins model simplified by Mougin and used by Ram and Verma.

Analysis

We consider a ferrofluid film of thickness $h(x,y,t)$ between two surfaces as in Fig.1. The upper surface has a porous matrix of thickness $H_0$ made-up of a naturally permeable material backed by a solid wall. It

![Fig. 1 — A ferrofluid film between two surfaces](image-url)
moves normally with a velocity $W_t$ towards the impermeable lower surface lying along the $xy$-plane and moves with a uniform velocity $U$ in the $x$-direction.

The basic equations governing the ferrofluid flow in the two regions are\textsuperscript{11}

**Film region:**

$$\frac{\partial \overline{u}}{\partial z} = \frac{1}{\eta} \left( \frac{1}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right) \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \quad \ldots \quad (1)$$

$$\frac{\partial \overline{v}}{\partial z} = \frac{1}{\eta} \left( \frac{1}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right) \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \quad \ldots \quad (2)$$

$$\frac{\partial}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) = 0 \quad \ldots \quad (3)$$

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \quad \ldots \quad (4)$$

**Porous region:**

$$\overline{u} = \frac{\phi_x}{\eta} \left[ \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) + \frac{\rho \alpha^2 \mu}{2} \frac{\partial}{\partial z} \left( H \frac{\partial \overline{u}}{\partial z} \right) \right] \quad \ldots \quad (5)$$

$$\overline{v} = \frac{\phi_y}{\eta} \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \quad \ldots \quad (6)$$

$$\overline{w} = \frac{\phi_z}{\eta} \frac{\partial}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \quad \ldots \quad (7)$$

$$\frac{\partial \overline{u}}{\partial x} + \frac{\partial \overline{v}}{\partial y} + \frac{\partial \overline{w}}{\partial z} = 0 \quad \ldots \quad (8)$$

where $(\overline{u}, \overline{v}, \overline{w})$ and $(\overline{u}, \overline{v}, \overline{w})$ are velocities and $p, P$ are the pressures of the fluid in the film and porous matrix respectively; $\phi_x, \phi_y, \phi_z$ are the permeabilities in the directions of the axes, $\eta$ is the fluid viscosity, $\rho$ is the fluid density, $\alpha^2$ is the porous material constant due to Jenkins. $\mu$ is the magnetic susceptibility, $\mu_0$ is the permeability of the free space and $H$ is the strength of the magnetic field with its curl and divergence vanishing.

**Solutions**

The no slip condition usually used at the porous-film interface made the solutions of the momentum equations easy. However, authors\textsuperscript{13-14} proved analytically and experimentally that the no slip condition failed when the porous region was made of naturally permeable material like foam. So, solving Eqs (1) and (2) under the boundary conditions\textsuperscript{14}

$$u = 0 \quad \text{when} \quad z = 0, \quad u = \frac{1}{s_1} \frac{\partial u}{\partial z} + U \quad \text{when} \quad z = h \quad \ldots \quad (9)$$

$$v = 0 \quad \text{when} \quad z = 0, \quad v = \frac{1}{s_2} \frac{\partial v}{\partial z} + U \quad \text{when} \quad z = h \quad \ldots \quad (10)$$

where $\frac{1}{s_1} = \frac{\sqrt{\eta_x \phi_x}}{5}$ and $\frac{1}{s_2} = \frac{\sqrt{\eta_y \phi_y}}{5}$

are the slip parameters, $\eta_x$ and $\eta_y$ being the porosities in the $x$ and $y$ directions; we obtain as in Appendix.

$$u = \frac{U_s h^2}{s_1 h + 1} + \frac{z^2 (s_1 h + 1) - (2zh + s_1 h^2 z)}{2s_1 (s_1 h + 1) \left( 1 - \frac{\rho \alpha^2 \mu H}{2 \eta} \right)} \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \quad \ldots \quad (11)$$

$$v = \frac{z^2 (s_1 h + 1) - (2zh + s_1 h^2 z) \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right)}{2s_1 (s_1 h + 1) \left( 1 - \frac{\rho \alpha^2 \mu H}{2 \eta} \right)} \quad \ldots \quad (12)$$

Substituting Eqs (11) and (12) into the continuity Eq. (4) and integrating it across the film thickness, we obtain

$$\frac{\partial}{\partial x} \left[ \frac{U_s h^2}{2(s_1 h - 1)} - \frac{h^3 (4 + s_1 h)}{12 \eta (1 + s_1 h) \left( 1 - \frac{\rho \alpha^2 \mu H}{2 \eta} \right)} \right] + \frac{\partial}{\partial y}$$

$$\left[ - \frac{h^3 (4 + s_1 h)}{12 \eta (1 + s_1 h) \left( 1 - \frac{\rho \alpha^2 \mu H}{2 \eta} \right)} \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]$$
since the lower surface is impermeable.

Substituting Eqs (5)-(7) into Eq. (8) and integrating it across the porous matrix \((h, h+H_0)\), we obtain

\[
\phi_z \frac{\partial}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \bigg|_{h} = \phi_z H_0 \frac{\partial^2}{\partial x^2} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) + \phi_z H_0 \frac{\partial^2}{\partial y^2} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) + \left( \phi_z - \phi_z \right) \frac{\rho \alpha^2 \mu}{2 \eta} \frac{\partial H}{\partial x}
\]

\[
\frac{\partial}{\partial x} \left[ \frac{H}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right] = \frac{\partial}{\partial y} \left[ \frac{H}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right] \tag{14}
\]

using Eq. (11) and Morgan-Cameron approximation.

Owing to the continuity of velocity components across the film-porous interface

\[
w_h = \bar{w}_h = W_\eta - \frac{\phi_z}{\eta} \left[ \frac{\partial}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]_h
\]

\[
= W_\eta - \frac{\phi_z}{\eta} \left[ \frac{\partial}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]_h + \frac{\rho \alpha^2 \mu}{2 \eta} \frac{\partial}{\partial x} \left[ H \frac{\partial \mu}{\partial z} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]_h + \frac{s_1 \phi_z}{2 \eta} \frac{\rho \alpha^2 \mu}{2 \eta} \frac{\partial}{\partial x} \left[ H \frac{\partial \mu}{\partial z} \right]_h + \frac{s_2 \phi_z}{2 \eta} \frac{\rho \alpha^2 \mu}{2 \eta} \frac{\partial}{\partial x} \left[ H \frac{\partial \mu}{\partial z} \right]_h
\]

\[
\frac{\partial}{\partial x} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]_h = \frac{\partial}{\partial y} \left[ \frac{H}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \frac{\partial}{\partial y} \left( p - \frac{1}{2} \mu_0 \mu H^2 \right) \right]_h \tag{15}
\]

using Eq. (11).

From Eqs (13)-(15), we obtain the Reynolds-type equation in this case as

\[
\frac{\partial}{\partial x} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right) + \frac{\partial}{\partial y} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right)
\]

\[
= \frac{\partial}{\partial y} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right)
\]

\[
\frac{\partial}{\partial y} \left[ \frac{p - \frac{1}{2} \mu_0 \mu H^2}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right) = 12 \eta W_h + 6 U \eta \tag{16}
\]

Discussion

Eq. (16) is the Reynolds-type equation in the present case. We can obtain various special forms from it.

(i) Setting \(s_1, s_2 \to \infty\) in Eq. (16) we obtain the no-slip version corresponding to the present case as

\[
\frac{\partial}{\partial x} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right) + \frac{\partial}{\partial y} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right)
\]

\[
= \frac{\partial}{\partial y} \left[ \frac{p - \frac{1}{2} \mu_0 \mu H^2}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right) + \frac{\partial}{\partial y} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right)
\]

\[
= 12 \eta W_h + 6 U \eta \tag{16}
\]

(ii) Setting \(\alpha^2 = 0\) in Eq. (16) we obtain the Reynolds equation using Neuringer-Rosensweig model for the ferrofluid flow considering slip velocity and anisotropic permeability as

\[
\frac{\partial}{\partial x} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right) + \frac{\partial}{\partial y} \left[ \frac{12 \phi_z H_0 + h^3 (4 + s_2 h)}{1 - \frac{\rho \alpha^2 \mu H}{2 \eta}} \right] \left( s_1 h + 1 \right)
\]

\[
= 12 \eta W_h + 6 U \eta \tag{16}
\]

(iii) Setting \(s_1, s_2 \to \infty\) in (ii) above we obtain the Reynolds equation using Neuringer-Rosensweig
model for the ferrofluid flow with no-slip condition and anisotropic permeability as

\[
\frac{\partial}{\partial x}\left(12\phi_yH_0 + h^3\right)\frac{\partial}{\partial x}\left(p - \frac{1}{2} \mu_0 \bar{H}^2\right) + \frac{\partial}{\partial y}\left(12\phi_yH_0 + h^3\right)\frac{\partial}{\partial y}\left(p - \frac{1}{2} \mu_0 \bar{H}^2\right) = 12\eta W + 6U\eta \frac{\partial h}{\partial x}
\]

(iv) Setting \(H=0\) in Eq. (16) we obtain the Reynolds equation for the conventional lubricant considering slip velocity and anisotropic permeability as

\[
\frac{\partial}{\partial x}\left[12\phi_yH_0 + \frac{h^3}{(s_1h + 1)}\right] + \frac{\partial}{\partial y}\left[12\phi_yH_0 + \frac{h^3}{(s_1h + 1)}\right] \frac{\partial p}{\partial y} = 12\eta W + 6U\eta \frac{\partial h}{\partial x}
\]

Conclusions

Eq. (16) is the most general form using Jenkins model for the ferrofluid flow. It can be used to study the effects of anisotropic permeability and Jenkins material constant on the performance of slider bearings and journal bearings.

Nomenclature

- \(h\) = film thickness
- \(H\) = strength of external magnetic field
- \(H_0\) = thickness of porous matrix
- \(p, P\) = pressures in film and porous regions
- \(t\) = time
- \((u,v,w)\) = fluid velocity in film region
- \((\bar{u}, \bar{v}, \bar{w})\) = fluid velocity in porous matrix
- \(U\) = sliding velocity of the upper surface
- \(W_0\) = squeeze velocity of the upper surface
- \(w, \bar{w}\) = values of \(w, \bar{w}\) at \(z=0\)
- \(x, y, z\) = coordinates
- \(\alpha^2\) = material constant of Jenkins model
- \(\eta\) = fluid viscosity
- \(\eta_0, \eta_1\) = porosities along \(x, y\) directions
- \(\mu_0\) = permeability of free space
- \(\mu_\tau\) = susceptibility of magnetic particles
- \(\rho\) = fluid density

References


Appendix

\[
\frac{\partial u}{\partial z} = A + \frac{z}{\eta_1} \frac{1}{1 - \rho \alpha^2 H^2} \frac{\partial}{\partial z} \left(p - \frac{1}{2} \mu_0 \bar{H}^2\right)
\]

and

\[
\frac{\partial u}{\partial z} = B + \frac{z^2}{\eta_1} \frac{1}{1 - \rho \alpha^2 H^2} \frac{\partial}{\partial z} \left(p - \frac{1}{2} \mu_0 \bar{H}^2\right)
\]

by integrating Eq. (1) once and twice respectively, \(A\) and \(B\) being constants of integration.

Using the first condition of Eq. (9), Eq. (A.2) yields

\[
B = 0
\]

Using the second condition of Eq. (9) one obtains from Eqs (A.1)-(A.3)
which yields

\[
A = \frac{1}{s_1 h + 1} \left[ s_1 U - \frac{h(2 + s_1 h)}{2\eta} \frac{\partial}{\partial x} \left( \frac{p - \frac{1}{2} \mu_0 \bar{W}^2}{1 - \frac{\rho \bar{c}^2 \bar{W} H}{2\eta}} \right) \right]
\]

Using Eqs (A.3)-(A.4), Eq. (A.2) yields Eq. (11) of the text. Similarly, one has Eq. (12).