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Free vibration analysis and optimal design of an adhesively bonded double containment cantilever joint

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This study presents three-dimensional free vibration and stress analyses of an adhesively bonded double containment cantilever joint. Modulus of elasticity, Poisson's ratio and density had negligible effect on natural frequencies and mode shapes of adhesive joint. ANN models predicted that support length and plate thickness played important role on natural frequencies, mode shapes and modal strain energy of adhesive joint whereas adhesive thickness has a minor effect. Genetic Algorithm combined with ANN model determined optimum geometrical dimensions, which are satisfying maximum natural frequency and minimum elastic modal strain energy conditions for each natural frequency and mode shape of the adhesively bonded double containment cantilever joint.

Keywords: ANN, Adhesive joint, Free vibration, Genetic algorithm, Mode shape, Natural frequency, Optimisation

Introduction

Adhesive bonding technique has been used successfully in joining materials[1-3]. Natural frequencies (NFs) and logarithmic decrement[4] depend on modal number and overlap length. He & Rao[5-6] presented an analytical model to study coupled transverse and longitudinal forced vibration of a simply-supported bonded viscoelastic adhesive lap joint under transverse distributed loads considering both shear and thickness deformation in adhesive layer. Rao & Zhou[7] studied effects of structural parameters and mechanical properties of adhesive layer on system modal loss factors and resonance frequencies for transverse vibration and damping of an adhesively bonded tubular lap joint.

Ko et al[8] determined NFs and mode shapes of laminated plates with a lap joint. He & Oyadiji[9] found that mode shapes of lap joint are more pointed for a relatively soft adhesive layer whereas fairly flat for a very stiff adhesive layer. Kaya et al[10] also investigated effects of various parameters on in-plane vibration characteristics of an adhesively bonded single lap joint subjected to two orthogonal in-plane loads. Khalil & Kagho[11] showed that defects caused a reduction in joint stiffness, which leads to a decrease in resonant frequency and an increase in damping capacity. Vaziri & Nayeb-Hashemi[12] investigated effects of tubular joint geometry parameters, material properties and viscoelastic adhesive properties on dynamic response of a tubular adhesive joint subjected to a harmonic axial load and found that system response was sensitive to adhesive loss factor. Yuceoglu et al[13] showed that material characteristics of adhesive layer have significant effect on deformations of bonded multilayer composite plates, their NFs and mode shapes.

This study investigates effects of the properties of adhesive layer and geometric dimensions on free vibrations of adhesively bonded double containment cantilever joint. In addition, an optimal design of this adhesive joint is searched based on first 10 NFs and modal strain energies of adhesive joint using artificial neural networks (ANNs) and genetic algorithm.

Materials and Methods

Joint Configuration and Modal Analysis

NFs and mode shapes of an adhesively bonded double containment cantilever joint were investigated (Fig. 1). Support and horizontal plate are made of aluminum and joined with an epoxy adhesive. Support, adhesive layer

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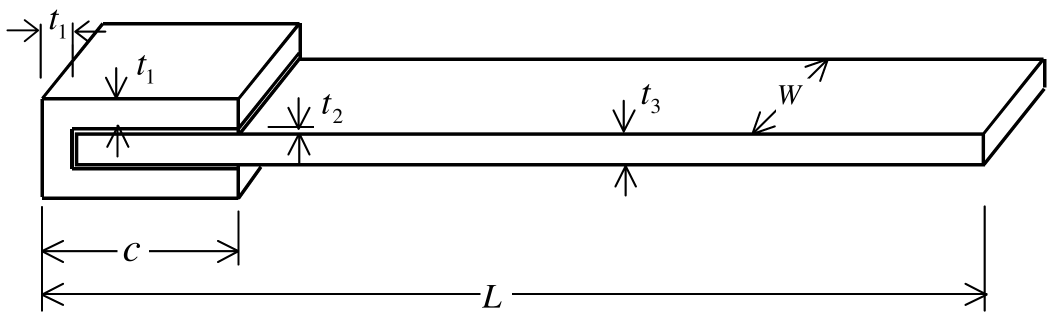

Fig. 1—Dimensions of an adhesively bonded double containment cantilever joint

and horizontal plate are assumed to have linear elastic properties. Aluminum and epoxy adhesive have: modulus of elasticity E, 68.95, 3.33 GPa; Poisson's ratio υ, 0.33, 0.30; and density, 2770, 1200 kg/m^3. A joint length L (140 mm), joint width W (25 mm), plate thickness t_3 (2 mm), adhesive thickness t_2 (0.2 mm), support length c (20 mm) and support thickness t_1 (2 mm) were taken constant. Adhesively bonded double containment cantilever joint was fixed through back surface of the support.

In case structure has constant stiffness, mass effects and no time varying boundary conditions such as forces, displacements, equation of motion for an undamped system is

$$[\mathbf{M}]\{\ddot{\mathbf{u}}\}+[\mathbf{K}]\{\mathbf{u}\}=\{0\} \qquad \ldots(1)$$

where nodal $\{\ddot{\mathbf{u}}\}$ and $\{\mathbf{u}\}$ are acceleration and displacement vectors, $[\mathbf{M}]$ is mass matrix and $[\mathbf{K}]$ is stiffness matrix. Finite element method (FEM)n:[14] yields

$$([\mathbf{K}]-\lambda[\mathbf{M}])\{\bar{\mathbf{D}}\}=\{0\}, \text{ where } \lambda=\omega^2 \qquad \ldots(2)$$

Solution of this eigenvalue problem

$$\det([\mathbf{K}]-\lambda[\mathbf{M}])=\{0\} \qquad \ldots(3)$$

Each eigenvalue λ_i is associated with an eigenvector $\{\bar{\mathbf{D}}\}_i$ which is called a natural mode. The block Lanczos eigenvalue extraction method was used for calculation of eigenvalues and eigenvectors since models have large degree of freedoms[14,15]. A 3D isoparametric structural finite element formulation was implemented to plate, support and adhesive layer[14]. Finite element includes three degree of freedoms (displacements) at each node. Adhesive free edges, in which stress concentrations occurred, were refined to predict accurately strain energy in each element. Since total number of degree of freedoms in FEM affects strongly solution time, which is necessary for eigenvalue extraction and strain energy calculations, a series of finite element model were tested.

Results and Discussion

Free Vibration Analysis (FVA)

First 10 NFs of adhesive joint were considered in FVA. Corresponding mode shapes of adhesive joint can be classified as transverse (bending), torsional and lateral modes. NFs of bending mode shape are 129.08, 790.47, 2138.55, 3853 and 5609.59 Hz and that of torsional mode shapes are 1370.02, 4170 and 7130.80 Hz (Fig. 2). Lateral mode shapes have NFs of 1030.93 and 5412.36 Hz (Fig. 2). First 2 NFs of bending mode (129.07 & 790.47 Hz) are smaller than those of torsional and lateral mode. Consequently, adhesive double containment cantilever joint tends strongly to a bending mode shape. In order to determine effects of mechanical and physical properties of epoxy adhesive on NFs and modal strain energy of adhesive joint, FVA was repeated for modulus ratios E_{ep}/E_{al} = 0.0005-1.0, density ratios ρ_{ep}/ρ_{al} = 0.05-1.0 and Poisson's ratio of epoxy adhesive ν_{ep} = 0.2-0.4. FVA and stress analyses indicated that Poisson's ratio, modulus of elasticity and density of epoxy adhesive have negligible effect on NFs, mode shapes and modal strain energies of double containment cantilever joint with an elastic adhesive layer.

Increasing support length c from 10 to 50 mm caused considerable variations in NFs and modal strain energies of adhesive joint. Thus, increases of 198, 150, 75, 41 and 69% in NFs $\omega_{1,2,5,6,9}$ of bending mode shapes, 104, 87 and 98% in NFs $\omega_{4,7,10}$ of torsional mode shapes and 87 and 20% in NFs $\omega_{3,8}$ of lateral mode shapes occurred as support length is increased. Similarly, corresponding modal strain energies of adhesive joint increased by 207, 192, 177, 87 and 72% for NFs $\omega_{1,2,5,6,9}$ of bending mode

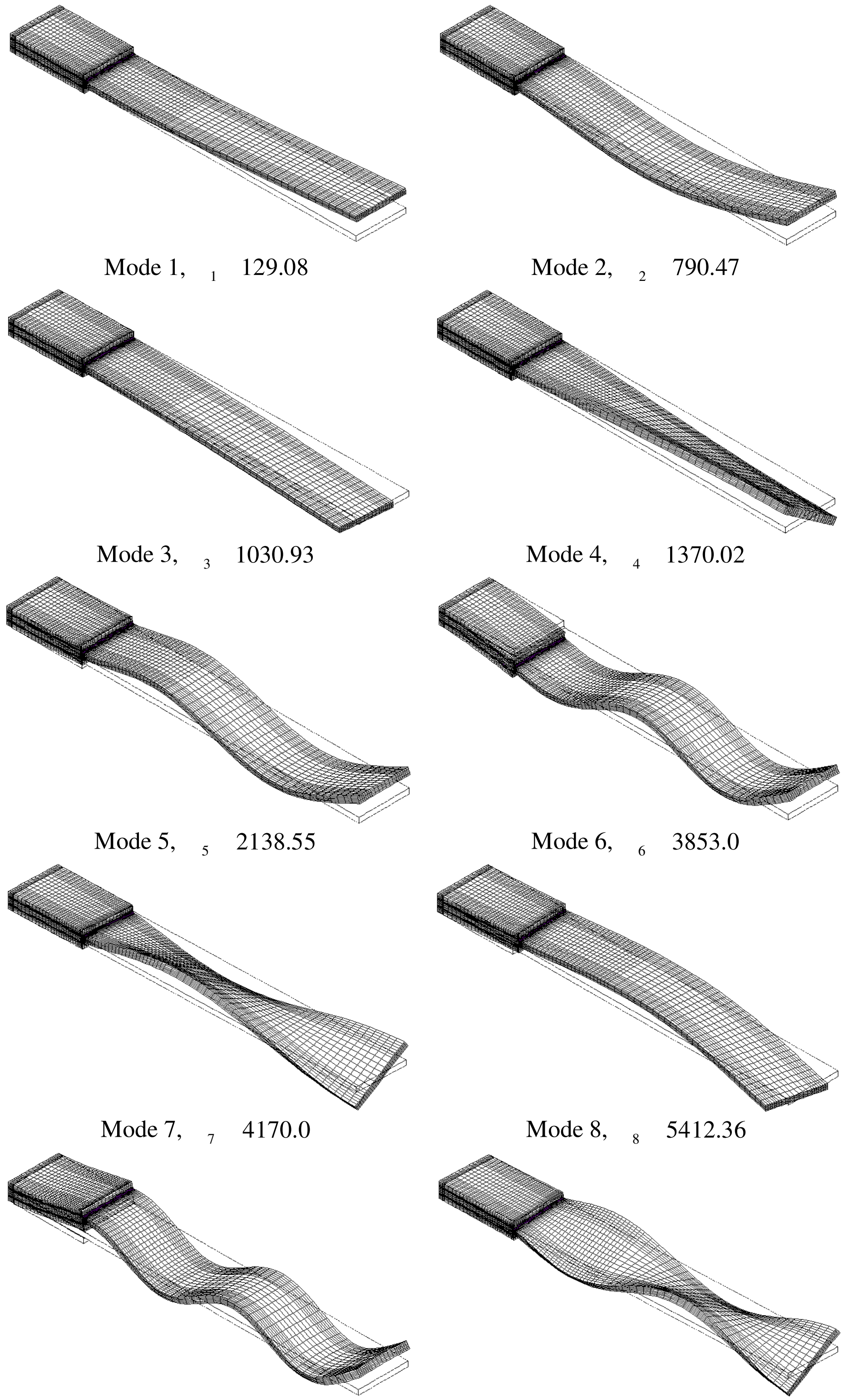

Fig. 2—First ten natural frequencies and mode shapes of adhesively bonded double containment cantilever joint (c = 25, t_2 = 0.5 and t_1 = t_3 = 5 mm)

shapes, 51, 26 and 230% for NFs $\omega_{4,7,10}$ of torsional mode shapes and 94 and 36% for NFs $\omega_{3,8}$ of lateral mode shapes. It is evident that increasing support length causes considerable increases on first 10 NFs and modal strain energies of adhesively bonded double containment cantilever joint. In particular, bending mode shapes of adhesive joint were dominant, and considerable increases in NFs and modal strain energies were observed for bending mode shapes. In practice, maximizing first NF of adhesive joint is a design objective, and FVA shows that a larger support length results in a higher NF, whereas it causes increases in modal strain energy of adhesive joint since containment support has a larger volume and mass and adhesive joint gains a stiffer structure. Since adhesive layer is assumed to be a weak link, adhesive joints are designed such that stress concentrations around adhesive free edges can be reduced by using an optimal support length[1-3]. Thus, support length has an effect of reducing peak adhesive stresses, consequently an effect of increasing joint strength. Increases in modal strain energy of adhesive joint appear since elastic deformations take place in horizontal plate and in containment support rather than in adhesive layer of a stiffer joint.

In order to determine effect of plate thickness on NFs and modal strain energies of adhesive joint, plate thickness t_3 was changed from 1.0 to 5.0 mm. As other dimensions were kept constant, increasing plate thickness caused increases of 1463, 619, 1823, 999 and 1036% in NFs $\omega_{1,2,5,6,9}$ of bending modes, 590, 2028 and 999% in NFs $\omega_{4,7,10}$ of torsional modes, and 614 and 2532% NFs $\omega_{3,8}$ of lateral modes. However, a uniform increase in NFs is observed until a plate thickness of 2.8, 3.2 and 4.1 mm for NFs of modes 2, 6 and 9, respectively, and then plate thickness does not contribute to increase in NFs. A similar effect was observed for modal strain energies for each mode shape of adhesive joint as increases of 24900, 9900, 16567, 8991 and 24900% for NFs $\omega_{1,2,5,6,9}$ of bending modes, 590, 2028 and 999% for NFs $\omega_{4,7,10}$ of torsional modes, and 614 and 2532% for NFs $\omega_{3,8}$ of lateral modes. Plate thickness had a considerable effect of increasing NFs and modal strain energies of adhesive joint. Plate thickness contributed to mass of horizontal plate and affected evidently vibration characteristics of adhesive double containment cantilever joint.

Adhesive thickness t_2 (0.1-0.5 mm) had a negligible effect of increasing NFs of adhesive joint (an increase of 1-1.7%), whereas a minor decrease (3%) was observed in NFs $\omega_{3,8}$ of lateral modes. Since adhesive material is less stiff than aluminum plate and containment support increasing adhesive thickness causes volume and mass of adhesive region in adhesive joint to increase. Consequently, horizontal plate gains more capability to vibrate laterally due to low stiffness of adhesive layer, and a decrease is observed in NFs of lateral mode shapes, which becomes minor because adhesive layer is still thinner than plate and support. Adhesive thickness exhibits a similar behavior for modal strain energy of adhesive joint for each mode shape. Thus, a negligible increase (1-1.5%) is observed whereas a minor decrease (1%) appears in modal strain energies at NFs $\omega_{3,8}$ of lateral modes.

Vibration characteristics and elastic deformation state of adhesive joint are found dependent on all mechanical properties of adhesive layer and geometrical dimensions. However, modulus of elasticity, Poisson's ratio and density of epoxy adhesive result in a negligible effect on NFs and modal strain energies. In addition, as plate thickness and support length are increased, evident increases in NFs and modal strain energies of bending, lateral and torsional modes are observed. However, increasing adhesive thickness exhibits a negligible effect and lateral modes of adhesive joint become more apparent. An optimal design of adhesively bonded double containment cantilever joint can be searched inside specified limits of design variables. In case mechanical properties of adhesive layer are not included due to their negligible effects on both vibration characteristics and modal strain energy, this optimisation procedure requires a search among 125000 models to be solved for design variables, such as a support length $10, 10.1 \le c \le 50\,\mathrm{mm}$, a plate thickness $1.0, 1.1 \le t_3 \le 5.0\,\mathrm{mm}$ and an adhesive thickness $0.1, 0.11 \le t_2 \le 0.5\,\mathrm{mm}$. Since three-dimensional free vibration and stress analyses are carried out using FEM, adhesive joint is to be divided into a number of small volume elements, which are connected at the corner and side nodes with three degree of freedoms. In addition, first ten eigenvalues of each model are extracted, and

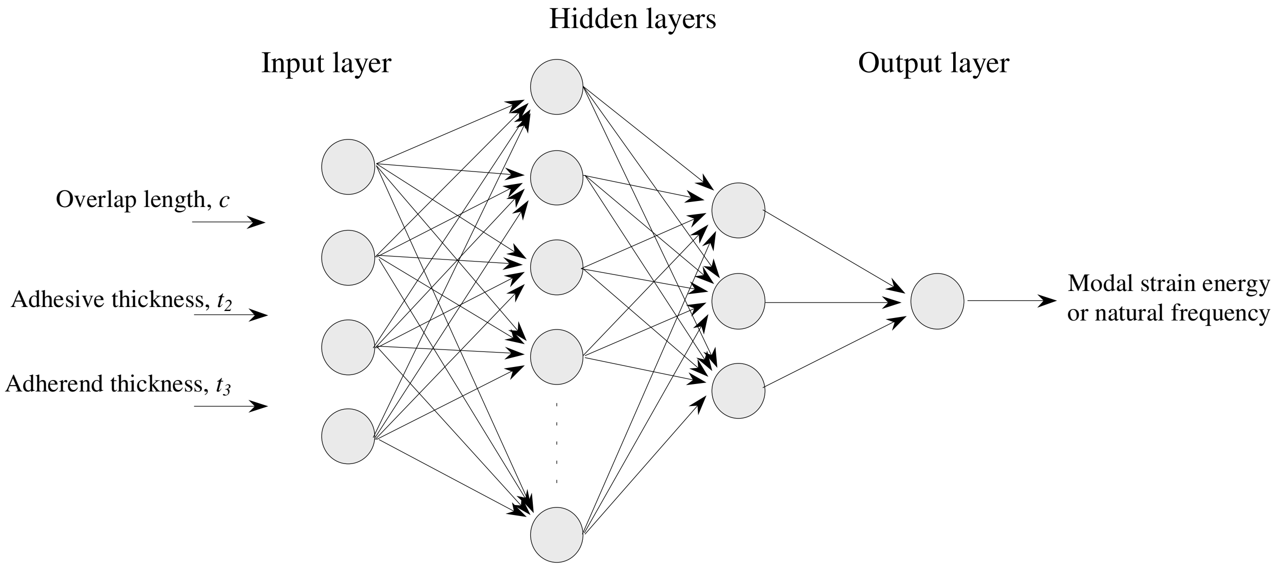

Fig. 3—Architecture of ANN model with two hidden layers

corresponding mode shapes of each NF and subsequent stress and strain distributions are calculated. Therefore, direct search of optimal design of adhesive joint requires long computation time, and is not efficient. Artificial neural networks (ANNs) could easily predict value of objective function without solving directly the problem for specified variables.

Artificial Neural Networks (ANNs)

NFs and corresponding modal strain energies of an adhesively bonded double containment cantilever joint will be estimated using a proposed algorithm of neural network (NN) model. For input and output data for free vibration of adhesive joint NN model, adhesive joint was clamped through back surface of its double containment support and free vibration and stress analyses were carried for support length $10 \le c \le 50\,\text{mm}$, adhesive thickness $0.1 \le t_2 \le 0.5\,\text{mm}$, and plate thickness $1.0 \le t_3 \le 5.0\,\text{mm}$, whereas mechanical properties of plate, support and epoxy adhesive are constant. These design variables were generated randomly between their lower and upper bounds. First 10 NFs of each new joint model were extracted and their corresponding strain energies were calculated using FEM. Design variables and analysis results were used as input for training season of NN. Two training seasons were carried out for NFs and modal strain energy. Input of NN is a random set of support length, plate thickness and adhesive thickness for each training season, whereas output of NN model is first 10 NFs and corresponding modal strain energies. This study uses Levenberg-Marquardt (LM) back-propagation learning algorithm[16] in a feed-forward, two hidden layers network.

Tangent sigmoid transfer function

$$f(x) = \frac{2}{1+e^{-2x}} - 1 \qquad \ldots(4)$$

Logistic sigmoid transfer function $f(x) = \dfrac{1}{1+e^{-x}}$

…(5)

Eq. (4) and Eq. (5) are used as activation functions of hidden layers and output layer, respectively. Values of training and test data were normalized to a range of 0-1. LM Back-Propagation training[16] was applied until satisfactory training is achieved. NN toolbox of MATLAB[19] was used to develop proposed model. Input data set includes 4100 patterns based on free vibration and stress analyses of adhesively bonded double containment cantilever joint for random values of present design variables. The 4000 patterns of input data were used for training NN model and 100 patterns selected randomly from this set were used as testing patterns to check correctness of predicted output by NN model.

Based on tests of a series of NN models, layer configuration composed of 4, 10, 3 and 1 neurons in each layer (1-4) appeared as an optimal topology of NNs, which is fast and predicts accurately NFs and strain energies (Fig. 3). In training phase, network was trained with values of input(s) and output(s) to adjust connection

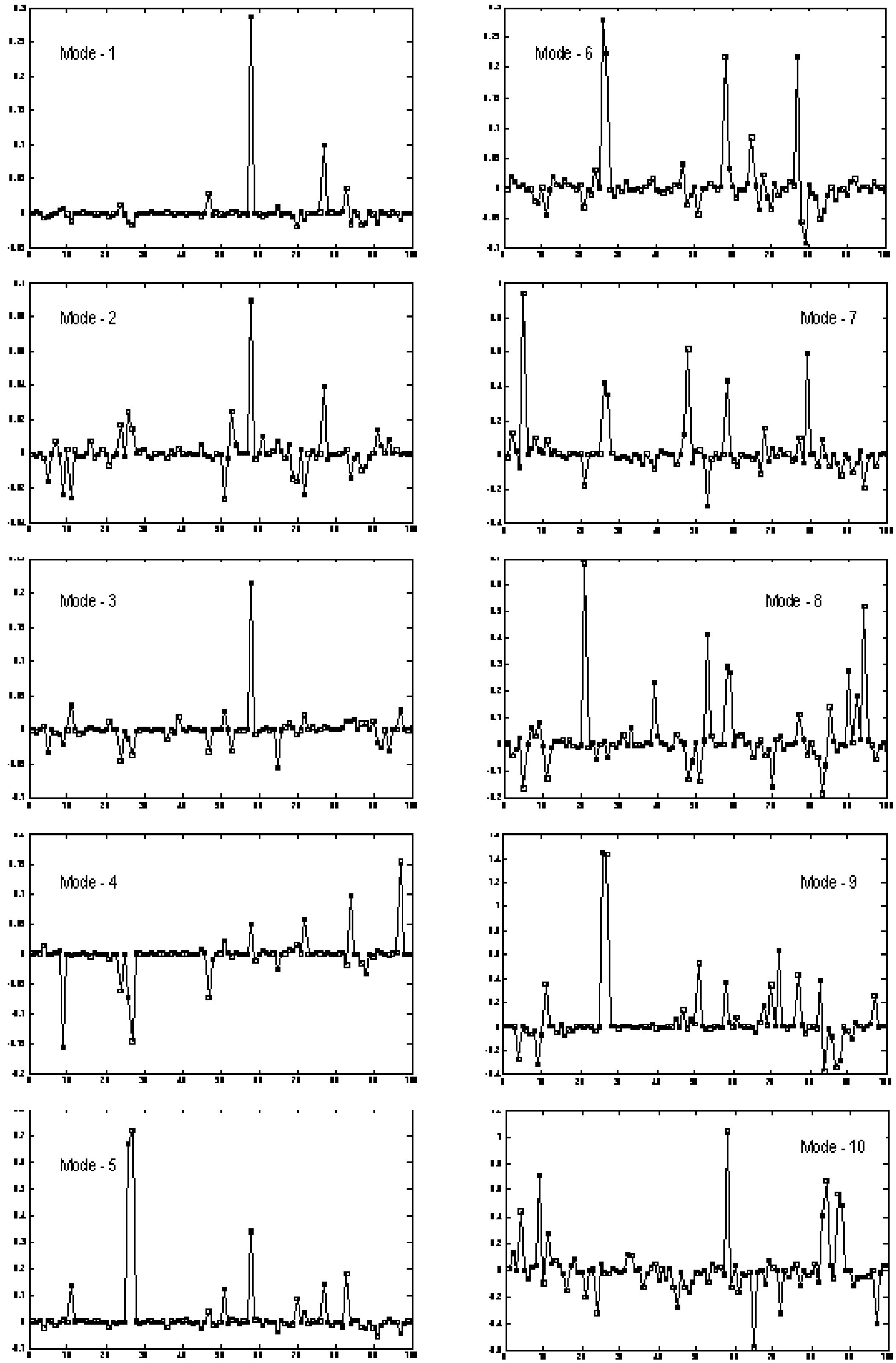

Fig. 4—Percentage error levels between natural frequencies of ANN and FEM for each of first ten modes of adhesively bonded double containment cantilever joint

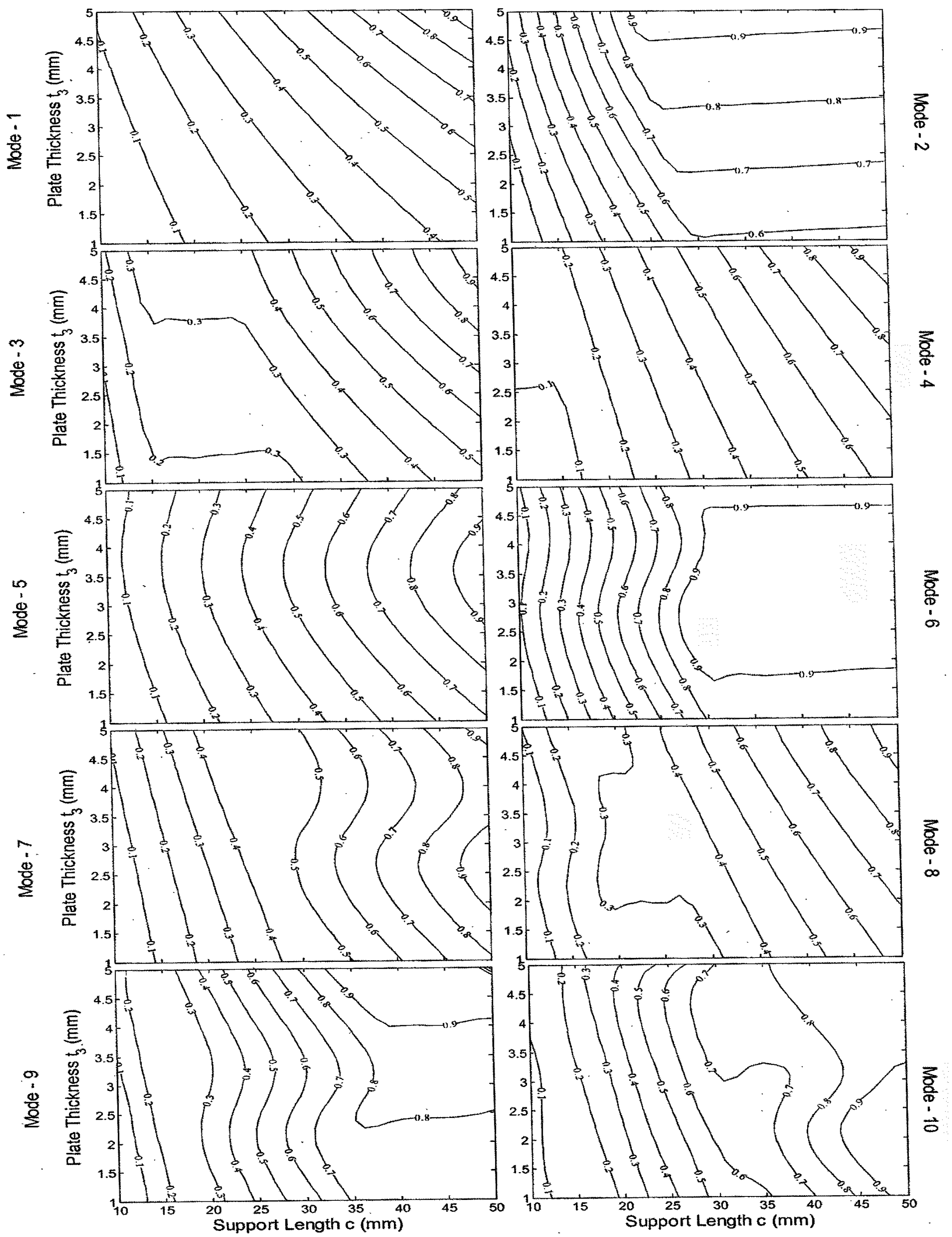

Fig. 5—Effects of plate thickness and support length on first ten normalized natural frequencies of adhesively bonded double containment cantilever joint

weights between input layer and hidden layer and between hidden layer and output layer, and then this procedure was repeated until outputs generated by network achieve an acceptable accuracy. The performance function was selected as mean square error (MSE) between outputs predicted by network and test target outputs. Training procedure elapsed a CPU time of 10 min on a PIV processor having a 3.0 Mhz CPU speed and 1 Gbyte Ram for a training cycle of 500 (epochs). MSE of training data could be reduced to an order of $10^{-8} – 10^{-5}$ depending on mode shape of adhesive joint. Comparison of NFs predicted by ANN model with

Table 1—Optimum overlap length c, adhesive thickness t_2 and plate thickness t_3 for each mode of adhesive joint (all dimensions in mm)

Modes	1	2	3	4	5	6	7	8	9	10
c	50.0	50.0	46.2	50.0	31.1	25.3	50.0	49.5	50.0	44.9
t_2	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
t_3	3.0	1.0	3.3	3.0	3.6	2.7	3.8	4.2	3.3	2.7

testing data shows that NN model was capable of predicting both NFs and strain energies accurately (Fig. 4).

Genetic Algorithms

Evolutionary computing[18] has become a growing area of artificial intelligence. Holland[19] invented and developed Genetic algorithms (GAs) as a part of evolutionary computing. Koza[20] first used to evolve programs to perform certain tasks. During solution of optimisation problem, an evolutionary process is carried out, which resuls in a best (fittest) solution (survivor). Genetic programming[18-20] requires evolution toward better solutions of a population of abstract representations (chromosomes) of candidate solutions (individuals) to an optimisation problem.

Optimal Joint Design

FVA of adhesively bonded double containment cantilever joint showed that bending mode shapes were dominant for low fundamental frequencies, and modulus of elasticity, Poisson's ratio and density of epoxy adhesive were of a negligible effect on NFs of adhesive joint. In addition, support length and plate thickness have a considerable effect on both NFs and strain energies of each mode whereas increasing adhesive thickness exhibits a minor effect. In order to determine couple effect of both plate thickness and support length on NF and modal strain energy, NN models were used.

First 10 NFs of adhesive joint were predicted by using NN model for various support lengths ($10 \le c \le 50$ mm) and plate thicknesses ($1.0 \le t_3 \le 5.0$ mm) whereas other design parameters were taken constant. These NFs were normalized with maximum NF value among predicted output data for each mode, and variations of normalized NFs are shown for all modes (Fig. 5). NFs exhibit uniform increases as support length and plate thickness are increased. For flexural modes 5, 6 and 9, plate thickness loses slightly its effect on NFs as plate thickness is increased, whereas support length is taken constant. However, both plate thickness and support length exhibit a similar effect of increasing NFs for flexural modes 1 and 2. After a support length of 35 mm, increasing plate thickness has a minor effect. For torsional modes 4, 7 and 10, plate thickness exhibits a minor effect of increasing NFs as support length is kept constant. This effect is evident especially for modes 7 and 10. Increasing plate thickness has a minor effect after a support length of 35 mm; thus, NFs of torsional modes do not increase. On the contrary, plate thickness corresponding to smaller support lengths (< 30 mm) has minor effect on NFs for lateral modes 3 and 8. ANN models indicate that support length is to be maximal for any plate thickness, or vice versa in order to maximize present 10 NFs (Fig. 5). In addition, strain energy levels of adhesive joint were predicted using trained NNs for each mode as both plate thickness and support length are increased. However, modal strain energy distributions are similar to those of normalized NFs as support length and plate thickness are varied in design ranges.

Trained NN models give accurate predictions promptly for NFs and strain energies, whereas FEM requires a new time-consuming calculation phase for each new set of design variables. Consequently, trained NNs were used to predict value of fitness function for random values of design variables generated by GA. GA toolbox of MATLAB[17] was used to solve optimisation problem based on free vibration characteristics of adhesively bonded double containment cantilever joint, defined as

$$f(\omega_i, U_i) = c_1 \times \omega_i - c_2 \times U_i \rightarrow \max$$

Table 2—Optimum adhesive thicknesses t_2 and plate thicknesses t_3 for various support lengths c for each mode of the adhesive joint

(mm)	Mode 1		Mode 2		Mode 3		Mode 4		Mode 5		Mode 6		Mode 7		Mode 8		Mode 9		Mode 10	
c	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3	t_2	t_3
10.0	0.10	3.60	0.10	2.89	0.10	4.11	0.10	3.04	0.10	4.13	0.10	3.03	0.50	3.85	0.10	4.71	0.10	3.48	0.50	3.07
15.0	0.10	3.54	0.10	2.83	0.10	4.02	0.10	3.04	0.10	4.04	0.14	2.95	0.10	2.79	0.10	4.64	0.10	3.40	0.10	3.07
20.0	0.10	3.48	0.10	2.78	0.10	3.93	0.10	3.04	0.10	3.95	0.23	2.84	0.10	4.90	0.20	4.76	0.10	3.33	0.10	3.05
25.0	0.10	3.42	0.39	2.65	0.10	3.85	0.10	3.04	0.29	3.83	0.50	2.72	0.10	2.72	0.10	4.47	0.10	3.44	0.10	3.01
30.0	0.10	3.35	0.41	2.56	0.50	3.77	0.10	3.04	0.37	3.67	0.50	2.59	0.10	2.62	0.10	4.64	0.10	3.77	0.10	2.95
35.0	0.43	3.25	0.44	2.47	0.49	3.66	0.10	3.04	0.46	3.39	0.16	2.55	0.10	2.49	0.10	4.49	0.10	3.73	0.13	4.00
40.0	0.45	3.16	0.48	2.38	0.49	3.53	0.10	3.04	0.50	3.04	0.10	2.97	0.50	2.33	0.10	4.43	0.10	3.66	0.10	4.06
45.0	0.50	3.08	0.50	2.30	0.50	3.43	0.10	3.04	0.10	3.04	0.12	2.93	0.16	3.95	0.10	4.24	0.10	3.49	0.14	2.90
50.0	0.50	3.00	0.50	2.23	0.50	3.29	0.10	3.03	0.10	3.21	0.10	2.68	0.17	3.95	0.10	3.97	0.10	3.30	0.11	3.10

Table 3—Optimum support lengths c and plate thicknesses t_3 for various adhesive thicknesses t_2 for each mode of the adhesive joint

(mm)	Mode 1		Mode 2		Mode 3		Mode 4		Mode 5		Mode 6		Mode 7		Mode 8		Mode 9		Mode 10	
t_2	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3	c	t_3
0.10	50.00	3.01	46.89	2.35	44.64	3.43	50.00	3.03	29.82	3.70	23.12	2.76	50.00	3.95	48.57	4.05	50.00	3.30	49.14	3.04
0.15	50.00	3.01	47.28	2.34	44.87	3.42	50.00	3.02	29.85	3.70	23.35	2.76	50.00	3.95	48.13	4.06	50.00	3.30	41.46	2.87
0.20	50.00	3.01	47.69	2.33	45.11	3.41	50.00	3.02	29.93	3.69	23.57	2.75	50.00	3.95	47.75	4.06	50.00	3.30	41.39	2.87
0.25	50.00	3.01	48.07	2.32	45.34	3.40	50.00	3.02	30.03	3.68	23.77	2.75	50.00	3.95	47.38	4.06	50.00	3.30	41.28	2.86
0.30	50.00	3.01	48.43	2.31	45.57	3.39	50.00	3.01	30.14	3.67	23.94	2.75	50.00	3.94	47.00	4.05	50.00	3.30	41.19	2.86
0.35	50.00	3.01	48.83	2.30	45.80	3.38	50.00	3.01	30.23	3.66	24.09	2.74	50.00	3.94	46.60	4.04	50.00	3.30	41.02	2.86
0.40	50.00	3.00	49.38	2.28	46.04	3.36	50.00	3.01	30.31	3.65	24.22	2.74	50.00	3.94	46.14	4.04	50.00	3.30	38.54	2.81
0.45	50.00	3.00	50.00	2.25	50.00	3.27	50.00	3.01	30.38	3.64	24.33	2.74	50.00	3.93	45.59	4.04	50.00	3.30	37.98	2.82
0.50	50.00	3.00	50.00	2.23	50.00	3.29	50.00	3.01	30.44	3.63	24.45	2.73	50.00	3.93	44.89	4.05	50.00	3.29	37.37	2.82

Table 4—Optimum support lengths c and adhesive thicknesses t_2 for various plate thicknesses t_3 for each mode of the adhesive joint

(mm)	Mode 1		Mode 2		Mode 3		Mode 4		Mode 5		Mode 6		Mode 7		Mode 8		Mode 9		Mode 10	
t_3	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2	c	t_2
1.0	50.00	0.50	50.00	0.50	50.00	0.50	45.26	0.50	40.54	0.50	30.92	0.50	50.00	0.50	50.00	0.10	50.00	0.50	50.00	0.12
2.0	50.00	0.50	50.00	0.50	50.00	0.10	50.00	0.10	36.66	0.50	50.00	0.50	50.00	0.36	50.00	0.10	50.00	0.10	43.71	0.10
3.0	50.00	0.50	50.00	0.50	50.00	0.50	50.00	0.10	32.95	0.50	12.39	0.10	50.00	0.28	50.00	0.10	50.00	0.10	26.17	0.10
4.0	50.00	0.32	35.00	0.35	45.79	0.50	50.00	0.10	28.74	0.29	19.43	0.17	50.00	0.17	48.88	0.10	45.73	0.10	40.90	0.11
5.0	17.01	0.10	11.81	0.10	31.11	0.10	50.00	0.10	23.39	0.10	12.59	0.10	50.00	0.11	43.50	0.10	33.34	0.10	10.01	0.10

where $c_1 = c_2 = 0.5$, ω_i is i th NF, and U_i is corresponding modal strain energy, subject to $10 \le c \le 50\,\text{mm}$, $1.0 \le t_3 \le 5.0\,\text{mm}$ and $0.1 \le t_2 \le 0.5\,\text{mm}$.

GA and NNs find optimal support lengths, adhesive thicknesses and plate thicknesses (Table 1) for each mode of adhesively bonded double containment cantilever joint. Support length c takes values (44.9 - 50 mm) close to its upper bound for bending modes (1, 2, 9), lateral modes (3, 8) and torsional modes (4, 7 and 10). Adhesive thickness t_2 is maximised to its upper bound of 0.5 mm. On the contrary, plate thickness t_3 is in a range of 3.0 - 3.8 (Fig. 5). In practice, optimal plate thickness, support length and adhesive thickness values (Table 1) may not be convenient. Thus, designers may need other dimensions of optimal adhesive joint design corresponding to a specified value of design parameters. Tables 2-4 show optimal values of adhesive thickness, support length and plate thickness, whereas other dimensions are kept constant. Support length appears dominant among design parameters followed by design parameter, whereas adhesive thickness has a minor effect.

Conclusions

FVA of an adhesively bonded double containment cantilever joint was carried out and an optimal design of adhesive joint was searched. Adhesive joint exhibited flexural (bending), lateral and torsional mode shapes among first 10 NFs. Bending mode shapes were dominant. Modulus of elasticity, Poisson's ratio and density had minor effect on NFs and on modal strain energies. Increasing support length and plate thickness resulted in considerable increase in NFs and modal strain energies, whereas adhesive thickness had a minor effect. GA was implemented successfully for determining an optimal joint for each mode, and ANNs were used to predict accurately all data, such as NFs and strain energies, which are used in optimisation problem since FEM requires time-consuming computations. Support length and plate thickness are dominant design parameters, which maximize NFs and minimize modal strain energies.

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