Indian Journal of Marine Sciences
Vol. 36(2), June 2007, pp. 122-127

The great Sumatra-Andaman earthquake of 26 December 2004 was predictable even from seismicity data of $m_b \geq 4.5$: A lesson to learn from nature

S.S.Teotia* & Dinesh Kumar

Department of Geophysics, Kurukshetra University, Kurukshetra-136119, India
*[Email-teotia_ss@rediffmail.com]

Received 11 October 2006; revised 11 April 2007

The spatial distribution of earthquakes is found to change before and after occurrence of an earthquake of given size. The occurrence of an earthquake of any size may be related with the self-organized criticality behavior of turbulence in solids. This change is reflected in the temporal variation of generalized dimension D_q or D_q spectra. Therefore, the study of temporal variations of D_q and D_q spectra may be used to study the changes in Seismicity structure before the occurrence of earthquakes and hence multifractal study holds promise in forcasting earthquake in the regions having potential to generate great earthquake. The study in this paper deals with multifractal analysis of seismicity data of the region which have resulted in great Sumatra-Andaman earthquake of 26 December 2004. The significant increase in D_{-2} and D_q spectra has been observed prior to occurrence of (m_b=9, M_w =9.1 to 9.3) great Sumatra-Andaman 26th December 2004 even with seismicity data having completeness of catalogue for $m_b \geq 4.5$. The monitoring of USGS global network holds promise to reveal changes in D_q prior to occurrence of great earthquakes even from earthquake catalogue which have their completeness for magnitudes (i.e. $m_b \geq 4.5$).

[Kew words: Seismicity, self-organized criticality, multifractal analysis, earthquake, Sumatra-Andaman]

Introduction

Fracture exhibits a fractal structure over a wide range of fracture scale, i.e from the scales of microfractures to megafaults[1-5]. Recent studies have shown that many natural phenomena such as the spatial distribution of earthquakes, fluid turbulence are heterogeneous fractals[6-12]. A number of studies have been made to investigate the temporal variation of heterogeneity in seismicity using multifractal analysis in various seismic regions[12-16] In such systems, the number of fractures that are larger than a specified size are related by power law to the size. Originally the notion of self-similarity was defined for sets. The generalization from fractal sets to multifractal measures involves the passage from objects that are characterized primarily by one number namely a fractal dimension to objects that are characterized by a function[8,10-11]. This is a transition from characterization by a single number (the exponent in the power law relation or the dimension) of geometrical objects to that by a function which may have any number of parameters. The characterization is not in terms of one or two or three dimensions, but by a family of generalized dimension D_q as a function of index(q), which goes from $-\infty$ to $+\infty$.

For some time it has been recognized that the parameters which describe seismicity in a region show a spatial and temporal evolution which may be associated with the process of generation of large earthquakes i.e. evolving of fracture from micro fracture to mega fault. The change in seismicity pattern of earthquakes is reflected in the generalized dimension D_q or D_q spectra of the seismicity. Therefore, the study of temporal variation D_q and D_q spectra may be used to study the changes in the seismicity structure before and after the occurrence of large earthquakes, which may prove to be of value in forecasting of occurrence of such events. Accordingly Hirata & Imoto[12] have performed multifractal analysis of microearthquake data of Kanto region using correlation integral method. Hirabayashi *et al.*[13] have done the multifractal analysis of seismicity in regions of Japan, California and Greece using fixed mass and fixed radius methods.. Li *et al.*[14] have performed the multifractal analysis of spatial distribution of earthquakes of Tanshan region (Ml> 1.8) using extended Grassberger-Procraccia method of dimension D_q estimation. Teotia *et al.*[15-16] have

*Corresponding author:
Phone: 91-01744-238410/238679 (#2725, 2726)
Fax: 91-01744-238277, 238198

studied the multifractal characteristics of seismicity of Himalaya region ($m_b \geq 4.5$) and have found steep slope in D_q spectra to be associated with high rates of energy release and the gentle slope corresponds to low rates of energy release. These studies have noted changes in temporal characteristic of the generalized dimension D_q and D_q spectra which were found to be associated with the occurrence of some large earthquakes in these regions. This question, however, needs further investigations, particularly in different tectonic domains of the world. In the present study we investigate the temporal behavior of generalized dimension D_q as well as D_q spectra for earthquakes in Sumatra-Andaman region ($m_b \geq 4.5$) during the years AD 1964-2004 to see the precursory behavior of change in D_q and D_q spectra from the data having magnitude threshold ($m_b \geq 4.5$) in the time series analysed.

Methodology

Figure 1 shows the study area along with the tectonics of the region. For the analysis of seismicity, the earthquake data set of the USGS[17] was used as shown in Fig. 2. The time window for analysis was chosen to be 1964-2004 and the USGS[17] earthquake catalogue for Sumatra-Andaman region for this period was used for the multifractal analysis. Figure 2 shows the Sumatra-Andaman region having seismicity of subduction zone from Myanmar to Sumatra. To gain an unbiased and homogeneous data set, we restricted the data by setting a lower limit and analysis time period and it is found to be complete for magnitude threshold $m_b \geq 4.5$. Under the assumption that

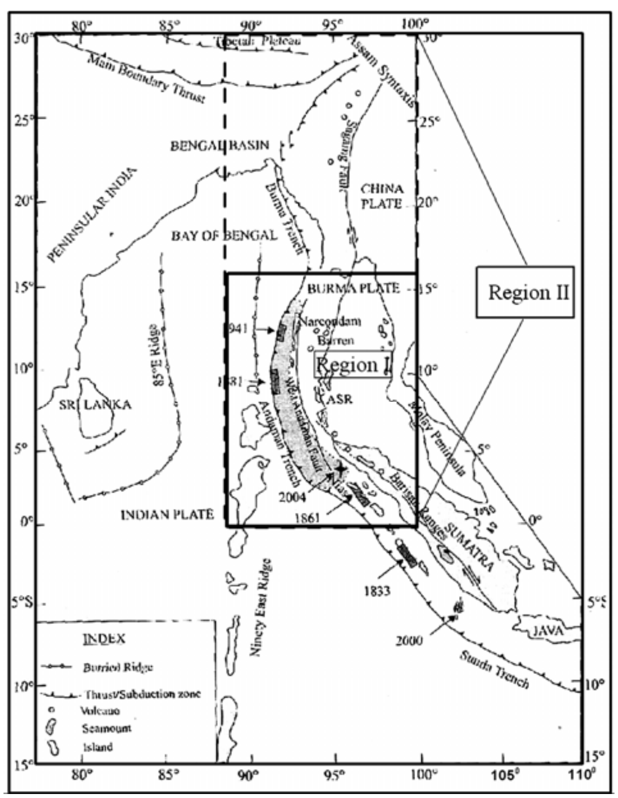

Fig. 1—The tectonic map of region along with area under study (After Kayal[32]).

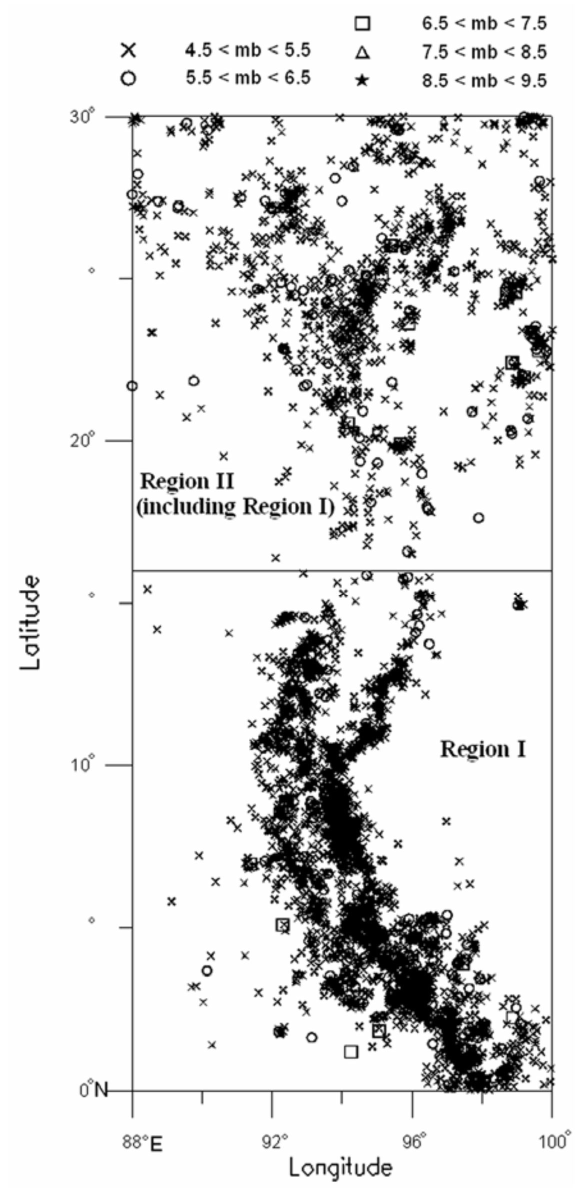

Fig. 2—Seismicity map showing epicenter distribution of earthquakes which occurred in between January 1964 to March, 2005. The analysed data set covers two regions with apexes latitude and longitude as (0°, 88°), (0°, 100°), (16°, 88°), (16°, 100°) in the first region and (0°, 88°), (0°, 100°), (30°, 88°), (30°, 100°) in the second region.

geological processes are steady over historical time and the observation capabilities are steady over the time window under consideration for $m_b \geq 4.5$, the temporal changes the frequency of earthquakes may be attributed to the aftershocks of the large earthquakes and the local temporal variation in the seismicity. Figure 3 shows the yearly change in number of shallow earthquakes for different magnitude thresholds. The number of earthquakes of $m_b \geq 4.5$ vs the time curve shows a poor trend in data set for Sumatra Andaman region. Thus, the available data set can be considered as fairly complete in the time series for magnitude threshold $m_b \geq 4.5$.The analysed data set covers two regions i.e. region I and region II with four apexes having latitude and longitude as (0º,88º), (0º, 100º), (16º, 88º), (16°, 100º) in region I and (0º, 88º), (0º,100º), (30º,88º), (30º, 100º) in region II. The total number of events occurred in smaller and bigger regions were 3652 and 4675 respectively during Jan.1964-March 2005. However in the present study the time window (Jan.1964-26th Dec., 2004) has been selected for the multifractal analysis in small as well as bigger regions. The great Sumatra earthquake of 26th Dec., 2004 is 1598th event in smaller region (region I) and 2719th event in bigger region (region II).

The available methods for calculating D_q are the box counting method[18,21], the fixed-radius methods[22] and the fixed-mass method[23,24]. These methods work well provided the number of data points are very large[22,24]. The extended[25,26]Garssberger & Procraccia method is used in this analysis, which can recover the dimension from a time series. It is described as:

$$\log C_q(r) = D_q \log r(r\rightarrow 0) \qquad \dots (1)$$

$$C_q(r) = \mathrm{Lim}\ r\rightarrow 0\ \{(1/N\sum[(1/N) \sum_{\substack{i=1 \\ i\neq j}}^{N} H(r-X_i - X_j)]^{q-1}\}^{1/(q-1)} \qquad \dots (2)$$

where r is the scaling radius, N is the total number of data points within a search region in a certain time interval (also called the sample volume): X_i is the epicentral location (given in latitude and longitude) of the ith event, X_j is the epicenter (given in latitude and longitude) of the jth event, $C_q(r)$ is the qth integral and H(.) is the heaviside step function.

In the procedure for estimating D_q spectra as a function of time, a time series of earthquake epicenters has to be formed and divided into sub-series (subsets). Let set $\{X_i, M_i\}$i=1, M be a complete set of earthquakes occurring in time period analysed, and M_i the magnitude of an earthquake occurring at time t_i. Thus the earthquake constitutes a time series of N elements. The time series consists of 1600 events in the smaller region and 2750 in the bigger region. We consider these time series as the original data sets for multifractal analysis of these regions. In the

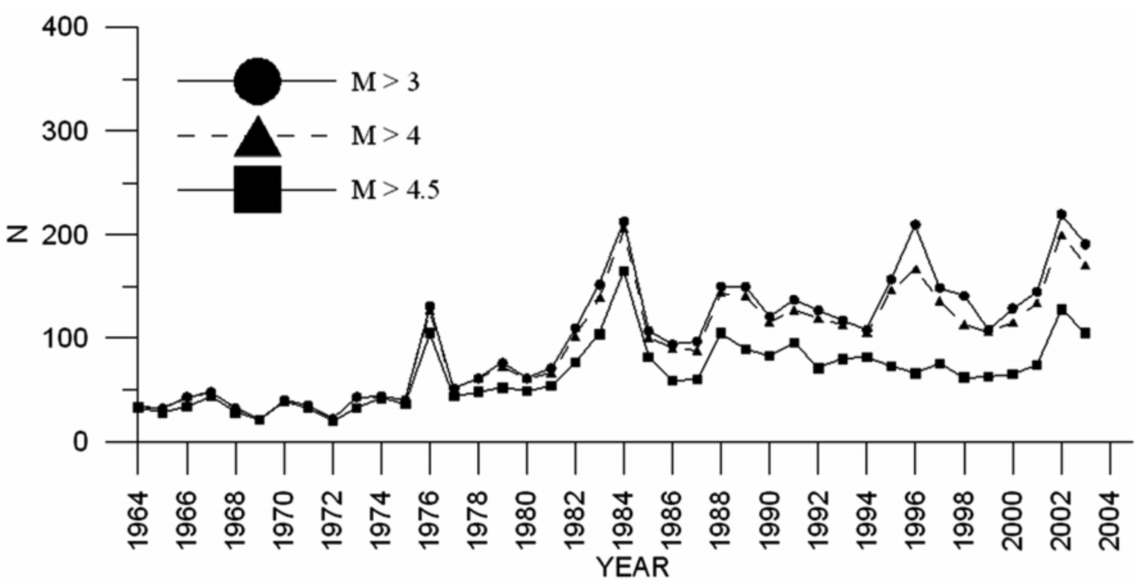

Fig. 3—Yearly change of the number of earthquakes from January 1964 to December 2003 different magnitude threshold in region having apexes (0°, 88°), (0°, 100°), (30°, 88°), (30°, 100°).

present case the original data sets for Sumatra-Andaman regions are divided into 24 subsets for smaller region and 45 subsets for bigger region. The subset in Sumatra-Andaman region consists of 500 events with an overlap of 450 events. The shift of 50 events has been used in the analysis. The correlation integral $C_q(r)$ is calculated using Eq. (2) for the epicentral distribution X_i of the subset. The distance r between two events is calculated by using spherical triangle[27-28]. For epicentral distribution having a fractal structure, the following power law relationship is obtained in the scaling region.

$$C_q(r) \sim r^{Dq}$$

An appropriate scaling region has been estimated before the computation of generalized dimension D_q. The scaling region is a linear segment in the graph of log r versus log $C_q(r)$. The scaling region may be characterized by the circular boundary defined by scaling radius around epicenter. We use the method of Li *et al.*[14] to determine the scaling region in Sumatra-Andaman region.

Results and Discussion

Observation of seismicity suggests a relationship between the distribution of earthquake magnitudes and the distribution of earthquake epicenters[28,29]. Figure 4 (A, B) shows the temporal variation of D_q for three different values of q(-2, 0, 2) for the total time series analyzed. The D_q values for each subset are plotted at the time where the last earthquake entered the subset. The first subset starts from 1963.202 years (i.e. 14th March 1964) and ends at 1982.964 years for Sumatra-Andaman region in the smaller region. The temporal changes observed in D_{-2} are larger and rough as compared to the variations in other two dimensions D_0 and D_2 (Fig.4). The evolving pattern of seismicity structure may emerge in significant trends in temporal variation of D_q and D_q spectra prior to occurrence of great a Sumatra earthquake of 26th December, 2004.The D_q spectra for smaller region is shown in this study (Fig.5). Therefore, if there is any organized pattern of clustering emerges prior to occurrence of great earthquake may be resolved even from the time series having earthquakes $m_b \geq 4.5$. There is a clear-cut trend present in temporal variation of D_q and D_q spectra for Sumatra-Andaman region as is shown in Fig.4 and Fig. 5. This trend may be because of the great Sumatra earthquake (m_b=9.0) providing a time

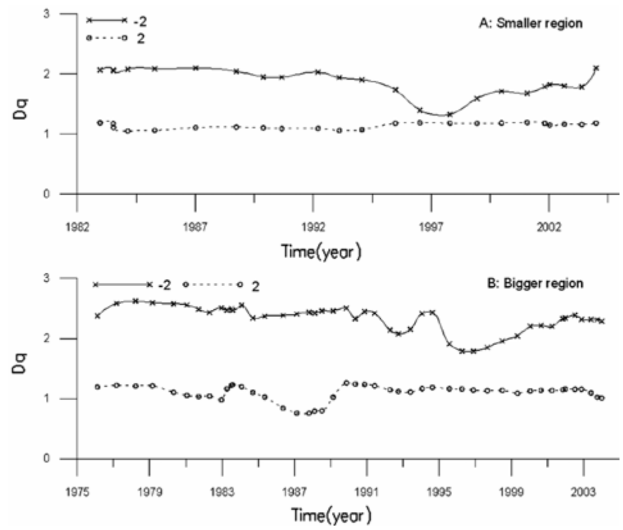

Fig. 4— A)The temporal variation of generalized dimension D_q for q, -2.0, 2.0 in order from top to bottom for the epicenter distribution in A)Region I and B) Region II.

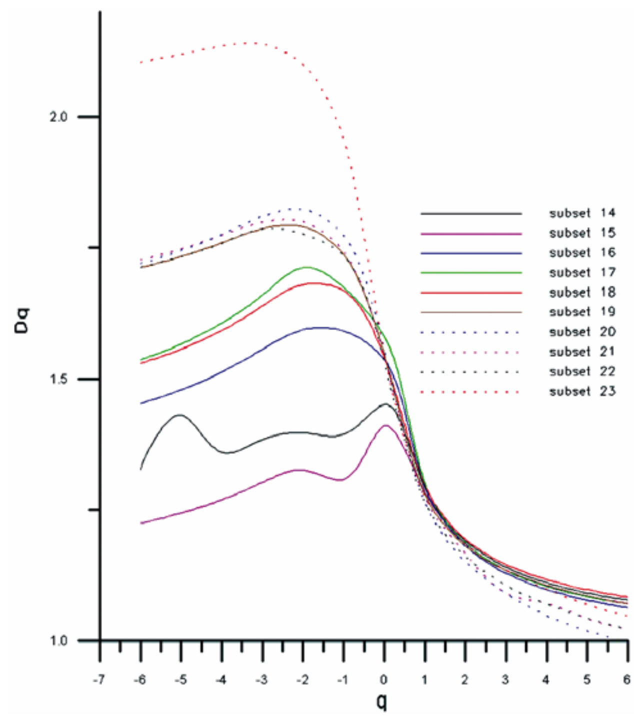

Fig. 5—Multifractal analysis of epicenter distribution of earthquakes occurring in subset 14 to subset 23 in Region I.The D_q spectra of 10 subsets i.e from subset 14 to subset 23. Subset 14 has gentle slope of D_q-q spectra. Subset 23 has steep slope of D_q-q spectra. The great Sumatra-Andaman earthquake of 26 December 2004 lies in subset 23.

series of better resolution in size ranging $4.5 \leq m_b \leq 9.0$ for understanding the evolving pattern of seismicity. These evolving patterns may result from the presence of significant clustering of events in space and time. The presence of required clustering in the data has resulted in clear-cut trends in temporal variation of D_q and D_q spectra. Figure 5 shows the consistent increase in slope of D_q spectra i.e. from subset 14 to subset 23. Subset 14 corresponds to seismicity having minimum difference in D_2 and D_{-2} which may have less clustering presence i.e. declustering in the region. The spatial distribution of seismicity in this subset may be considered as relatively homogeneous seismicity in the region. The Sumatra earthquake is 498th event of the subset 23 in the smaller region. Therefore, the D_q spectra of subset 23 is mainly defined by earthquake epicenters of events occurring prior to 26th December, 2004 as huge stresses are required in the region for such a great earthquake to occur which may result in high degree of clustering in seismicity pattern as evident from the steep D_q –q spectra in subsets during and prior to occurrence of great Sumatra earthquake of 26th December, 2004. The seismicity evolves from homogeneous to heterogeneous distribution of epicenters from subset 14 to subset 23. The significant trend in temporal variation in generalized dimension is also available even in bigger region. This may be due to clustering pattern of seismicity involving larger area in great earthquakes. This study suggests that temporal variation of D_q and D_q spectra may serve as precursor in prediction of earthquake provided the seismicity data of potential regions represents the time series having presence of sufficient clustering of earthquake epicenters, which could emerge as a self organized pattern[30] in the zone of preparation of great size earthquake. In case of great earthquake like Sumatra the zone of preparation of self emerging earthquake epicenters pattern may be clearly reflected in small as well as bigger areas. Both smaller region as well as bigger region show significant trend in generalized dimension of D_q after year 1996 as seismicity pattern evolves from declustering to clustering scenario in the regions as regions may prepare for huge build up of strain energy (see Fig.4). This may be indicative of the fact that great earthquake like Sumatra have much bigger rupture zone involving self emerging patterns of earthquake epicenters. Such changes are also evident in plot of cumulative seismic moments as a function of time for 29 years history of the Harvard CMT catalog, which appears to have high cumulative global earthquake seismic moment release for the preceding decade[31]. One can see the change in moment rate from 1995 onward in the cumulative plot of seismic moments[31]. This further supports the increase in clustering (i.e. steep D_q spectra) in the region due to higher amount of energy release as was observed in Himalaya[15]. The change in clustering in region I is visible in seismicity structure of subset 14 and subset 23 (see Fig. 6)

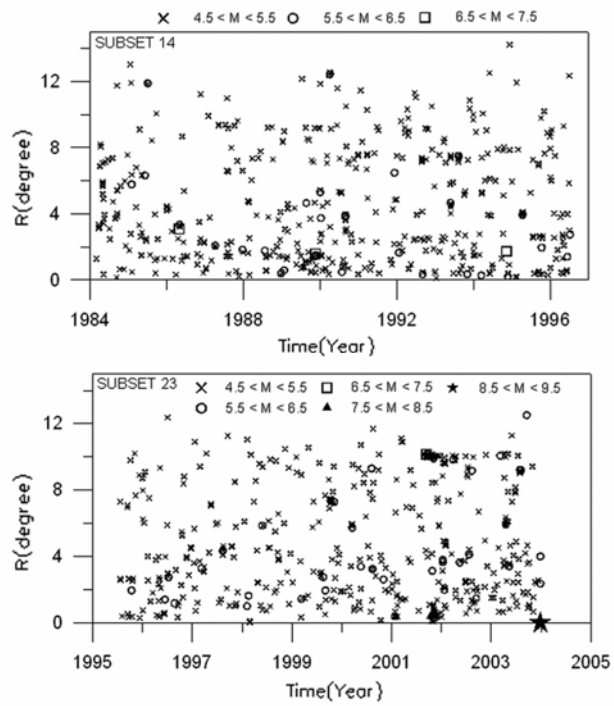

Fig. 6—Seismicity structure of subset 14 and subset 23 in R-T domain in Region I. R is the distance from the epicenter of great Sumatra earthquake of 26th December 2004 to epicenters of other earthquakes occurring in these subsets and T is the corresponding time of the events in respective subsets.

Conclusion

Studies by seismologists have shown that the occurrence of earthquakes are fragmented, contain gaps, and have heterogeneous distribution both in space and time. The present study suggests that multifractal analysis is more useful for the study of heterogeneous fractals of earthquakes. The spatiotemporal variation in the occurrence of earthquakes may be studied in the framework of mutifractal i.e. by studying the temporal variation of generalized dimension D_q and D_q –q spectra. We suggest that the regions having potential to generate great earthquake (like Himalaya, Alaska and Chile regions etc) be systematically analysed by even earthquake catalogue $m_b \geq 4.5$ for detection of changes in temporal variation in D_q and D_q spectra for possible prediction of great earthquake. The temporal variation of generalized dimension involves significant trends in both smaller and bigger area around the epicenter of great Sumatra earthquake. This can also serve as precursor for prediction of Tsunami if such changes are observed prior to great earthquakes in the seismotectonic regions which may result in Tsunami as has been observed prior to occurrence of great Sumatra 26th December 2004 earthquake. The results of this study along with GPS data may be helpful in real time monitoring of evolving scenario of fracture process in the areas where probability of occurrence of great earthquake is high.

Acknowledgement

Authors are thankful to Kurukshetra University for providing the necessary help and support for carrying out this work. Authors are also thankful to late Prof. K.N. Khattri, for his valueable suggestions.

References

1 Brown, S.R. & Scolz, C.H., Broadband width study of the topography of the natural surfaces, *J. Geophys. Res,* 90 (1985) 12575-12582.

2 Scholz, C.H.& Aviles, C.A., The fractal geometry of faults and faulting in earthquake Source Mechanics edited by Das.S., Boatwright, J. and Scholz, C.H, Maurice Ewing Ser.6, (American Geophysical Union, Washington, D.C.), 1986, pp. 147-155.

3 Okubo, P.G. & Aki, K., Fractal geometry in San Andreas fault system, *J. Geophys. Res*., 92 (1987) 345-355.

4 Allegre, C.J., Le Mouel, J.L. & Provost, A., Scaling rules in rock failureand possible implications for earthquake prediction, *Nature*, 297 (1982) 47-49.

5 Hirata, T., Omori's power law aftershock sequences of microfracturing in rock fracture experiment, *J. Geophys. Res.*, 92 (1987) 6215-6221.

6 Frisch, U., Sulem, P.L. & Nelkin, M., A simple dynamical model of intermittent fully developed turbulence, *J. Fluid Mech.*, 87(1978) 719-736.

7 Stanley, H.E. & Meakin, P., Multifractal phenomena in physics and chemistry, *Nature,* 335 (1988) 405-409.

8 Mandelbrot, B.B., Intermittent turbulence in self-similar cascades:divergence of high moments and dimension of carrier, *J. Fluid Mech.,* 62 (1974) 331-358.

9 Mandelbrot, B.B., The fractal geometry of nature (W.H.Freeman, New York), 1982, p. 448.

10 Frisch, U. & Parisi, G., Fully developed turbulence and intermittency. In: *Turbulence and predictability in geophysical fluid dynamics and climate dynamics*, edited by Ghil, M. *et al*., (North Holland, Amsterdam), 1985, p. 84.

11 Mandelbrot, B.B., Multifractal measures, especially for the geophysicist, *Pure Appl. Geophys*. 131(1989) 5-42.

12 Hirata, T. & Imoto, M., Multifractal analysis of spatial distribution of microearthquakes in the Kanto region, *Geophys. J. Int*., 107 (1991) 155-162.

13 Hirabayashi, T., Ito, K. & Yoshii, T., Multifractal analysis of earthquakes, *Pure Appl. Geophys*., 138 (1992) 591-610.

14 Dongsheng, Li., Zhaobi, Z. & Binghong, W., Research into the multifractal of earthquake spatial distribution, *Tectonophysics*, 223 (1994) 91-97.

15 Teotia, S.S., K.N. Khattri & Roy P.K., Multifractal analysis of seismicity of Himalayan region, *Curr Sci*, 73 (1997) 359-366.

16 Teotia, S.S., *Multifractal analysis of earthquakes: An overview*, edited by V.P. Dimri, (Oxford & IBH publishing Co. Pvt. Ltd, New Delhi9) 2000, pp. 161-170.

17 USGS home page, [http://neic.usgs.gov/mneis/epic].

18 Grassberger, P., Generalised dimension of strange attractor, *Phys. Lett. Ser. A*., 97(1983) 227-230.

19 Hentchel, H.G. & Procraccia, I., The infinite number of generalzed dimensions of fractals and strange attractors, *Physica D*, 8(1983) 435-455.

20 Halsey, T.C., Kadanoff, L. P., Procraccia, I. & Shraiman, B.I., Fractal measures and their singularities: The characterization of strange sets, *Phys. Rev. A*., 33 (1986) 1141-1151.

21 Greenside, H.S., Wolf, A., Swift, J. & Pignataro, T., Impracticality of a box counting algorithm for calculating the dimensionality of strange attractors, *Phys. Rev. A*, 25 (1982) 3453-3459.

22 Jensen, M.H., L.P., Libichaber, A., Procraccia, I. & Stavans, J., Global universality at the onsetof chaos: Results of a forced Rayleigh-Bernard experiment, *Phys. Rev. Lett*., 55 (1985) 2798-2801.

23 Grassberger, P., Baddii, R. & politi, Scaling laws for invariant measures on hyperbolic attractors, *J. Stat. Phys.,* 55 (1988) 135-178.

24 Baddi, R. & Broggi, G., Measurement of the dimension spectrum f(α): Fixed-mass approach, *Phys. Lett. A*, 131 (1988) 339-343.

25 Grassberger, P. & Procraccia, I., Measuring the strangeness of strange attractors, *Physica D*, 9 (1983) 189-208.

26 Pawelzik, K. & Schuster, H.G., Generalised dimension entropies from a measured time series, *Phys. Rev*., 35 (1987) 481-484.

27 Bullen, K.E. & Bolt, B.A., *An introduction to theory of seismology*, (Cambridge University Press, New York), 1985.

28 Hirata, T., A correlation between the b value and the fractal dimension of earthquakes, *J. Geophys. Res*., 94 (1989) 7507-7514.

29 Henderson, J., Main, I.G., Pearce, R.G. & Takeya, M., Seismicity in north-eastern Brazil: Fractal clustering and the evaluation of the b value, *Geophys. J. Int*., 116 (1994) 217-226.

30 Bak, P. & Tang, C, Self-organized Criticality, *Phys. Rev A*, 38 (1989) 364-371.

31 Thorne Lay, Hiroo Kanamori, Charles J. Ammon, Meredith Nettles, Steven N.Ward, Richard C Aster, Susan L. Beck, Susan L Bilek, Michael R.Brudzinski, Rhett Butler, Heather R. DeShon, Goran Ekstrom, Kenji Satake & Stuart Sipkin, The great Sumatra-Andaman earthquake of 26 December 2004, *Science*, 308 (2005) 1127-1133.

32 Kayal, J.R., Gaonkar, S.G., Chakraborty, G.K. & Singh, O.P., Aftershock and seismotectonic implications of the 13 September 2002 Earthquake (Mw 6.5) in the Andaman Sea basin, *Bull. Seismological Soc. America*, 79 (2004) 989-1005.