Experimental investigation of saturation depth of 0.662 MeV gamma rays in copper

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The effect of target thickness on 0.662 MeV multiply Compton scattered gamma photon has been studied experimentally. An intense collimated beam, obtained from 6-Ci$^{137}$Cs source, is allowed to impinge on a rectangular copper target of varying thickness and the scattered photons are detected by a properly shielded NaI(Tl) scintillation detector, having dimensions 51$^\phi$ mm × 51 mm, placed at 90° to the incident beam. The subtraction of events under the analytically reconstructed full energy peak from the events recorded under the observed inelastic peak results in multiply scattered events having energy same as in single scattering. The target thickness at which the number of multiply scattered events saturates has also been determined. The signal-to-noise ratio is found to be decreasing with increasing target thickness. Monte Carlo calculation supports the present experimental results.

**Keywords**: Compton scattered events, Saturation thickness, Signal-to-noise ratio
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1 Introduction

The Compton profile, a means for determining the electron momentum distribution in an atom, requires that the photon scattered by a sample should have undergone only one elastic collision. However, in the collisions between photons and free electrons, higher orders processes occur due to large number of secondary radiation produced in the first encounter and are known as multiply Compton scattered radiation. In gamma ray interaction study, these radiations may also reach the detector along with singly scattered radiation and get counted. Therefore, multiply Compton scattering is one of the principle difficulties for interpreting data, which is present within the same energy range as the single scattering. In these multiple scattering processes, the unique relation between photon energy and electron momentum is also lost resulting in incorrect evaluation of Compton profiles.

There exist different approaches to account for the study of multiple scattering of gamma rays in a material, which are based on the geometrical arrangement of the radioactive source and detector, scatterer dimensions and interaction probability of radiation in a particular energy range. In the previous studies, multiple scattering has been evaluated both analytically and numerically, but due to complicated nature of the scattering process and differing geometrical constraints, analytical approaches$^{1-5}$ to the study of multiple scattering can’t generally provide all the information required to correct the data for this contamination. Therefore, Monte Carlo methods$^6$ are commonly used to predict the distribution of multiple scattering, which should be subtracted off the Compton data. There has been no experimental study of the accuracy of the Monte Carlo simulations and their validity has been inferred from the single profiles so deduced. The Monte Carlo studies$^{5,7}$ relates multiple scattering to the sample thickness. Different scattering geometries have been suggested to minimise the amount of multiple scattering but no systematic study on the shape of the energy distribution of multiply scattered radiation in the various experimental conditions exists.

Experimentally at 0.662 MeV, measurements$^8$ are made to determine the contribution of multiply scattered gamma rays in backward direction as a function of scatterer thickness by subtracting analytically evaluated contribution of singly scattered gamma rays. Pitkanen et al.$^9$ have measured the spectrum of 662 keV gamma rays multiply scattered by the Nickel sample at scattering angle of 104°. The effect of collimator size on multiple scattering has also been studied by Pitkanen et al.$^{10}$ for 60, 159, 412 and 662 keV incident photons. Although there is experimental and theoretical work available on

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multiple scattering of gamma rays but the experimental data is confined to measurements in backward hemisphere only.

Our recent measurements\textsuperscript{11} investigate the effect of the detector collimator and the sample thickness on 0.662 MeV multiply Compton scattered photons from cylindrical samples of aluminium, and confirm that in order to increase the signal-to-noise ratio, multiple scattering background should be minimised and the said objective can be achieved using a narrow detector collimation. The effect of sample thickness on the multiply Compton scattered gamma rays for the fixed set of values of source collimator size, source-target distance and target-detector distance for 0.662 MeV gamma rays scattered from the copper sample at scattering angle of 90° has been studied.

2 Experimental Details

An intense beam of gamma rays incident on a thick target results in multiply scattered events in addition to singly scattered ones. The principle of the present measurements is to record these events using a gamma detector placed at 90° to the incident beam. The experimental arrangement used in the present measurements is shown in Fig. 1. The radioactive source of $^{137}$Cs of strength 6-Ci (1 Ci = 37 Gbq) is placed in the cavity of a rectangular lead container of dimensions 200 mm×160 mm×160 mm especially prepared for the source. Keeping in mind the biological effects of radiation, a cylindrical beam collimator consisting of a brass pipe and fitted with aluminium windows on both ends can be filled with a column of mercury between measurements and is used to open and close the incident beam. The collimator of diameter 7 mm placed next to the cylindrical collimator provides a narrow beam of gamma photons. The distance between the source collimator and the front face of the target is kept 350 mm resulting in an angular spread of ±0.6° due to the source collimator (3.5 mm radius) on the copper target. The source assembly is placed at a height of 380 mm on an aluminium table fixed to one side of the scattering table to avoid scattering of radiation from the scattering table.

An intense collimated beam of gamma rays from $^{137}$Cs radioactive source is made to impinge on rectangular target of copper (Z=29). The scattered gamma rays are detected by a properly shielded NaI(Tl) scintillation detector having dimensions 51 mm×51 mm placed at 90° to the incident beam. The scattered beam is further collimated by a cylindrical collimator made of lead, lined with aluminium having radius 2 mm and thickness 17 mm placed at a distance of 25 mm in front of the gamma detector. The axes of the source collimator, gamma detector and cylindrical collimator pass through the centre of the target. The distance between the front face of the copper target and the detector collimator is kept 375 mm, so the angular spread about the median ray in direction of gamma ray detector is limited to ±0.3°. The scintillation detector is properly shielded by a cylindrical lead shielding having the inner side covered with 2 mm thick iron and 1 mm thick aluminium, with iron facing lead to absorb K X-rays emitted by lead shielding. The experimental data are accumulated on the PC based Multi channel analyser (MCA).

3 Method of Measurements

The methodology employed in the present measurements to determine multiply-Compton scattered events, having same energy as in single-Compton scattering, is based upon reconstruction of singly scattered spectrum using experimentally determined parameters such as the full width at half maximum (FWHM) and detector efficiency of the gamma detector. For 0.662 MeV gamma rays impinging on the rectangular samples of copper, the dominant mode of interactions is Compton scattering. The energy loss in this process depends on the incident photon energy ($E_0$) and scattering angle ($\theta$), and is given by the relation

\[ E = \frac{E_0}{1 + \frac{E_0}{m_0 c^2 (1 - \cos \theta)}} \]  \hspace{1cm} \text{...(1)}

![Fig. 1 — Experimental set-up. S-source; A-aluminium window; M-mercury collimator; C-fine beam collimator; Sc-rectangular copper target; Pb-lead](image-url)
where $E$ is the energy of the scattered photon and $m_o c^2$ is the rest mass energy of electron.

The Klein-Nishina cross-section, providing the probability for scattering at an angle $\theta$, is given by

$$\frac{d\sigma(E, \theta)}{d\Omega} = \frac{r_o^2}{2} \left[ \frac{E}{E_o} \right]^2 \frac{E + E_o}{E} \left[ \frac{E}{E_o} - \sin^2 \theta \right]$$ \hspace{1cm} \text{(2)}$$

where $r_o$ is the classical electron radius.

Using these relations, the number of photons scattered singly from the whole sample of thickness $X_o$ can be written as:

$$N(E) = \frac{1}{X_o} \int_0^X I_o \alpha \left[ \frac{d\sigma}{d\Omega} \right] \times d\Omega \cdot n_o \exp(-\mu x) \cdot \exp(-\mu_1 r) \cdot d x$$ \hspace{1cm} \text{(3)}$$

where $I_o$ and $\alpha$ are intensity and total cross-section for the incident beam, respectively. $n_o$ is number of electrons/cm$^3$ of the medium. The quantities $\mu$ and $\mu_1$ are the total attenuation coefficients of the medium at energies $E_o$ and $E$, respectively. The parameter $d\Omega$ is the solid angle subtended by the detector collimator at the scattering point of the scatterer. The parameter $r = \text{distance (AB)}$ as shown in Fig. 2, which also confirms that with increase in the thickness of the target, the angle $\theta$ remains the same, but $\theta_1$ goes on changing. The two specified angles are related by the following equation

$$\cot \theta_1 = -\frac{x}{R_o \sin \theta} + \cot \theta$$ \hspace{1cm} \text{(4)}$$

where $x$ and $R_o$ are the distances of the scattering point and detector collimator centre from the front surface of copper target, respectively. The spread in the scattering angle, corresponding to different points in the scatterer along the direction of propagation of primary gamma beam, results in the scattered energy that goes on changing. The change in scattered energy results in varying detector efficiency and FWHM of the gamma detector. These facts are taken into consideration for calculating the values of $N(E, x)$ at any energy $E$ for different values of $x$, $\theta_1$ and correspondingly different values of scattered energy $E$.

When incident on the detector, $N(E)$ gives rise to a pulse-height distribution whose photo-peak at energy $E$ can be represented by a Gaussian distribution

$$Y(E) = Y_o \exp \left\{ -\frac{(E - E_o)^2}{b} \right\}$$ \hspace{1cm} \text{(5)}$$

where $b = (\Delta E)^2 / \ln(2)$; $Y_o$ is the number of counts at peak energy $E_o$, and $\Delta E$ is the FWHM of the detector corresponding to the energy $E$.

Then, the Gaussian distribution, $Y(E)$, corresponding to each energy $E$ is calculated using the values of $Y_o$. Then, $Y(E)$ is numerically integrated to obtain the total number of photons at the desired energy. The resulting distribution is the analytically estimated singly scattered intensity consisting of singly scattered photons only.

4 Results and Conclusions

Measurements of the scattered photons are carried out as a function of sample thickness, for the rectangular copper targets of thickness in the range 0.1-3.0 cm. A typical observed experimental spectrum, recorded by NaI(Tl) detector, is obtained by irradiating a typical sample of copper for 10 Ks (curve-a of Fig. 3). Also the background spectrum is recorded for the same period of time to permit registration of events unrelated to target (curve-b of Fig. 3). The observed experimental spectrum obtained by subtracting the events under curve-b from those under curve-a, consists of both singly and multiply scattered events. The subtraction of reconstructed singly scattered spectrum (curve-d of Fig. 3) from this observed experimental spectrum (after subtracting background) in the range 210-330 keV results in multiply scattered photons only. This procedure is then repeated for different thicknesses of the copper sample.
Fig. 3 — (a) An observed experimentally scattered spectrum with copper sample of thickness 10 mm for 10 ks, (b) The background spectrum unrelated to target, (c) Experimentally observed scattered spectrum (after subtracting background) consists of multiply and singly scattered photons, (d) Analytically reconstructed singly scattered spectrum

The plot of observed number of multiply scattered events, having the same energy as in singly scattering, as a function of thickness of the copper sample is shown in Fig. 4. The multiple scattering increases with increase in sample thickness and saturates after a particular value, called the saturation thickness whose measured value (Fig. 4) in the present measurements being 2.6 cm (=1.69 mfp, 1 mean free path equals 1.537 cm in copper for 0.662 MeV incident energy). Initially, the numbers of multiply scattered photons coming out of the scatterer increase with increase in sample thickness because of much greater scattering centres provided for the interaction of incident gamma rays with the sample material. However, after saturation thickness, the number of photons coming out of the scatterer does not increase further with increase in sample thickness because at this value of the thickness, the probability for absorption within the target sample gets enhanced. So a stage is reached when the thickness of the sample becomes sufficient to compensate the increase and decrease of the multiply scattered photons. Hence, the number of multiply scattered photons coming out of the scatterer saturates. The present experiment is simulated with the Monte Carlo package developed by Bauer and Pattison\textsuperscript{12}. The results based upon Monte Carlo calculations for multiply scattered intensity, at scattering angle of 90° for 0.662 MeV incident photons, are also given in Fig. 4. The simulated data of multiply scattered intensity increases with increase in target thickness and saturates at 2.6 cm (Fig. 4) thickness of the copper target. This behaviour supports the present experimental data.

The photons scattered by the sample should have undergone only one elastic collision. But in actual practice, the scattered beam contains invariably photons scattered more than once when the scatterer is of finite dimensions both in depth and lateral extension. So the multiple scattering of photons acts as a noise in Compton profiles and cross-section measurements. Therefore, only the singly scattered photons entering the detector opening are desired and the multiply scattered photons act as noise to the original signal. A parameter called signal-to-noise ratio is plotted as a function of target thickness (Fig. 5) for the copper target. It is clear that when the target thickness is large, the signal to noise ratio is low indicating the presence of more multiply scattered events in comparison to the singly scattered events. Since multiple scattering backgrounds are to be avoided so a high signal-to-noise ratio is must, which is obtained by using thin targets only.

Apart from signal-to-noise ratio, another parameter called multiply scattered fraction (MSF) giving the effect of multiply scattered radiations on the original signal (singly scattered radiations), is given by:
where $N_m$ and $N_s$ are the multiply and singly scattered radiation photon flux, respectively detected by the gamma detector. The singly scattered radiations $N_s$ are calculated by assuming that the scattered radiations from the sample having least thickness (1 mm) are free from multiply scattered contamination and contain only singly scattered events. The variation of MSF against the energy window width $\Delta E$ for different sample thickness values is shown in Fig. 6. It has been found that MSF increases and saturates with increase in energy window width $\Delta E$. This behaviour agrees qualitatively with the work of Barnea et al.\textsuperscript{13}, both experimentally and by Monte Carlo radiation transport calculations, as a function of scattering angle, scattering material and object thickness at 662 keV. Our present measurements also confirm that the number of multiply scattered events, having energy equal to singly scattered events, saturates with increasing sample thickness and a narrow beam incident on thin samples results in higher signal-to-noise ratio. The Monte Carlo calculations also support the present experimental data. The present measurements also support the work carried out by Parmesh et al.\textsuperscript{8} for saturation thickness and Barnea et al.\textsuperscript{13} for multiply scattered fraction. Here, it is also important to note that attempts on this objective have been very rare. So our present findings will serve very good reference for further comparison with experimental data of this process. It is further required to obtain more experimental data on multiply Compton scattering at different incident photon energies in scattering materials having different Z-number for better understanding of this process.

References
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