Theoretical studies on shuttle checking: Part I—Mathematical modelling of checking system

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A generalized mathematical model for shuttle checking is considered from which the change in the velocity of shuttle during impact with swell fixed fulcrum and picking stick is derived. The relative contribution of impact and frictional rubbing by swell to shuttle braking is separated. The dynamic displacement of swell and picking stick is also obtained. This information is helpful in designing the profile, mass, etc. of the swell and the picking stick.

Keywords: Shuttle checking, Coefficient of restitution, Dynamic displacement of swell

1 Introduction

Extensive work has been done on the checking of shuttles, both experimentally and theoretically by considering suitable models for the swell mechanism. Some of the models are not realistic since they consider either frictional braking or impact braking. Shuttle checking by swell involves both impact and frictional rubbing. It has been shown that the dynamic behaviour of swell gives a true picture about the performance of swell. Vincent in a review on ‘Checking Shuttles in High Speed Looms’ states that the study of complex impact phenomenon during shuttle checking has not yet developed to the point where design standards based on analytical considerations can be firmly established.

In the present study, an attempt has been made to predict the reduction in shuttle speed and the number of impacts made by the shuttle with swell and picker/picking stick assembly under dynamic condition by considering a mathematical model of the checking system. The relative contribution of impact and frictional rubbing by swell to shuttle braking is separated. The effect of variables like inertia of swell, picking stick, coefficient of restitution, damping, etc. on the shuttle retardation is studied. This paper deals only with the mathematical modelling of checking system. Application on shuttle deceleration with swell and picker/picking stick would be dealt in subsequent papers.

2 Mathematical Modelling of Checking System

Checking of shuttle, as it enters the box, occurs in two stages. In the first stage, it makes oblique lateral impact(s) with box wall/swell and also rubs against these. In the second stage, there occurs axial impact(s) between shuttle and picker/picking stick, the latter ultimately brings the shuttle to rest. The pivoted swell and picking stick are stationary during the initial shuttle impact but set in dynamic motion for subsequent impacts.

The classical theory of impact is based on the law of conservation of momentum. All impacts are basically a process of energy exchange. During impact, the deformation of colliding bodies takes place and the force between them increases with time, and before separation the deformation is recovered based on the elastic nature of the bodies and the force between the bodies decreases with time. This is shown in Fig.1. The force \( F \) is lower during the rebounding phase by a factor \( e \) (the coefficient of restitution) due to inelastic collision.

3 Impact Model

The impact of shuttle with swell (fixed fulcrum) and picker/picking stick is studied with the help of a generalized model for oblique lateral impact between a ball and a pendulum (Fig. 2), the former representing shuttle and the latter swell or picking stick. The change in the velocity of impacting bodies shuttle and swell, shuttle and picker/picking stick is deduced based on whether the swell or picking stick is...
Fig. 1—Contact force vs time for colliding bodies

Fig. 2—Ball/pendulum impact and vibration model

stationary or in motion.

Assumptions

— Initial velocity of shuttle makes an angle α with the inclined surface AB which is at an angle β with the radius \( \vec{r} \) through point C.
— The angular movement of pendulum is small and does not affect the angle α.
— The pendulum has sufficient rigidity to overcome the buckling.
— Friction during impact is neglected because the duration of contact between the impacting bodies is very small. During the impact, the ball exerts the force \( F \) on the pendulum normal to the line of contact and the force \( F \) is reacted on the ball according to Newton’s third law of motion. Newton’s equations of motion are:

\[-F = m \frac{dv}{dt} \quad \cdots \quad (1)\]

\[F \times \vec{r} = I \frac{d\omega}{dt} \quad \cdots \quad (2)\]

where \( m \) and \( V \) are the mass and velocity of the ball; and \( I \) and \( \omega \) are the moment of inertia and angular velocity of the pendulum.

The initial velocity of the ball \( V_1 \) in Fig. 2. The component \( V_1 \) perpendicular to the radius \( \vec{r} \) is responsible for its movement. It changes to \( V \), pendulum rotates by an angle \( \theta' \) but it is taken as \( V_1 \), sin \( \alpha \) \( \cos \beta \) due to the second assumption. The reduction in the ball velocity is due to its reduction in the \( V \),

Integration of Eqs (1) and (2) at the end of compression and rebounding phase gives:

\[m(V \sin \alpha - V_1) = - F \int_0^t \quad \cdots \quad (3)\]

\[I(\omega - \omega_0) = \frac{1}{2} \int F \cdot dt = -De \quad \cdots \quad (4)\]

\[m(V_1 \sin \alpha - V \sin \alpha) = - \int F \cdot dt = -D \quad \cdots \quad (5)\]

\[I(\omega_1 - \omega) = Be \times r \quad \cdots \quad (6)\]

Also, at the end of compression phase

\[V \sin \alpha \cos \beta = \omega \times r \quad \cdots \quad (7)\]

Suffix 1 and 2 refer to the velocity before and after impact and \( V \) and \( \omega \) are the velocities at \( t = t' \).

Eqs (3) to (7) can be solved for \( V_1 \) and \( \omega_1 \) and give:

\[V_2 \sin \alpha = \frac{V_1 \sin \alpha \cos \beta (\text{mr})}{[I \cos \beta + \text{mr}^2 \sin \alpha \cos \beta]} \cdots \quad (8)\]

\[\omega_2 = \frac{\omega_1 (I - \text{mr}^2) + V \text{mr} \sin \alpha \cos \beta (1 + e)}{[I + \text{mr}^2]} \cdots \quad (9)\]

3.1 Dynamic Response of Pendulum

The pendulum will be at rest for initial impact and so \( \omega_1 = 0 \), but for subsequent impacts, it will have initial angular velocity \( \omega_1 \). This can be determined by considering the vibrational response of pendulum as shown in Fig. 2. The friction or damping in the physical system is represented by an equivalent dashpot in the model.

The equation of motion of the system after being excited by an impulse is given by:

\[\ddot{\theta} + C_\theta \dot{\theta} + K_\theta \theta + P = 0 \quad \cdots \quad (10)\]

where \( P \) is the force due to precompression of spring; \( K_\theta \), the spring constant; and \( C_\theta \), the dashpot constant.

The impulse would cause a change in the velocity without any appreciable change in its displacement.
The general solution of equation (10) is given by:
\[ \theta(t) = \theta_h + \theta_p \]  
where \( \theta_h \) and \( \theta_p \) are the harmonic and particular solutions.

For underdamped case:
\[ \theta_h = e^{-\omega_a t} \left[ C_1 \sin \omega_c t + C_2 \cos \omega_c t \right] \]
where \( \omega_a, \omega_c, (\zeta = C/\omega_c) \) (\( \zeta \) is the critical damping and the natural frequency),
\[ \omega_a = \sqrt{\omega_T^2 - \omega_c^2}, \]
Conditions at \( t = 0, \theta = 0, \) and \( \dot{\theta} = \dot{\theta}(0) \) give
\[ \theta_h = e^{-\omega_a t} \frac{\theta(0)}{\omega_a} \sin \omega_c t \]

For overdamped case:
\[ \theta_h = e^{-\omega_a t} [A e^{\omega_c t} + B e^{-\omega_c t}] \]
where A and B are the constants of differential equation.

Conditions at \( t = 0, \theta = 0, \) and \( \dot{\theta} = \dot{\theta}(0) \) give
\[ \theta_h = e^{-\omega_a t} \frac{\theta(0)}{2\omega_a} \left[ e^{\omega_c t} - e^{-\omega_c t} \right] \]

4 Shuttle and Swell Velocity after Impact

The shuttle nose and swell profile contact is shown in Fig. 3. It may be seen that \( \nu \sin \alpha \) component is involved in impact and based on fourth assumption. The reduction in shuttle speed will be due to the reduction in this component.

The angle \( \beta \) will be almost equal to \( \alpha \). Taking the moment of inertia of swell \( I_1 \) and moment arm \( r_1 \), velocity of shuttle and swell before and after impact \( V_1, V_2 \) and \( \omega_1, \omega_2 \) and the coefficient of restitution between shuttle and swell \( e_1 \), the Eqs (8) and (9) reduce to:

When the swell is in motion:
\[ V_2 \sin \alpha = \frac{(mr_1^2 - I_1 e_1 \cos \alpha) V_1 \sin \alpha + (1 + e_1) I_1 \omega_1 r_1}{(mr_1^2 + I_1 \cos \alpha)} \]
\[ \omega_2 = \frac{(1 + e_1) mr_1 V_1 \sin \alpha \cos \alpha + (I_1 \cos \alpha - mr_1^2 e_1) \omega_1}{mr_1^2 + I_1 \cos \alpha} \]

Eqs (13) and (14) are also applicable for initial impact with stationary swell by substituting \( \omega_1 = 0 \).

The resultant shuttle velocity \( V_2 \) is given by:
\[ V_2 = \sqrt{(V_1 \sin \alpha)^2 + (V_1 \cos \alpha)^2} \]

5 Shuttle and Picker Velocity after Impact

In this case \( \alpha = 90^\circ \) and \( \beta = 0^\circ \). Taking moment of inertia of picking stick as \( I_2 \), the moment arm (the distance between picker center and picking stick fulcrum) as \( r_2 \), velocity of shuttle and picking stick before and after impact \( U_1, U_2 \) and \( \Omega_1, \Omega_2 \) and the coefficient of restitution between shuttle and picker as \( e_2 \), the Eqs (8) and (9) reduce to:

When the picking stick is in motion:
\[ U_2 = \frac{(mr_1^2 - I_1 e_1) U_1 + (1 + e_1) I_1 \Omega_1 r_1}{(mr_1^2 + I_1 \cos \alpha)} \]
\[ \Omega_2 = \frac{(1 + e_1) mr_1 U_1 + (I_2 - mr_1^2 e_2) \Omega_1}{(mr_1^2 + I_2)} \]

Eqs (16) and (17) are also applicable for initial impact with stationary picker/picking stick by substituting \( \Omega_1 = 0 \).

6 Dynamic Response of Swell/Picking Stick System

The velocity of swell and picker for impact with shuttle when the former are moving is determined by the derivative of dynamic displacement at the appropriate time.

6.1 Dynamic Response of Swell

Fig 4 shows the swell system as spring mass and dashpot. The springs will be precompressed before vibration. Taking moment of inertia of swell \( I \), damping ratio \( \zeta_1 \), \( K_2^2 = K_1 I_1 + K_2 I_2 \), and \( P = F_{pe} (I_1 + I_2) / 2 \), Eqs (10)-(12) give the vibration response of swell.
For underdamped case:
\[ \theta(t) = e^{-\zeta \omega_n t} \left( \theta(0) \sin(\omega_d t) \right) \] ...
(20)

For overdamped case:
\[ \theta(t) = e^{-\zeta \omega_n t} \left( \theta(0) \left( e^{\omega_n t} - e^{-\omega_n t} \right) \right) \] ...
(21)

In the above equations, \( \theta(0) \) is the angular velocity before vibration commences and is obtained from Eqs (14) and (17) by using \( \omega_n = \Omega_1 = 0 \).

7 Conclusion
The equations have been obtained for the change in the speed of shuttle/swell and shuttle/picker after impact and also for the dynamic response of swell and picker. The latter gives the angular velocity at any time \( t \) when impact is being considered. Together, they predict the reduction in speed of shuttle due to swell (fixed fulcrum) and picker/picking stick.

References