Er-concentration/minimum pump power optimization technique for design of broadband single link using Er-doped dispersion compensating optical amplifiers

Pramod R.Watekar*a, M.L.N.Goswami*a, J.C.Biswas*b & H.N.Acharya*a

*aOptical Fiber Unit, Central Research Facility, Indian Institute of Technology, Kharagpur 721302, India
*bDepartment of Electronics and Communication Engineering, Indian Institute of Technology, Kharagpur 721302, India

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A design of broadband (C-band and L-band) link for dense wavelength division multiplexing (DWDM) system employing Er-doped dispersion compensating optical fiber amplifier is presented using a procedure based on the optimization of Er-concentration with minimum pump power requirement in the optical fiber. The design has been done using simple step index fiber at a pump wavelength of 980 nm. For 45 km of link-length only one pumping scheme has been used. With the channel separation of 50 GHz, a wide band has been obtained for bit rate above 10 Gb/sec. The procedure and results are useful for designing dispersion-less communication fiber link along with amplification.

It is a well-known that an 1.3 μm optimized optical fiber link can be upgraded to an 1.55 μm optimized one using dispersion compensating fibers (DCF). The core-clad refractive index difference is raised for obtaining high negative dispersion that contributes to increased losses in the fiber. One then needs Er-doped amplifiers to compensate these losses. The Er-doped dispersion compensating amplifier (DCFA) is a DCF with Er-ions embedded in the core. Thus, we get simultaneous dispersion compensation as well as amplification. Designs of DCF with high negative dispersion as well as that of Er-doped amplifier have been carried out in1-4. The purpose of this paper is to design and present a procedure for DCFA-based DWDM C-band and L-band links.

Design Optimization
The amplifier is designed considering the optimization of gain, pump power and Er-concentration including the effects of the spot size. A single link DCFA needs a single pumping scheme to achieve required gain at an optimum length. The designed link is shown in Fig. 1.

Dispersion compensation in optical fibers

At a wavelength of 1550 nm, the dispersion coefficient of conventional fiber ($D_c$) is nearly 16 ps/km-nm. Thus, to compensate this dispersion over the length of $L_c$, we need DCF of length $L_c$, so that total pulse spreading is zero:

$$L_c = D_c L_c/D_c$$

where $L_c$ is the DCF length, $L_c$ is the conventional fiber length and $D_c$ is dispersion coefficient of DCF in ps/km-nm. An important parameter, the modified figure of merit (MFOM) in ps/km-dB is given as:

$$MFOM = \frac{|D_c| L_c}{\alpha_s L_c + \alpha_d}$$

where $\alpha_s$=bending loss + Rayleigh scattering loss and $\alpha_d$ is the splice loss.

Optical fiber amplifier modeling
Considering a 3-level system and uniform doping of Er-ions in core, the optical fiber amplifier is modeled using rate equations of pump and signal powers6,7 and radial electrical field envelopes8. The simplified equations are given as:

![Fig. 1 — Optical fiber link using DCFA](image-url)
\[
\frac{dP_p}{dz} = -\frac{\sigma_{pa} N_0 P_p(z)}{w} \left[ \left( 1 - \frac{u}{w} \right) \ln \left( \frac{1 + w}{1 + w \exp(-a^2/\Omega_p^2)} \right) \right] - \alpha_p P_p(z) \\
+ \frac{u}{w} \left[ \ln \left( \frac{1 + w}{1 + w \exp(-a^2/\Omega_p^2)} \right) \right] - \alpha_p P_p(z)
\]

\[
\frac{dP_s}{dz} = \frac{\sigma_{sa} N_0 P_s(z)}{w} \left[ \left( 1 + \frac{v}{w} \right) \ln \left( \frac{1 + w}{1 + w \exp(-a^2/\Omega_s^2)} \right) \right] - \alpha_s P_s(z) \\
+ \frac{v}{w} \left[ \ln \left( \frac{1 + w}{1 + w \exp(-a^2/\Omega_s^2)} \right) \right] - \alpha_s P_s(z)
\]

where \( z \) is distance and normalized Gaussian envelope approximation has been used for the pump and signal, i.e.

\[
f_p(r) = \frac{\exp(-r^2/\Omega_p^2)}{\pi \Omega_p^2} \quad \text{for the pump input and}
\]

\[
f_s(r) = \frac{\exp(-r^2/\Omega_s^2)}{\pi \Omega_s^2} \quad \text{for the signal input}, \ \Omega \ \text{stands}
\]

for the Gaussian spot size. Cross-section parameters \( \sigma_{sa}, \sigma_{sa}, \sigma_{pa} \) and \( \sigma_{pc} \) are the signal absorption, signal emission, pump absorption and pump emission cross sections respectively and \( a \) is core radius. Others parameters are defined as follows:

\[
\eta = \sigma_{sa}/\sigma_{sa}
\]

\[
v = \eta P_s(z)/P_{s0}
\]

\[
u = \frac{\eta}{1 + \eta} P_s(z)
\]

\[
w = \frac{P_p(z)}{P_{p0}} + \frac{P_s(z)}{P_{s0}}
\]

\[
P_{s0} = \pi \Omega_s^2 \frac{hv_s}{\sigma_{sa} + \sigma_{sa} \nu_p} \quad \text{and} \quad P_{s0} = \pi \Omega_s^2 \frac{hv_p}{\sigma_{pa} \nu_p}
\]

\( t_\nu \) is spontaneous lifetime of Er-ions, \( v_s \) and \( v_p \) is signal and pump frequencies respectively, \( h \) is Planck’s constant, \( \alpha_p \) and \( \alpha_s \) are the attenuation coefficient at the pump and signal wavelength respectively.

For simplicity, we have considered 980 nm pumping scheme only. The amplifier can be modeled as a three level system. As the relaxation rate at 3\(^{rd}\) level is very rapid, we can generalize the total Er-ions density \( N_0 \) as sum of steady state atomic population densities of the ground state \( N_g \) and metastable state \( N_2 \). These parameters are given as:

\[
N_1 = N_0 \frac{1 + \eta P_p f_s}{1 + \eta l_{s0} f_s} \quad \text{and}
\]

\[
N_2 = N_0 \frac{1 + \frac{P_p}{l_{p0}} f_p + \frac{P_s}{l_{s0}} f_s}{1 + \frac{P_p}{l_{p0}} f_p + \frac{P_s}{l_{s0}} f_s}
\]

where, \( l_{p0} = \frac{P_{p0}}{\pi \Omega_p^2} \) and \( l_{s0} = \frac{P_{s0}}{\pi \Omega_s^2} \).

The rate equation for noise power is given as:

\[
\frac{dP_n}{dz} = 2 h \nu \Delta \nu (a^2 f_s N_0 \pi \sigma_s (f_p l_{p0} P_p(z) \sigma_{sa} + f_p l_{s0} P_s(z) \sigma_{sa} + \sigma_{pc}))
\]

\[
\quad \times (f_p l_{p0} P_p(z) + l_{s0} (f_s P_s(z) \sigma_{sa} + \sigma_{pc}))(f_p l_{s0} P_s(z) + l_{p0} (f_s P_p(z) \sigma_{sa} + \sigma_{pc}))
\]

\[
\quad + \frac{a^2 f_s l_{s0} N_0 \pi (f_p P_p(z) \sigma_{sa} - l_{p0} \sigma_{pc})}{f_p l_{s0} P_p(z) + l_{p0} (f_s P_p(z) \sigma_{sa} + \sigma_{pc})} P_p(z)
\]

where \( h \) is Planck’s constant, \( \nu \) is the signal frequency and \( \Delta \nu \) is the bandwidth over which spontaneous emission measurement is taken. In all the above equations, \( P \) stands for power and suffix \( s, p \) and \( n \) stands for signal, pump and noise respectively. The noise figure (NF) is calculated as the ratio of two ratios respectively i.e., input signal to noise ratio and output signal to noise ratio.

**WDM in the DCFA Link**

The Er-doped optical fiber amplifier gives the amplification spectrum in C-band and L-band range. DCFA with conventional fiber allows use of WDM transmission on all long haul submarine and terrestrial networks. The bit rate-length product for the optical fiber is given as\(^9\):
\[ B \times L = \frac{250}{D \Delta \lambda} \text{ Gb/s-km} \] ... (7)

where \( B \) is bit rate in Gb/sec, \( L \) is optical fiber link length in km, \( D \) is total dispersion in the fiber link and \( \Delta \lambda \) is spectral width of source in nm. We have considered minimum signal power of 1 \( \mu \)W at EDFA, therefore to have bit error rate less than \( 10^{-9} \), it is necessary to have input power at conventional fiber end (see Fig. 1) more than value given by\(^{10}\):

\[ P = \frac{1 \mu W}{10^{-\alpha d/10}} \] ... (8)

where \( \alpha \) is attenuation in dB/km that is considered to be 0.2 dB/km. It is 7.9 \( \mu \)W for 45 km of commercial single mode fiber.

**Petermann spot sizes**

Here, we consider definitions of Petermann II and III spot sizes\(^{10}\) and their relation to bending and splice loss.

(a) Petermann II spot size is defined using a near field and is expressed as:

\[ W_{d}^2 = \frac{2}{\pi} \int_{0}^{\infty} E^2(r) r dr \] ... (9)

where \( E(r) \) is a radial electrical field in the core of optical fiber and \( r \) is the radial parameter. The splice loss between two optical fibers is related to this spot size. The splice loss \( \alpha_d \) (dB) due to transverse offset \( d \) is expressed as\(^{11}\):

\[ \alpha_d = 4.34 \left( \frac{d}{W_d} \right)^2 \] ... (10)

(b) Petermann III spot size is given as:

\[ W_{\infty}^2 = \frac{\lambda}{\pi n_{oc}(\beta - \frac{2\pi}{\lambda} n_{cl})} \] ... (11)

where \( n_{oc} \) and \( n_{cl} \) are the core and clad refractive indices respectively, \( \beta \) is the propagation constant of the fundamental mode and \( \lambda \) is the signal wavelength. This spot size is related to the pure bend loss of the fiber. The pure bend loss (dB/m), in a fiber with radius of curvature \( R \), is given by\(^{6,12}\):

\[ \alpha_b = A \sqrt{\frac{\pi V^8}{16aR_s W_3}} \exp \left( -\frac{4R_s W_3 \Delta}{3aV^2} \right) \] ... (12)

where,

\[ A = \frac{\left( \int_{0}^{\infty} (1 - g(r)) r E(r) dr \right)^2}{\int_{0}^{\infty} r E^2(r) dr} \]

and \( a \) is the core radius, \( V \) and \( W \) are normalized parameters, \( \Delta \) is the refractive index difference \( (= n_{core}^2 - n_{clad}^2) \) and \( g(r) \) is the profile function \( (= n_{core}^2 - n(r)^2) \) with \( n(r) \) being refractive index profile of the optical fiber. The value of \( \frac{W_{\infty}}{W_d} \) is the figure to determine the optimum losses, preferred value being as nearer to unity.

**Design Criteria and Design Procedure for DCFA and Link**

The design criteria for DCFA is based on two points: (i) Selection of fiber parameters such that negative dispersion is higher at 1.55 \( \mu \)m and, (ii) Er-ion concentration in the DCFA is optimized so that minimum pump power is required to attain required gain. Then one can find bit rate at different wavelengths and number of channels supported.

The use of 1.55 \( \mu \)m signal wavelength on 1.3 \( \mu \)m optimized optical fiber gives positive dispersion that can be compensated by a dispersion compensating fiber with length \( L_c \) as calculated by Eq.(1). The optical amplifier rate equations can be solved to get evolution of signal power, pump power and noise power\(^{8}\) with respect to fiber length. The required gain at \( L_c \) is fixed by choosing suitable Er-ion concentration in the core and the pump power. The Er-ion concentration has to be optimized for
minimum pump power to achieve the gain that is fixed earlier.

For broadband system, the dispersion at 1.53 μm to 1.6 μm in C-band and L-band is calculated for the conventional fiber as well as dispersion compensating fiber. The required length of DCFA is found so that total dispersion in the link is nearly zero. The maximum possible bit rates for these parameters are found from Eq.(7). Then signal power to be applied at the input of conventional fiber to minimize bit error rate is calculated using Eq.(8).

**Results and Discussion**

We have considered a step index fiber for designing DCFA, as it is most widely used for dispersion compensation schemes. The other constraints used are:

(a) Total dispersion \( D_T \leq -16 \) ps/km-nm
(b) Petermann II spot size \( W_d=4 \) μm
(c) Gain at \( L_g=1 \) dB so that at least original signal strength is recovered after suffering losses in the link
(d) Pump power \( P_p \leq 100 \) mW to avoid nonlinear effects
(e) Conventional fiber length=45 km

Thus the link is considered to be containing 45 km of conventional fiber followed by DCFA and the target gain at the end of DCFA is 1 dB. It is obvious that to solve the rate equations of optical amplifier, we require normalized mode intensity profiles \( f_o \) and \( f_i \). These are obtained numerically by using a method described already\(^9\). We have performed calculations for pump power requirements as well as for noise figure corresponding to 980 nm wavelength only because the noise figure is better at 980 nm wavelength pump. The values of the emission and absorption cross-sections and fluorescence lifetime have been taken from the earlier experimental measurements of Er-doped fibers fabricated at our laboratory. These values are \( 6.2 \times 10^{-25} \) m\(^3\) for \( \sigma_{\text{em}} \) and \( 6.8 \times 10^{-25} \) m\(^3\) for \( \sigma_{\text{a}} \) at the signal wavelength. The fluorescence lifetime is 12 msec. The signal input power (at DCFA) is considered to be 1 μW at 1.55 μm wavelength. The length of the conventional fiber is assumed to be 45 km that is then compensated by DCFA.

Fig. 2 shows the variation of dispersion, DCF length required to compensate for 45 km of conventional fiber, and spot size ratio \( \frac{W_o}{W_d} \) with core radius and normalized refractive index difference \( \Delta' = \frac{n_{\text{core}}^2-n_{\text{clad}}^2}{2n_{\text{clad}}^2} \) at Petermann II spot size of 4 μm.

It can be observed that the negative dispersion can be increased by a large value of refractive index difference \( \Delta' \) and a smaller value of core radius. However, by increasing negative dispersion, i.e., decreasing the DCF length \( L_c \), the spot size ratio value goes far from unity and modified figure of merit (MFOM) degrades due to increasing bending loss (Fig. 3). The bending and splice loss increase quite sharply as shown in Fig. 3. The effect of bending radius over the MFOM is shown in Fig. 4 for radius

![Fig. 2](image1)

**Fig. 2**—Variation of dispersion, length of DCFA and spot size ratio with respect to fiber core radius: (i) \( \Delta' \), (ii) \( W_o/W_d \), (iii) \( D_c \), (iv) Length of DCFA \( L_c \) required to compensate dispersion in 45 km of commercial fiber

![Fig. 3](image2)

**Fig. 3**—Variation of losses in the DCFA with respect to fiber core radius and normalized refractive index: (i) Bending loss at radius of curvature of bend=10 cm, (ii) Transverse offset loss at transverse offset of 1 μm, (iii) Angular offset loss at angular offset of 0.5 degrees, (iv) \( \Delta' \)
**Fig. 4**—MFOM variation with radius of curvature of bend in DCFA: (i) $R_c=3.75$ cm, (ii) $R_c=15$ cm, (iii) $R_c=15$ cm, (iv) $\Delta'$

**Fig. 5**—Optimized Er-ion concentrations for minimum pump powers: (i) $\Delta'$, (ii) $P_p$, (iii) $N_{io}$, (iv) Length of DCFA, $L_c$

**Fig. 6**—Variation of dispersion and spot size ratio for 1.53 $\mu$m to 1.6 $\mu$m band: (i) Dispersion, (ii) $W_s/W_d$, (iii) $\Delta'$

**Fig. 7**—Range of DCFA length for 1.53 $\mu$m to 1.6 $\mu$m band

**Fig. 8**—Bit rate in Gb/sec for 1.53 $\mu$m to 1.6 $\mu$m band

**Fig. 9**—Bandwidth supporting 10 Gb/sec bit rate (core radius $a=1.03$, $\Delta'=0.014$, $P_p=8$ $\mu$W, channel spacing=50 MHz)
of curvature of bending, $R_c=3.75$ cm, 10 cm and 15 cm. Hence, it is important to avoid curvature with radius of less than 10 cm to avoid bending losses.

The optimum values of pump power and Er-ion concentration to achieve goal of 1dB gain at DCFA length $L_c$, increases with increasing the DCFA length (Fig. 5). The suitable choice will be then to select DCFA parameters such that sufficiently high value of negative dispersion is obtained and which will require minimum pump power ($P_p$) and optimum Er-ion concentration ($N_o$). The noise figure is calculated in 1.00 nm bandwidth ($\Delta v = 128$ GHz) around 1.55 $\mu$m and it is found to be 3.2 to 3.4.

To calculate the bit rate supported by the link as shown in Fig. 1 and designed as above, the band of wavelengths from 1.53 $\mu$m to 1.6 $\mu$m is considered. The negative dispersion as well as spot size ratio variation with respect to this wavelength band is shown in Fig. 6. The DCFA length required compensating 45 km of conventional fiber for different core radius and core-clad index difference is shown in Fig. 7. The bit rate supported by link is calculated for different wavelengths in a band of 1.53 $\mu$m to 1.6 $\mu$m (Fig. 8). As a specific case, for the parameters of $a = 1.03$, $\Delta' = 0.014$, $P_p = 8$ $\mu$W and channel spacing of 50 MHz, the bandwidth supporting bit rate of 10 Gb/sec is found to be 3700.00 GHz (29.8 nm) and maximum number of channels are found to be 74 as shown in Fig. 9.

Conclusions

We have presented a design of optical fiber link for broadband application using Er-doped dispersion compensated optical fiber amplifier. The optimization is based on Er-ion concentration and minimum pump power requirement. The work will be useful for broadband optical communication.

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References