A geometric model of woven geotextile tape fabric to predict tensile property

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A geometric model has been presented to predict the strength and stretch behavior of woven tape fabrics that are usually utilized in geotextile applications. Model input parameters include fabric geometric parameters, tape bending effect, consolidations, and tensile properties. It is possible to predict the strength of the commercial geotextile fabric within 10% of the experimentally determined values in most of the cases. Some discrepancies have been noted that are probably due to the high variation in tape tensile properties and estimation in the angle $\theta$. 

Keywords: Geometric parameters, Geotextile, Tensile properties.

1 Introduction

Geotextile fabrics are used in civil engineering applications, such as structural and environmental processes. These fabrics can be woven, nonwoven or composite that exhibit certain properties including permeability and high tensile strength. The important factors in manufacturing geotextiles are fiber (polymer) type and fabric design. At present, a major use of geotextiles is within foundation components or load supporting parts of a structure which include buildings, embankments, roads and railways, and hence the strength of these fabrics is very important. The woven geotextiles used in our study had a plain weave construction that was produced from polypropylene tapes. The geometry of a fabric has significant effects on fabric's mechanical and physical behavior. The study of woven fabric geometry started with Pierce\(^1\) who assumed yarns as flexible circular cylinders interwoven in a regularly recurring pattern to form fabric. Pierce\(^1\) developed a geometrical model to determine the various fabric parameters; however, consideration was not given to the internal forces. Although Pierce's model has been found to be valuable for very open structures, the assumption of the circular cross-section, flexibility and incompressibility was found to be unrealistic, which limited the application of this model. It was realized that in more tightly woven fabrics, the inter-thread forces set up during weaving cause considerable flattening of the thread. In order to avoid complexity of the elliptical yarn section, Pierce used an approximation treatment\(^2\). In the approximation, the original equations derived from the circular cross-sections were used by replacing the diameter of the circular cross-section yarn with the minor diameter (axis) of the appropriate elliptic section. Pierce's approximation was adequate for open weave fabrics, but could not be applied to jammed structures.

Grosberg\(^3\) described the tensile property of the woven fabric as an anisotropic phenomenon, which makes the prediction of tensile properties more complex. Lack of symmetry in the fabric causes a difference in modulus between the warp and weft directions. The variation in modulus becomes larger when the extension of the fabric takes place at an angle and also involves a different mechanism of deformation\(^3\). Kawabata \textit{et al.}\(^4\) described a biaxial-deformation theory for plain weave fabrics. In this theory, warp and weft yarns are considered perfectly flexible and the forces due to yarn bending are negligible. The theory introduced the compressibility of the yarn caused by a lateral compressive force. It was shown that the compressive properties of the yarns had an effect on the tensile properties of the fabric. The model developed by Kawabata \textit{et al.}\(^5\) has been shown to predict the tensile behavior of the fabrics that is in agreement with the experimental results; however, the model did not include the bending of the yarn that would have an effect on the tensile property of the fabric.

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In the uniaxial-deformation theory of plain weave fabrics, Kawabata et al.\(^6\) assumed that the yarn in the direction of the applied force is perfectly flexible and also considered the effect of bending on the transverse yarn. The same theory used to solve the problem of biaxial tension was not applicable to this situation because no tension was applied to the transverse yarns in the case of uniaxial loading. The uniaxial-deformation model proposed by Kawabata et al.\(^5\) appeared to predict the fabric's tensile behavior well except for high and low extension regions where discrepancies have been observed between the experimental and theoretical predictions. Reallff et al.\(^6\) have reported a micromechanical model to describe the tensile behavior of woven fabrics. The inputs of this model\(^6\) included original fabric geometry, experimentally determined bending behavior, consolidation response, flattening response and tensile response of the yarns. This model has been used to describe the load versus fabric strain curve, and also the normal force versus fabric strain curve at the crossover points. The various models to predict the tensile behavior of woven fabrics, proposed by the different researchers\(^1,5,6\), were based on cylindrical yarns. In the present work, a model has been developed following the similar approaches of both Kawabata et al.\(^5\) and Reallff\(^6\) and using Pierce's\(^1\) original geometric parameters for geotextile fabric produced from the rectangular tape yarns. The primary objective of the study was to develop a geometric model of a woven tape fabric to predict tensile strength.

2 Model Development

The model was developed to predict the tensile behavior of the geotextile fabric through the analysis of fabric geometry, and the interaction of the geometric elements to the applied stress on the fabric. The tape parameters considered were denier, tape width, tape thickness, tape strength, tape elongation and flexural rigidity. The fabric parameters were warp density, weft density, and warp and weft crimp. Geotextile fabrics produced from polypropylene tape yarn were collected from different industrial locations to validate the model. These fabrics had a range of warp and weft density and tensile properties as follows:

- Warp density, ends per inch: 12-28
- Weft density, picks per inch: 11-18
- Warp breaking load, g: 1519-4791
- Weft breaking load, g: 1887-9055
- Warp elongation, %: 29.5-11.8
- Weft elongation, %: 27.8-8.6

One of the major differences in our model arises from the rectangular cross-section of the tape with negligible thickness in all practical purposes. This difference in cross-section yields different properties such as modulus and moment of inertia. Flattening of yarn was not considered because the original thickness was so small (in microns), which would have a negligible effect on the ultimate prediction model.

3 Inputs and Outputs of the Model

The model inputs include the original fabric geometry\(^1,5,6\), tape bending behavior, consolidation response of the tapes and tensile response of the tape. The outputs of the model are the fabric load, fabric strain and normal force at the crossover points.

4 Incompressible Tapes and Fabric—No Jamming Situation

4.1 Fabric Structural Parameters

The tapes in the loaded direction of the fabric are straightened at a low strain region, which is the crimp interchange section. Normal forces that are perpendicular to the loading direction act on the tapes at the crossover points (\(F_N\)). These forces can be determined from the bending rigidity of the tape and the amount of tape deflection caused by these forces. The amount of deflection (\(h\)) can be measured from the unit cell geometry of deformation used by Kawabata et al.\(^5\) and Reallff et al.\(^6\) as illustrated in Fig. 1. The woven geotextiles that were modeled here had plain weave structure and, hence, the Kawabata's deformation geometry was, in general, applicable. In Fig. 1, the solid line represents the initial geometry, while the dotted line is the fabric geometry after deformation. The \(X_1\) and \(X_2\) are the axes in the warp and weft directions respectively. The deflection distance \(h\) is calculated considering the solid line and dotted-line triangles as in Fig. 1b. We can obtain:

\[
h = l_{m1} - (h_{m1} - h_1) \quad \text{from Fig. 1b. Using the triangles to solve} \quad h_{m1} \text{and} \quad (h_{m1} - h_1) \text{and utilizing the geometric relationship in Fig. 1, the following equation is obtained:}
\]

\[
h = \frac{l_{m1}}{2} \cos \theta_{m1} - \frac{l_{m1}^2}{2} \cos \theta_1 \quad \ldots (1)
\]
From the original geometry (Fig. 2), \( l_{0w} \) and \( Y_{0w} \) are known and, hence, \( \theta_{0w} \) can be determined as follows:
\[
S \sin \theta_{0w} = \frac{Y_{0w}}{l_{0w}}
\]
where \( Y_{0w} \) is the initial tape spacing (1/thread per inch).

4.2 Bending Effects in the Model
A beam model was utilized to determine the normal forces on the tapes at the crossover points. The beam model was applicable for this model because of the rectangular cross-section of the beam and the zero slopes at the walls and at the center of the beam. The slopes match those of the tapes at the crossover points in the fabric, which is also the reason for Realff et al.\(^6\) to use this model. Using the approach of Gere and Timoshenko\(^7\) for a prismatic beam and a beam that deflects a distance \( 2h \), the following relationship can be derived:
\[
2h = \frac{F_N (2l_{02})^3}{192 EI}, \quad \text{or} \quad F_N = \frac{192 EI \cdot 2h}{(2l_{02})^3}
\]
where \( F_N \) is the normal force; \( EI \), the flexural rigidity; \( l_{02} \), the initial unit length (Fig. 2) of the weft tape; and \( h \), the tape deflection.

This equation was used to calculate the bending coefficient \( (B_e) \) which is similar to the procedure used by Kawabata et al.\(^4\) to calculate their constants.

However, they considered bending of fibers instead of the yarn. Realff et al.\(^6\) measured the bending responses experimentally using a four-point bending tester. In this study, the bending coefficient \( (B_e) \) was calculated using the following relationship:
\[
B_e = \frac{192 EI}{(2l_{02})^3}
\]

4.3 Consolidation Effect
After crimp interchange and in the moderate strain region, the tapes in the loading direction are extended. In this region, tapes undergo consolidation or a reduction in width while the length increases. This increase in length and reduction in width is associated
with an increase in normal force. This increase in normal force takes into account the width before and after consolidation and the flexural rigidity. Reallf et al.⁶ used a similar equation where yarn diameter was used instead of the tape width to calculate the normal force due to consolidation \( F_{N(\text{consolidation})} \) as follows:

\[
F_{N(\text{consolidation})} = \frac{4 E I}{l_{02}} \left[ \frac{l_{01}}{W_{x1}} - \frac{l_{01}}{W_0} \right]
\]

where \( l_{01} \) is the initial unit length of warp tape; \( l_{02} \), the initial unit length of weft tape; \( W_{x1} \), the tape width after consolidation; and \( W_0 \), the tape width before consolidation.

### 4.4 Fabric Response to Extension—No Jamming

A tape within the fabric fails when it reaches its average extension, which is determined by an individual tape tensile test. Three equations for the plain weave tape fabric breaking load \( F \) and normal force \( F_N \) at the crossover points are proposed here that are similar to the models proposed by Kawabata et al.³ and Reallf et al.⁶. Tapes used in these fabrics are generally very thin; consequently, the effect of flattening due to the normal force becomes negligible. Therefore, the effect of flattening was not considered in the following model for the plain weave tape fabric.

\[
F = g_1(\lambda_{x1}) \sqrt{1 - \left( \frac{l_{01} \cos \theta_{01}}{4 g_1(\lambda_{x1}) + B l_{01} \lambda_{x1}} \right)^2} \left( \frac{\lambda_{x1}}{\lambda_0} \right)^2
\]

\[
F_N = B \left( \frac{l_{01}}{2} \cdot \cos \theta_{01} - \frac{l_{01} \lambda_{x1}}{2} \cdot \cos \theta_{1} \right) + F_{N(\text{consolidation})}
\]

where \( \lambda_1 \) is the fabric stretch \((1 + \text{strain})\); and \( \lambda_{x1} \), the tape stretch \((1 + \text{strain})\).

### 5 Fabric Breaking Load and Stretch (Experimental vs Model Predicted) Results

Several commercial tape fabrics (plain weave) were collected from different producers to compare the values of model predicted fabric strength with the experimentally determined results. The tape's tensile testing was performed on an Instron Single-end Tester at a constant rate of extension (10 inches/min) using a 5-inch gauge length. An average breaking load of the tapes from each fabric was determined. A grab tensile test was performed on the tape fabrics according to the ASTM Standard D5034 on a MTS Alliance RT/5 Tensile Tester. Bending coefficient \((B_c)\) was

<table>
<thead>
<tr>
<th>Fabric No.</th>
<th>Experimental Load, g</th>
<th>Predicted Load, g</th>
<th>% Difference</th>
<th>Experimental Stretch</th>
<th>Predicted Stretch</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>48.126</td>
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<td>0.77</td>
<td>1.2016</td>
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<td>2</td>
<td>42.290</td>
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<td>1.1610</td>
<td>1.303</td>
<td>10.90</td>
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<td>3</td>
<td>40.075</td>
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<td>4.51</td>
<td>1.1001</td>
<td>1.151</td>
<td>4.42</td>
</tr>
</tbody>
</table>

*These fabrics were produced with double tape (2-ply).
determined from Eq 4. Flexural rigidity ($EI$) was obtained from the modulus of elasticity that was derived from the load-elongation diagram. The moment of inertia ($I$) of the tape was then calculated from the following equation:

$$I = \frac{\text{Width} \times \text{Thickness}^3}{12} \quad \ldots (9)$$

The initial angle between the warp tape axis and the axis along the intersection of the warp and weft tapes was calculated using Eq 2. The angle after deformation ($\theta_i$) was approximated by examining the fabrics after deformation under a microscope. In Table 1, both experimentally determined and model predicted values of fabric load and stretch are presented. It was possible to predict fabric load and stretch within 10% of the experimentally determined values on a tensile tester. In a few cases, however, a larger discrepancy between the experimental and the model predicted values was found, which is shown in Table 1 as positive or negative variations. There are three possible sources that can be attributed to these discrepancies. Both breaking load and elongation of the commercial tapes used in these fabrics were highly variable, which created difficulty in accurately determining the tape strength and strain. There may be some error in estimating $\theta_i$ of the deformed fabric in absence of a more accurate method. Some of the variation may also be due to the fact that two fabrics caused jamming under load, which was not considered in our model. The model should be modified for those fabrics that would jam during extension, to yield better results.

For the various fabric geometry and tape parameters, normal forces at the crossover points near break were calculated. It is an important parameter because the normal force near break will affect the load transfer to the tapes in the area of the break. The normal force values presented in Table 2 were not experimentally verified and were calculated based on the beam model as described earlier.

### 5 Conclusions

A geometric model for predicting the strength and stretch of woven tape fabric has been reported. Original fabric geometric parameters, bending response and tape consolidation (width change effect due to extension) have been calculated to determine tape fabric strength and stretch. Model predicted load at break and stretch values were generally found to be within 10% of the experimentally determined fabric load and stretch; however, some exceptions were observed. To make the model more universal, the fabric jamming effect needs to be considered, particularly for tightly woven fabrics. It is believed that this could provide a guidance to design tape fabrics for specific application.

### References