Journal of Scientific & Industrial Research
Vol. 67, October 2008, pp.759-773

Optimal control of a fed-batch fermenter using parameterized data-driven models[#]

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Received 20 November 2007; revised 11 July 2008; accepted 18 July 2008

An optimal control approach is proposed for semi-batch processes based on parameterized data-driven (PDD) model structures. Orthonormally parameterized input trajectories, initial states and process parameters are inputs to the model, which predicts output trajectories in terms of Fourier coefficients. Two model structures (linear and quadratic) are incorporated into PDD modeling approach, and a previously proposed model structure of artificial neural networks (ANNs) is considered for comparison. Proposed PDD modeling approach using newly proposed model structures is capable of capturing nonlinear and time-varying behavior inherent in fed-batch systems fairly accurately, and results of operating trajectory optimization using all models are found to be comparable to the results obtained using exact first principles model.

Keywords: Artificial neural networks, Fed-batch fermentation, Linear model, Optimal control, Orthonormal polynomial approximations, Parameterized data-driven modeling, Quadratic model

Introduction

A shift in the focus of chemical industry from mass production of low value products to small-scale production of high value or specialty chemicals has generated more interest in optimal operation of batch or semi-batch processes because even a marginal improvement in product yield can lead to a considerable improvement in profitability[1]. Most of the available optimization and model based control schemes for batch / semi-batch reactors have been formulated based on mechanistic models[2]. Control approaches for fermenters have been categorized[3] as open-loop trajectory tracking, singular control and other optimal control approaches. Development of optimal control approaches for fed-batch fermentation processes include genetic algorithm based optimization method for single-objective[4-6] and multi-objective problems[7], differential evolution based optimization methods[8] and hybrid method of evolutionary algorithms[9]. From a practical viewpoint, simplified data-driven nonlinear models can provide an attractive alternative for formulating fed-batch fermentation optimization and control problems. The use of neural networks (NNs) either directly or as hybrid models in combination with first principles models have been discussed[3] mainly for control of fermentation processes. Krothapally & Palanki[10] have proposed the use of feed forward NNs for end point optimization, where end-point optimization is solved using first principles model for a range of model parameters; this data is used to train a network, which is then used to generate a new optimal solution for a new set of model parameters. Van Can *et al*[11-12] have developed a gray-box model combined with macroscopic balances for bioprocesses. Costa *et al*[13] employed NNs combined with process theoretical knowledge for fed-batch fermenter optimization and control. Schenker & Agrawal[14] used a combination of two NNs with a mechanistic model in a model predictive control framework for semi-batch reactors.

Tian *et al*[15] proposed the use of an augmented recurrent NN model for optimal control of a fed-batch bioreactor using successive quadratic programming. Chen *et al*[16] used genetic algorithms to find optimal control policies using a recurrent NN model. Another approach[17] employed NN model for re-optimization for generation of optimal input policy for a fed-batch fermentation process. In all these NN models, a difference equation structure approach assumes a stationary relationship between inputs and outputs, which is valid in continuous

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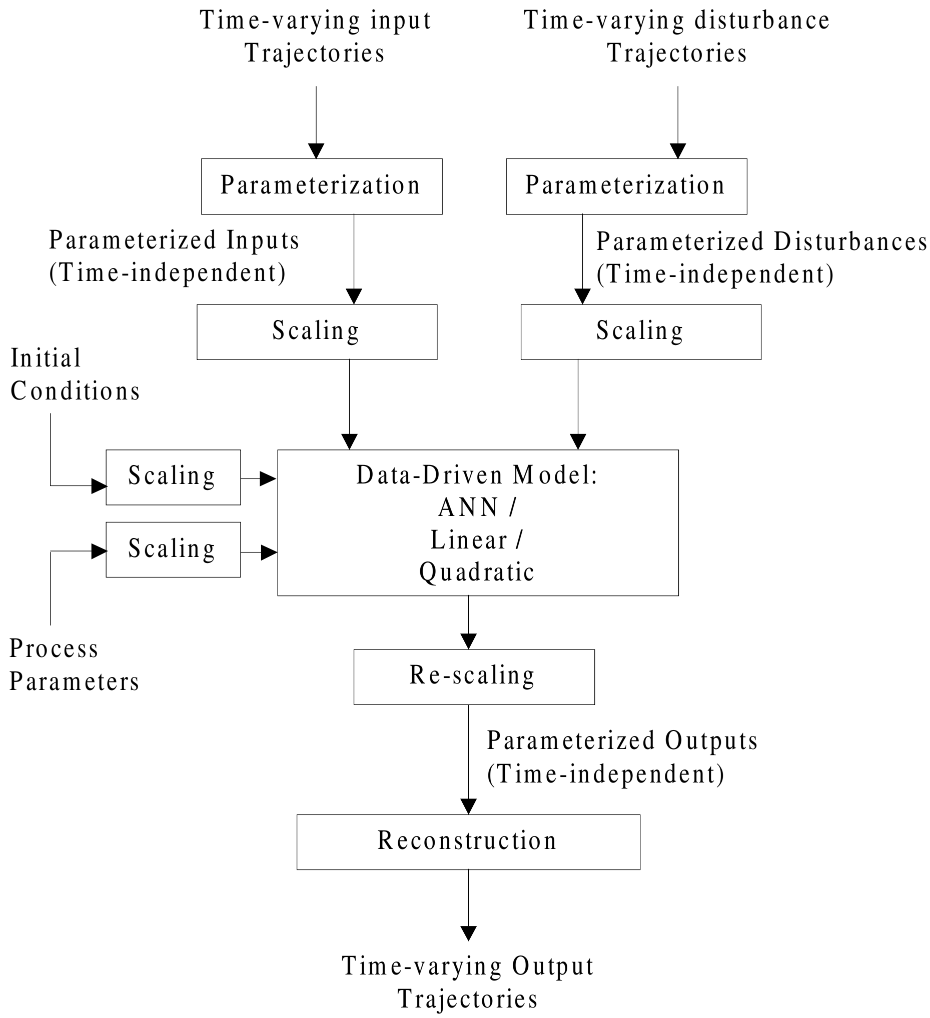

Fig. 1—Parameterized Data-Driven (PDD) model structure for a batch / semi-batch process

processes, but becomes questionable in batch or semi-batch processes. Rani & coworkers[18-21] proposed a novel modeling approach, by proposing a parameterized data-driven (PDD) model for semi-batch processes (Fig. 1).

In present study, modeling approach proposed by Rani & Patwardhan[18] is further extended to incorporate data-driven model structures other than NNs. Parameterized input trajectories, initial states and process parameters are considered as inputs to data-driven models, whereas parameterized output trajectories are outputs. Two model structures (linear and quadratic) have been considered in addition to ANN model. The models so developed are incorporated into an optimal control problem formulation and their performance is evaluated by comparison with ANN model based approach[18] as well as to the exact model based approach for a typical fed-batch fermenter simulation case study.

Proposed Model Structures

Consider a general multiple-input multiple-output (MIMO) lumped parameter process modeled by a vector differential equation as

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x},\mathbf{u},\mathbf{d},\mathbf{p}) \quad , \ \mathbf{x}(t_0) = \mathbf{x}_0 \qquad \ldots(1)$$

where $\mathbf{x}(t) \in R^n$ represent system states, $\mathbf{u}(t) \in R^m$ represent manipulated inputs, $\mathbf{d}(t) \in R^d$ represent disturbances for $t \in [t_0, t_f]$, $\mathbf{p} \in R^p$ represent model parameters and '$\mathbf{f}$' represents an n-dimensional function vector. The system outputs, $\mathbf{y} \in R^r$, are given as

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t)) \qquad \ldots(2)$$

where '$\mathbf{g}$' represents r-dimensional function vector mapping states to measured outputs. The model describing above system can be represented using an algebraic map $\Psi\{.\}$ as

$$\mathbf{y}(\Im_t) = \Psi[\mathbf{y}_0, \mathbf{u}(\Im_t), \mathbf{d}(\Im_t), \mathbf{p}] \quad \ldots(3)$$

where $\Im_t \equiv \{\tau : t_0 \le \tau \le t\}$ and y_0 denote initial value of output vector corresponding to initial state vector x_0. Output [$\mathbf{y}(\Im_t)$], input [$\mathbf{u}(\Im_t)$] and disturbance [$\mathbf{d}(\Im_t)$] profiles over time interval [$t_0 \le \tau \le t$] are further approximated after appropriate transformation of time coordinates as finite dimensional vectors using orthonormal polynomial functions over time interval [0, 1] as

$$u_i(t) \approx \sum_{j=1}^{s_i} a_{ij}\phi_j(t) \qquad i = 1,\ldots,m \quad \ldots(4)$$

$$d_i(t) \approx \sum_{j=1}^{q_i} b_{ij}\phi_j(t) \qquad i = 1,\ldots,d \quad \ldots(5)$$

$$y_k(t) \approx \sum_{j=1}^{l_k} c_{kj}\phi_j(t) \qquad k = 1,\ldots,r \quad \ldots(6)$$

where $\phi_j(t)$ represents j^{th} polynomial basis function, a_{ij}, b_{ij} and c_{kj} represent polynomial coefficients for input, disturbance and output trajectories respectively, s_i, q_i, and l_k denote number of polynomial basis functions employed to approximate input, disturbance and output trajectories respectively.

Three different model structures [one proposed in an earlier study[18], while two new structures (linear and quadratic forms)] are considered in this study to approximate nonlinear function in PDD model. In all three models, prior to model development, input data are scaled to a range of –5 to +5, whereas parameterized output trajectories are scaled to a range of 0 to 1.

Artificial Neural Networks (ANN)

An ANN map, which relates input trajectories (**U**), disturbance trajectories (**D**), initial states ($\mathbf{x}_0$) and certain process parameters (**p**) to output trajectories (**Y**), is constructed.

$$\mathbf{Y} = \Omega[\mathbf{U}, \mathbf{D}, \mathbf{x_0}, \mathbf{p}] \quad \ldots(7)$$

where, $\mathbf{Y} = [\mathbf{Y}_1^T, \mathbf{Y}_2^T, \ldots, \mathbf{Y}_r^T]^T$, $\mathbf{U} = [\mathbf{U}_1^T, \mathbf{U}_2^T, \ldots, \mathbf{U}_m^T]^T$, $\mathbf{D} = [\mathbf{D}_1^T, \mathbf{D}_2^T, \ldots, \mathbf{D}_d^T]^T$,

$\mathbf{U}_i \equiv [a_{i1}\; a_{i2} \ldots . a_{is_i}]^T \quad i = 1,\ldots,m$,

$\mathbf{D}_i \equiv [b_{i1}\; b_{i2} \ldots . b_{iq_i}]^T \quad i = 1,\ldots,d$,

$\mathbf{Y}_k \equiv [c_{k1}\; c_{k2} \ldots . c_{kl_k}]^T \quad k = 1,\ldots,r$

$\Omega(.)$ represents ANN map that approximates operator $\Psi\{.\}$ in PDD model, Eq. (3). Resulting ANN based model can be trained using database containing several batch or semi-batch runs. Two different types of systems can be modeled using proposed modeling approach : i) single-rate systems, where sampling is sufficiently fast and elements of finite dimensional input vector are Fourier coefficients (FCs) as defined above; and ii) multi-rate systems, where sampling of input moves is at a slower frequency than outputs and elements of finite dimensional input vector are piecewise constant manipulated input values at input sampling instants defined as $\mathbf{U}_i \equiv [u_{i,1}\; u_{i,2} \ldots . u_{i,s_i}]^T$.

Linear Model

In linear model structure, for single-rate case, **U**, **D**, $\boldsymbol{x}_0$ and $\boldsymbol{p}$ are related to **Y** as

$$\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{b} \quad \ldots(8)$$

where **X** and **Y** are defined as

$$\mathbf{X} = [\mathbf{U}, \mathbf{D}, \mathbf{x}_0, \mathbf{p}]^T \quad \ldots(9)$$

X is a vector of dimension *nx* x 1 and coefficient matrix **A** and bias vector **b** are of dimensions *ny* x *nx* and *ny* x 1, respectively. Dimensions *ny* and *nx* are defined as

$$ny = \sum_{k=1}^{r} l_k \quad \ldots(10)$$

$$nx = \left(\sum_{i=1}^{m} s_i\right) + \left(\sum_{i=1}^{d} q_i\right) + n + p \quad \ldots(11)$$

Definitions of $\mathbf{U}_i$ for single-rate and multi-rate cases are as defined earlier.

Quadratic Model

Second alternate model for incorporation into PDD model structure is a quadratic model, which is defined for single rate case as

$$\mathbf{Y} = \mathbf{AX}_1 + \mathbf{BX}_2 + \mathbf{c} \qquad \ldots(12)$$

where,

$$\mathbf{X}_1 = \mathbf{X} = [\mathbf{U}, \mathbf{D}, \mathbf{x}_0, \mathbf{p}]^T = [[\mathbf{U}_1^T \ldots \mathbf{U}_m^T][\mathbf{D}_1^T \ldots \mathbf{D}_d^T][x_{10} \ldots x_{n0}][p_1 \ldots p_p]]$$
$$= [\{[a_{11} \ldots a_{1s_1}] .. [a_{m1} \ldots a_{ms_m}]\}\{[b_{11} \ldots b_{1t_1}] .. [b_{d1} \ldots b_{dt_d}]\}\{x_{10} \ldots x_{n0}\}\{p_1 \ldots p_p\}] \qquad \ldots(13)$$

and

$$\mathbf{X}_2 = [\{[a_{11}^2 .. a_{1s_1}^2] .. [a_{m1}^2 .. a_{ms_m}^2]\}\{[b_{11}^2 .. b_{1t_1}^2] .. [b_{d1}^2 .. b_{dt_d}^2]\}\{x_{10}^2 .. x_{n0}^2\}\{p_1^2 .. p_p^2\}] \qquad \ldots(14)$$

Elements of $\mathbf{X}_1$ are same as those in $\mathbf{X}$ in linear model, whereas elements of vector $\mathbf{X}_2$ are squares of each element of $\mathbf{X}_1$. Dimensions of **A**, **B** and **c** are *ny* x *nx*, *ny* x *nx* and *ny* x 1, respectively. In multi-rate case, Eq.(12) remains same, whereas Eqs (13) and (14) are modified as

$$\mathbf{X}_1 = \mathbf{X} = [\mathbf{U}, \mathbf{D}, \mathbf{x}_0, \mathbf{p}]^T = [[\mathbf{U}_1^T .. \mathbf{U}_m^T][\mathbf{D}_1^T .. \mathbf{D}_d^T][x_{10} .. x_{n0}][p_1 .. p_p]]$$
$$= [\{[u_{1,1} .. u_{1,s_1}] .. [u_{m,1} .. u_{m,s_m}]\}\{[b_{11} .. b_{1t_1}] .. [b_{d1} .. b_{dt_d}]\}\{x_{10} .. x_{n0}\}\{p_1 .. p_p\}] \qquad \ldots(15)$$

and

$$\mathbf{X}_2 = [\{[u_{1,1}^2 .. u_{1,s_1}^2] .. [u_{m,1}^2 .. u_{m,s_m}^2]\}\{[b_{11}^2 .. b_{1t_1}^2] .. [b_{d1}^2 .. b_{dt_d}^2]\}\{x_{10}^2 .. x_{n0}^2\}\{p_1^2 .. p_p^2\}] \qquad \ldots(16)$$

Difference between Eqs (13) – (14) and (15) – (16) is only in the definition of elements of Au:$\mathbf{U}_i$.

Optimal Control using Data-driven Models

Batch / semi-batch optimization problem was formulated using proposed PDD modeling approach (single-rate as well as multi-rate) data-driven models as

$$\max_{\mathbf{U}, \mathbf{p}} \; I = G[\mathbf{y}(t_f)] + \int_0^{t_f} F(\mathbf{y}(t), \mathbf{u}(t), \mathbf{p})\, dt \qquad \ldots(17)$$

subject to

$$\mathbf{Y} = \Psi[\mathbf{x_0}, \mathbf{U}, \mathbf{D}, \mathbf{p}] \qquad \ldots(18)$$

$$\mathbf{g}(\mathbf{y}(t), \mathbf{u}(t), \mathbf{p}) \le \mathbf{0}$$
$$\mathbf{h}(\mathbf{y}(t), \mathbf{u}(t), \mathbf{p}) = \mathbf{0} \qquad \ldots(19)$$

where optimization is carried out in terms of coefficient vector U and model parameter vector p for a given initial state x_0. In multi-rate model, vector U denotes manipulated input levels. This problem is a finite dimensional optimization problem subject to algebraic equality and inequality constraints and can be solved by standard nonlinear programming method, like successive quadratic programming.

While performing iteration, for a given guess of input polynomial coefficients (or discrete input levels), say $U^{(k)}$, input trajectory vector $[u(t)]^{(k)}$ can be generated as a polynomial (or piecewise constant) approximation and resulting output trajectory vector $[y(t)]^{(k)}$ can be generated as a polynomial approximation from output polynomial coefficients computed using PDD model, and used to compute objective function values $I^{(k)}$ and for checking constraint violations. Some of typical inequality constraints encountered are upper and lower bounds on input trajectories,

$$u_i^{\min} \le u_i(t) \le u_i^{\max} \qquad i = 1, \ldots, m \qquad \ldots(20)$$

Since $u_i(t)$ is a continuous function, it is difficult to deal with above constraints directly in single-rate model. However, when batch or semi-batch system is computer controlled, resulting optimal input trajectory is discretized and implemented as a piecewise constant trejectory. Thus, it is sufficient to check constraints violations only at each sampling instant and Eq.(20) becomes

$$u_i^{\min} \le u_i(t_k) \le u_i^{\max} \qquad k = 1, \ldots, \left(\frac{t_f}{T_s}\right) \qquad \ldots(21)$$

This results in a finite number of constraints on input coefficients. Further, in fed-batch fermentation systems, volume constraints exist since product is not withdrawn until the end of batch and capacity of fermenter is limited. These are expressed as integral constraints over flow rates of streams entering the fermenter as

$$\int_0^{t_f} u_i(t)\, dt \le f_i^c \qquad i = 1, \ldots, m_c \qquad \ldots(22)$$

where $\{u_i : i = 1, \ldots, m_c\}$ represent manipulated inlet flows to fed-batch fermenter. Second constraint can be simplified considerably for single-rate model, since analytical integration of input trajectories is possible. Thus, Eq. (22) can be written as

$$\int_0^{t_f} u_i(t) = \sum_{j=1}^{s_i} a_{ij} t_f \int_0^1 \phi_j(\tau) d\tau \leq f_i^c \quad \ldots(23)$$

where Ä = t / t_f, and t_f is batch duration. Since first orthonormal polynomial is unity, by definition of orthonormal polynomials,

$$\int_0^1 \phi_j(\tau) d\tau = \int_0^1 \phi_1(\tau)\phi_j(\tau) d\tau = 0 \quad \text{if} \quad j \neq 1$$
$$\int_0^1 \phi_1(\tau) d\tau = 1 \quad \ldots(24)$$

Therefore, Eq.(23) can be simplified to

$$\int_0^{t_f} u_i(t) = a_{i1} t_f \leq f_i^c \quad \ldots(25)$$

In multi-rate systems, input constraints can be imposed directly on input levels as

$$u_i^{\min} \leq u_i(t_k) \leq u_i^{\max} \qquad k = 1, \ldots, s_i \quad \ldots(26)$$

Second constraint can be converted into a summation over the batch duration as

$$\sum_{j=1}^{s_i} u_{i,j} \, \Delta t_i \leq f_i^c \quad \ldots 27)$$

In the case where t_f is not an exact multiple of Δt_i for any i, Eq. (27) is modified slightly as

$$\left(\sum_{j=1}^{s_i - 1} u_{i,j} \, \Delta t_i \right) + u_{i,s_i} \, \Delta t_i^l \leq f_i^c \quad \ldots(28)$$

where $\Delta t_i^l = t_f - (s_i - 1)\Delta t_i$; $i = 1, \ldots, m$. During model development stage, data on different input and output trajectories from various batches is parameterized into corresponding FCs, combined with initial conditions and process parameters to develop PDD model (Fig. 2a). Developed PDD model is employed to find optimal input trajectories, corresponding predicted output trajectories and optimal process parameters that maximize a given objective function at final time for a given set of initial conditions (Fig. 2b).

Results and Discussion

In order to illustrate applicability of proposed PDD modeling and optimization approach, a fed-batch fermentation system is considered as a simulation case study.

System Description

Example considered in present study is production of α-amylase in a fed-batch fermenter with two different substrates as carbon and nitrogen. Dynamic model for this system[22] is represented as

$$\frac{dX}{dt} = \mu(S_1, S_2) X - \frac{F_2 X}{V} \quad \ldots(29)$$

$$\frac{dS_2}{dt} = \frac{F_2 (S_{2F} - S_2)}{V} - \sigma_2(S_1, S_2) X \quad \ldots(30)$$

$$\frac{dP}{dt} = \pi(S_1, S_2) X - kP - \frac{F_2 P}{V} \quad \ldots(31)$$

$$\frac{dV}{dt} = F_2 \quad \ldots(32)$$

where $\mu(S_1, S_2) = \dfrac{0.086 S_1 S_2}{2.0 + S_1 + 0.0303 S_1^2}$,

$\pi(S_1, S_2) = 117.7 e^{-0.311 S_2} \mu(S_1, S_2)$,

and $\sigma_2(S_1, S_2) = \mu(S_1, S_2)/1.05$. Also, X is cell concentration, S_1 and S_2 are first and second substrate concentrations, P is product concentration, V is reaction mixture volume, μ, σ_2, and π are specific growth rates for cell, second substrate and product respectively, F_2 is second substrate feed flow rate, S_{2F} is concentration of second substrate in feed stream, and k is reaction rate constant (Table 1). Second substrate feed flow rate F_2 is chosen as manipulated input, whereas productivity (PV), defined as the product concentration P times the reaction mixture volume V, is considered as output variable.

Development of PDD Models

For preliminary data generation step, 5 typical input trajectories (Fig. 3a) are employed for second substrate flow rate (F_2). The data generation for this system is

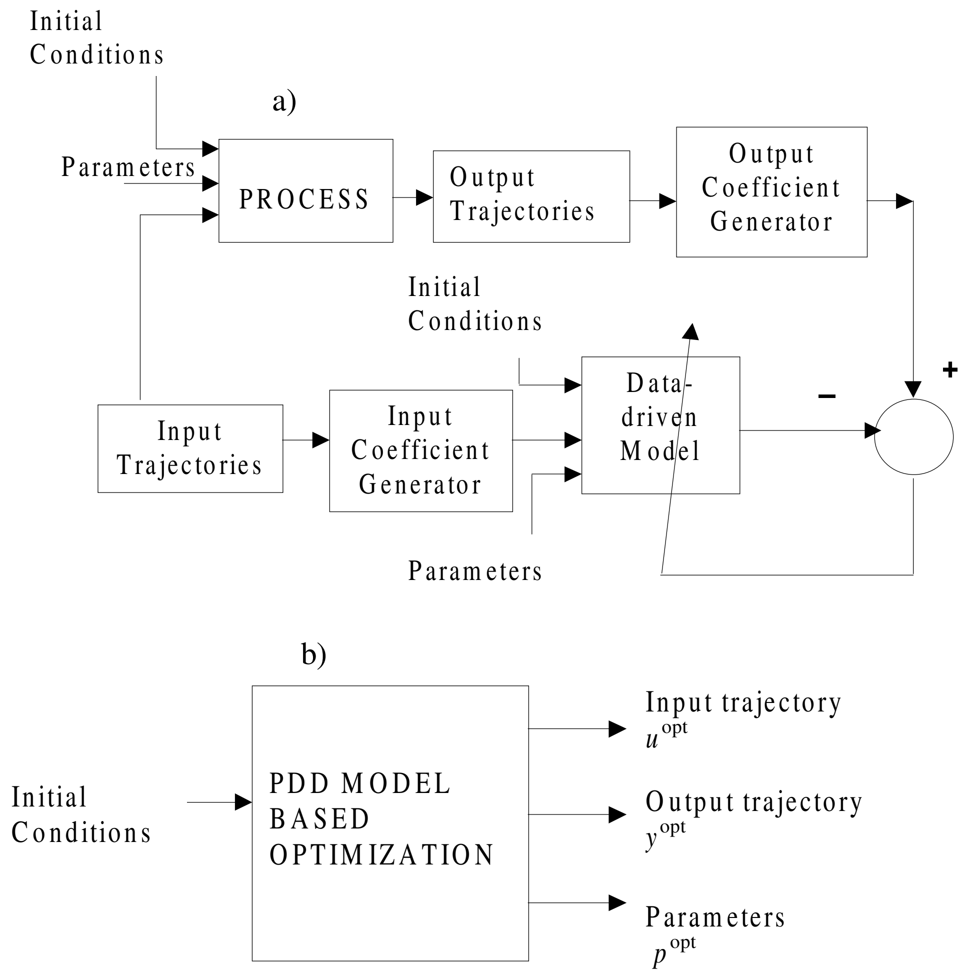

Fig. 2—Modeling and optimization schemes using Parameterized Data-Driven (PDD) model: (a) Model development; (b) Optimization

carried out using typical input trajectories together with random variations in initial conditions and in process parameters to be optimized. Variations introduced in initial conditions and process parameters as well as measurement noise are chosen from a normal distribution with a zero mean and standard deviations (Table 1). This simulation exercise results in total of 5500 batch runs of fed batch fermenter. Input data (range, –5 to +5) and output data (range, 0 to 1) are split into two parts – 5000 data sets as modeling part and 500 data sets as the validation part; each data set consists of a single batch run. Based on input and output trajectories, root mean square error (RMSE) and Akaike information criteria are computed, and order for input polynomial is chosen as 7, whereas for output polynomial, order is chosen as 6. For single-rate case, three data-driven models (linear, quadratic and ANN), have polynomial input FCs (7), initial conditions (3) and process parameters (2) as inputs and output FCs (6) as outputs. In multi-rate case, number of piecewise constant input values considered are 18, with 3 initial conditions and 2 process parameters as model inputs (Fig. 3b). Specific equations for linear, quadratic and ANN models are reported separately for single-rate and multi-rate cases together with dimensions of associated parameters (Table 2). Combined design and training algorithm[23,24] is used to train ANN model, whereas one-shot least squares method[25] is employed to generate linear and quadratic models.

In order to confirm prediction capability of single-rate models, output trajectories of modeling data as well as validation data (not used to train networks) are predicted using developed models. Since physical proximity of predicted output trajectories cannot be observed from predicted output FCs, comparison is made on the basis of difference between predicted output trajectory and its polynomial approximation on a time-scale over batch duration. From validation dataset, consisting of 500 simulation runs, only worst-case

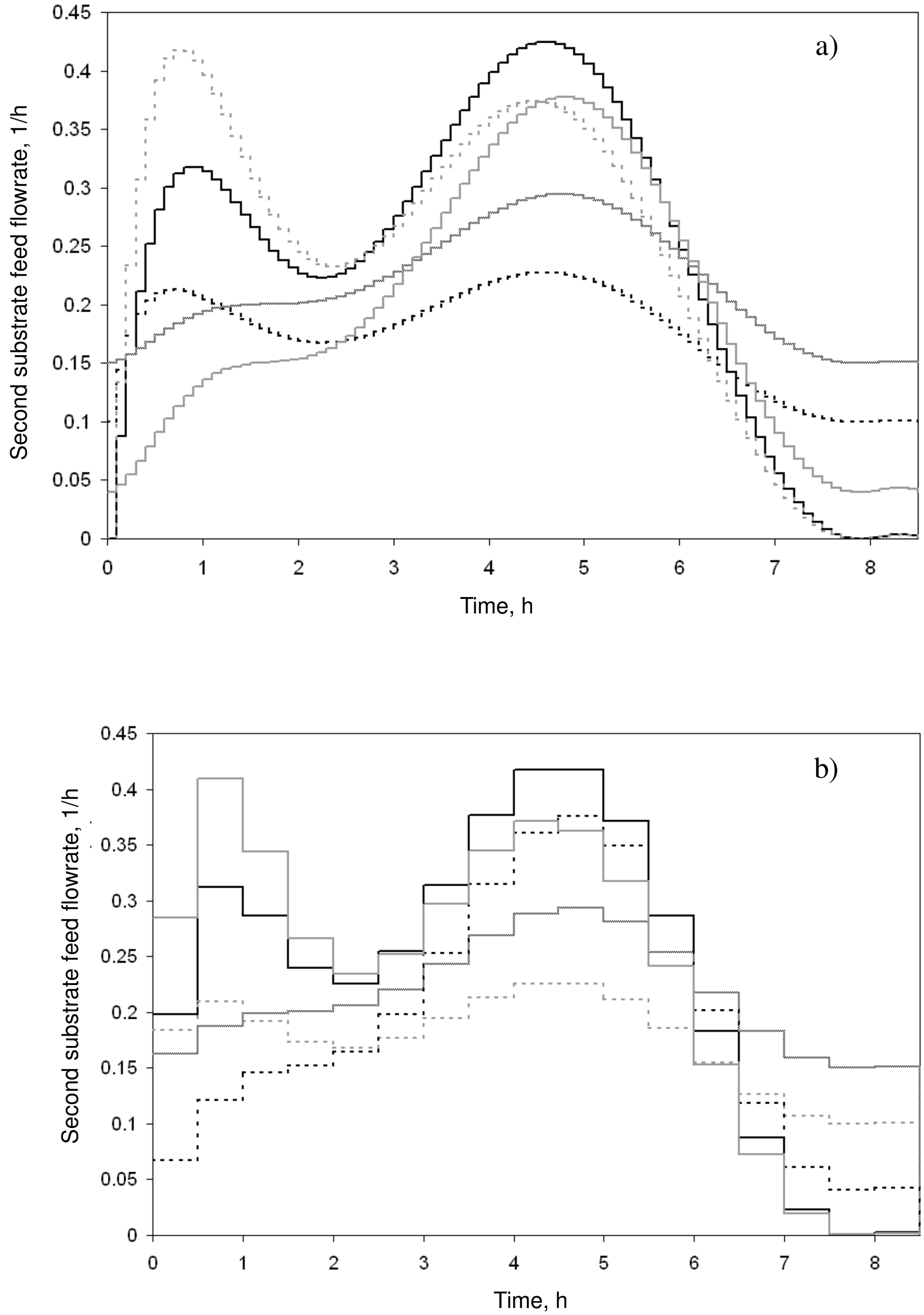

Fig. 3—Input trajectories for fed-batch fermenter: (a) Single-rate; (b) Multi-rate

predictions for single-rate case using linear (Fig. 4a), quadratic (Fig. 4c) and ANN (Fig. 4e) models predicted output trajectory with reasonable accuracy.

In multi-rate case, for 5 typical input trajectories for second substrate flow rate (F_2) employed in data generation (Fig. 3b), it is assumed that 18 manipulated input moves are made during each batch. These input trajectories together with random variations in initial conditions and process parameters are then employed to generate datasets necessary for modeling and validation of multi-rate models using 5500 simulation

Table 1—Model parameters and initial conditions for fed-batch fermenter

Model parameters / Initial Conditions			
k	0.18	X_0	4.667 g l^{-1}
S_1	8.12 g l^{-1}	S_{20}	2.667 g l^{-1}
S_{2F}	10.0 g l^{-1}	P_0	100.0 units l^{-1}
T_f	8.5 h	V_0	3.0 l
T_s	0.1 h	Δt_i	0.5 h
Standard deviations of variations in variables / parameters			
X_0	0.15 g l^{-1}	S_{20}	0.10 g l^{-1}
S_1	0.20 g l^{-1}	P_0	2.0 units l^{-1}
S_{2F}	0.25 g l^{-1}	P	1.0 units l^{-1}
		V	0.025 l
Bounds / Constraints/ Ranges of Variables			
	Minimum		Maximum
F	0.0 l h^{-1}	F	1.0 l h^{-1}
V			5.0 l
S_1	7.3354 g l^{-1}	S_1	8.9520 g l^{-1}
S_2	8.7743 g l^{-1}	S_2	10.9370 g l^{-1}
X_0	4.1099 g l^{-1}	X_0	5.2658 g l^{-1}
S_{20}	2.2732 g l^{-1}	S_{20}	3.0225 g l^{-1}
P_0	91.99 units l^{-1}	P_0	108.36 units l^{-1}

runs for fermenter. Variations in initial conditions, process parameters and measurement noise are identical to those employed in data generation step for single-rate models. Productivity profiles in validation dataset for worst case predictions using linear (Fig. 4b), quadratic (Fig. 4d) and ANN (Fig. 4f) models, for multi-rate case along with process productivity profile indicate better prediction accuracy by: ANN model > quadratic model > linear model.

Optimal Control

Optimal problems formulated using all three models are solved using ‘constr’ function available in Optimization Toolbox of MATLAB 5.2, which is based on successive quadratic programming approach. To explore validity of proposed approach, a comparision is made with the results obtained by employing exact first principles model and control vector parameterization (CVP) strategy. Polynomial (or piecewise constant) input approximations used in CVP formulation with first principles model is identical with the polynomial (or piecewise constant) input approximations used in the development of single-rate (or multi-rate) PDD model. Optimization exercise in each case yields: i) open loop optimal manipulated input policy together with optimum process parameters; and ii) open loop optimal output trajectory, which may be used as a setpoint trajectory in on-line feedback control of the semi-batch process, for a given set of initial conditions. A few typical variations are introduced in initial conditions and performance of proposed approach is compared with the results obtained based on the exact process model based optimization for single-rate and multi-rate cases.

In this system, for a given set of initial conditions, it is desired to find optimum values of starch (first substrate) concentration (S_1), feed caesenate (second substrate) concentration (S_{2f}), and second substrate flow rate (F_2) trajectory to maximize productivity at a fixed final time ($P(t_f)V(t_f)$). Therefore, single output trajectory is chosen as productivity, and single input trajectory is chosen as second substrate flow rate. In initial batch, S_1 and S_{2f} are process parameters to be optimized and are incorporated into model as additional inputs. Initial conditions are initial cell concentration, initial second substrate concentration and initial product concentration in reaction mixture. No measured disturbance is considered in present study. Thus, optimal control problem can be stated as

$$\underset{\mathbf{U},\mathbf{p}}{\text{Max}} \; P(t_f)V(t_f) \qquad \ldots(33)$$

Table 2—Details of the Models for fed-batch fermenter

Model	Equations	Parameters
Single-rate common equations	$\mathbf{X}=\{X_i\}\equiv[a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ X_0\ S_{20}\ P_0\ S_1\ S_{2f}]^T$ $x_i=\left[\left(X_i-X_i^{min}\right)*10/\left(X_i^{max}-X_i^{min}\right)\right]-5,\ i=1,2,...,12$ (Input scaling) $\mathbf{x}=\{x_i: i=1,2,...,12\}, \mathbf{y}=\{y_k: k=1,2,...,6\}=\psi(\mathbf{x})$, (different for each model) $Y_k=Y_k^{min}+y_k*(Y_k^{max}-Y_k^{min}),\quad k=1,2,...,6$ (Output scaling) $\mathbf{Y}=\{Y_k: k=1,2,...,6\}\equiv[c_1\ c_2\ c_3\ c_4\ c_5\ c_6]^T$	
Linear	$\mathbf{y}=\psi(\mathbf{x})=\mathbf{Ax}+\mathbf{b}$	A: 6x12, b: 6x1 Total parameters: 78
Quadratic	$\mathbf{y}=\psi(\mathbf{x})=\mathbf{Ax}_1+\mathbf{Bx}_2+\mathbf{c}$ $\mathbf{x}_1=\mathbf{x}=\{x_i: i=1,2,...,12\}$ $\mathbf{x}_2=\{x_i^2: i=1,2,...,12\}$	A: 6x12, B: 6x12 c: 6x1 Total parameters: 150
ANN	$\mathbf{x}=\{x_i: i=1,2,...,13\}\quad x_{13}=1$ (bias input) $H_j=f\left\{\sum_{i=1}^{13} w_{ij}x_i\right\}, j=1,2,...,20; H_{21}=1$ (bias input) $y_k=f\left\{\sum_{j=1}^{21} v_{jk}H_j\right\},\quad k=1,2,...,6$ $\mathbf{y}=\{y_k: k=1,2,...,6\}$ where $f\{x\}=1/(1+\exp(-x))$	W={w_{ij}}: 13x20 V={v_{jk}}: 21x6 Total parameters: 386
Multi-rate common equations	$\mathbf{X}=\{X_i\}\equiv[u_1\ u_2\ ...\ u_{18}\ X_0\ S_{20}\ P_0\ S_1\ S_{2f}]^T$ $x_i=\left[\left(X_i-X_i^{min}\right)*10/\left(X_i^{max}-X_i^{min}\right)\right]-5,\ i=1,2,...,23$ (Input scaling) $\mathbf{x}=\{x_i: i=1,2,...,23\}, \mathbf{y}=\{y_k: k=1,2,...,6\}=\psi(\mathbf{x})$, (different for each model) $Y_k=Y_k^{min}+y_k*(Y_k^{max}-Y_k^{min}),\quad k=1,2,...,6$ (Output scaling) $\mathbf{Y}=\{Y_k: k=1,2,...,6\}\equiv[c_1\ c_2\ c_3\ c_4\ c_5\ c_6]^T$	
Linear	$\mathbf{y}=\psi(\mathbf{x})=\mathbf{Ax}+\mathbf{b}$	A: 6x23, b: 6x1 Total parameters: 144
Quadratic	$\mathbf{y}=\psi(\mathbf{x})=\mathbf{Ax}_1+\mathbf{Bx}_2$ $\mathbf{x}_1=\mathbf{x}=\{x_i: i=1,2,...,23\}$ $\mathbf{x}_2=\{x_i^2: i=1,2,...,23\}$	A: 6x23 B: 6x23 Total parameters: 276
ANN	$\mathbf{x}=\{x_i: i=1,2,...,24\}\quad x_{24}=1$ (bias input) $H_j=f\left\{\sum_{i=1}^{24} w_{ij}x_i\right\}, j=1,2,...,28; H_{29}=1$ (bias input) $y_k=f\left\{\sum_{j=1}^{29} v_{jk}H_j\right\},\quad k=1,2,...,6$ $\mathbf{y}=\{y_k: k=1,2,...,6\}$ where $f\{x\}=1/(1+\exp(-x))$	W={w_{ij}}: 24x28 V={v_{jk}}: 29x6 Total parameters: 846

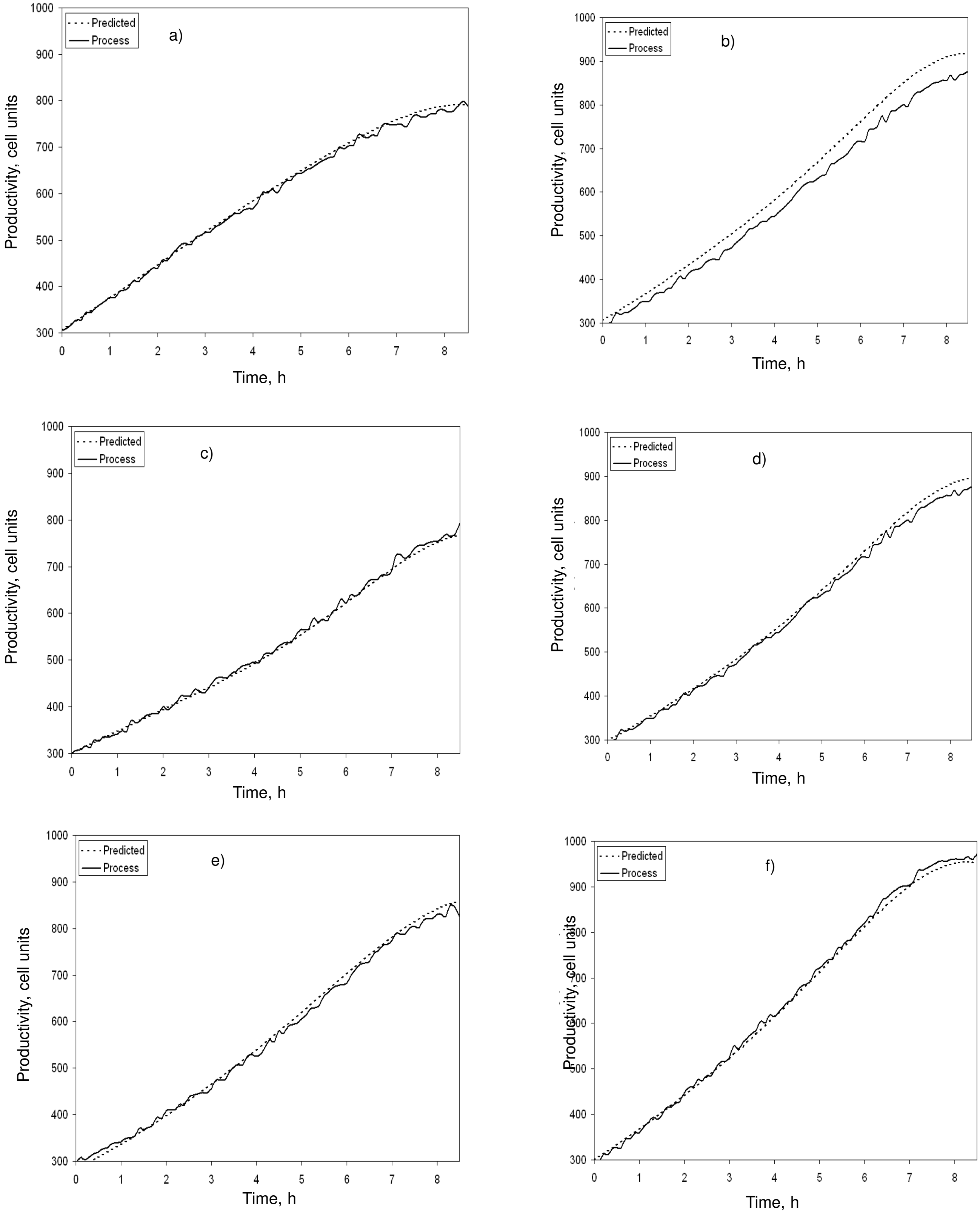

Fig. 4—PDD predictions for fed-batch fermenter testing data – Worst cases for: Linear model – (a) Single-rate (b) Multi-rate; Quadratic model – (c) Single-rate (d) Multi-rate; ANN model – (e) Single-rate (f) Multi-rate

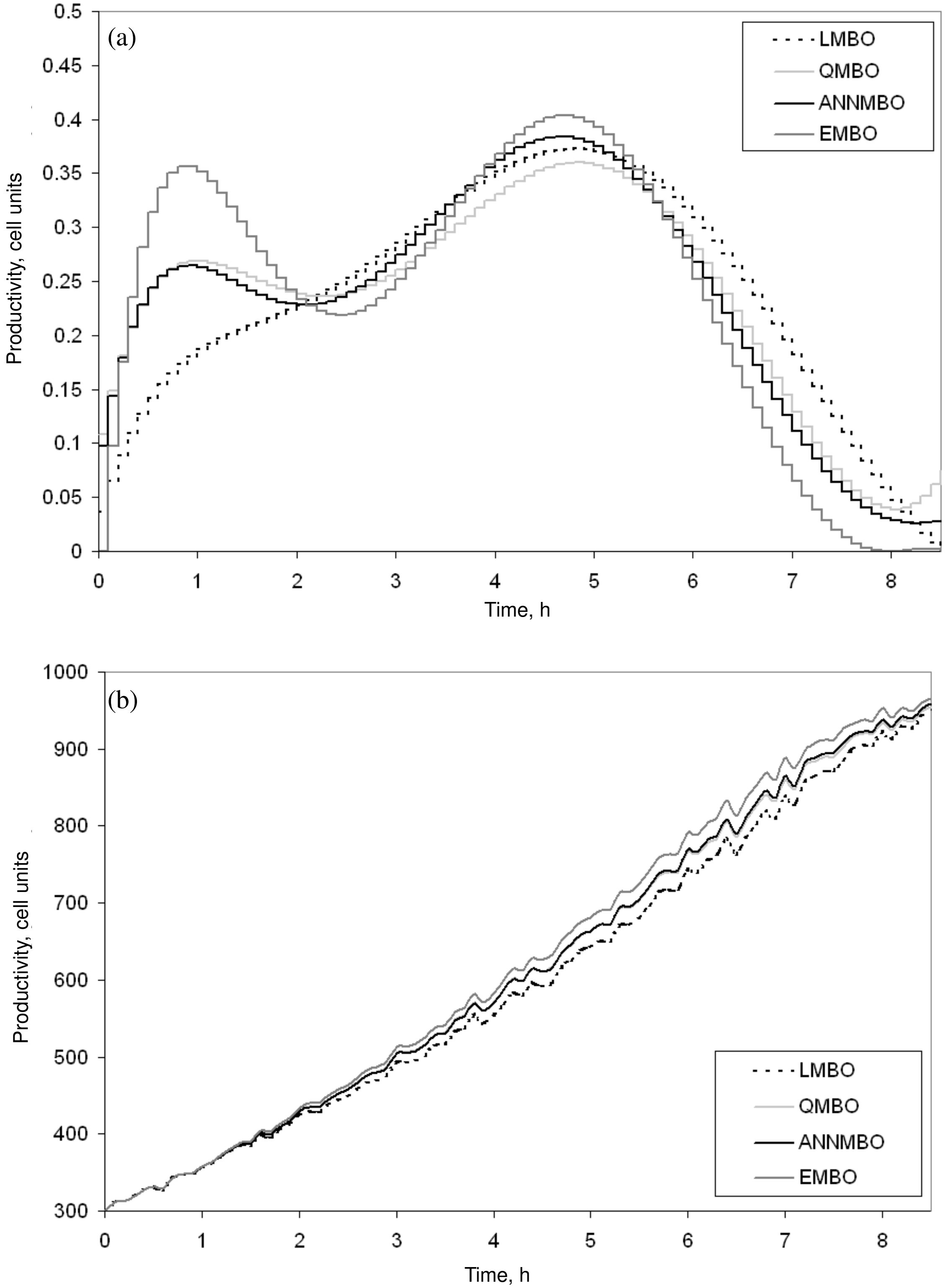

Fig. 5—Fed-batch fermenter optimization results – Single-rate case: (a) Optimal input trajectories, (b) Output trajectories by open-loop implementation

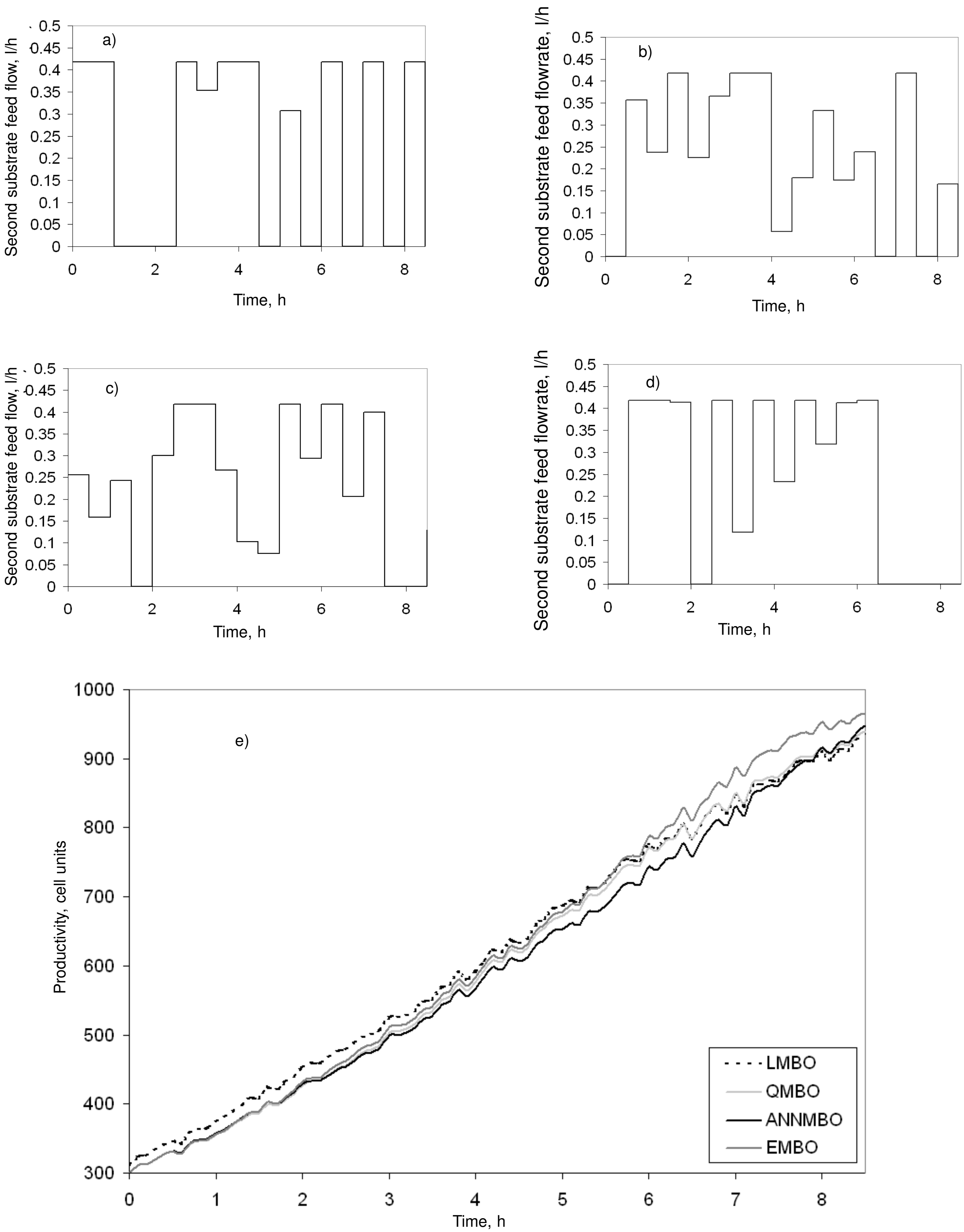

Fig. 6—Fed-batch fermenter optimization results – Multi-rate case: Optimal Input trajectories using (a) LMBO, (b) QMBO, (c) ANNMBO and (d) EMBO; (e) Output trajectories by open-loop implementation

Table 3—Optimization results of fed-batch fermenter system using PDD model

Opt. Scheme	Cases	Single-rate				Multi-rate			
		S_1 g l^{-1}	S_{2f} g l^{-1}	$PV(t_f)$	D-EMBO%	S_1 g l^{-1}	S_{2f} g l^{-1}	$PV(t_f)$	D-EMBO%
L	Nom.	8.9520	10.9370	949.8	1.6	7.3354	10.9370	913.3	5.5
M	A	8.9520	10.9370	922.7	2.0	7.3354	10.9370	889.7	5.6
B	B	8.9520	10.9370	975.9	1.3	7.3354	10.9370	935.8	5.4
O	C	8.9520	10.9370	958.3	1.5	7.3354	10.9370	919.9	5.6
	D	8.9520	10.9370	940.8	1.7	7.3354	10.9370	906.0	5.4
Q	Nom.	7.3354	10.9370	955.2	1.0	8.9520	10.9370	939.6	2.8
M	A	7.3354	10.9370	929.9	1.3	8.9520	10.9370	915.7	2.8
B	B	7.3354	10.9370	979.4	0.9	8.9520	10.9370	962.4	2.7
O	C	7.3354	10.9370	963.1	1.0	8.9520	10.9370	946.9	2.8
	D	7.3354	10.9370	946.7	1.1	8.9520	10.9370	931.8	2.7
AN	Nom.	8.9520	10.9370	958.8	0.7	8.9520	10.9370	948.1	1.9
N	A	8.9520	10.9370	939.8	0.2	8.9520	10.9370	933.5	1.0
M	B	8.9520	10.9370	985.1	0.3	8.9520	10.9370	964.9	2.5
B	C	8.9520	10.9370	968.6	0.5	8.9520	10.9370	951.4	2.3
O	D	8.9520	10.9370	949.3	0.8	8.9520	10.9370	946.3	1.2
E	Nom.	8.1245	10.9370	965.3	-	8.1252	10.9370	966.3	-
M	A	8.1244	10.9370	941.8	-	8.1244	10.9370	942.5	-
B	B	8.1244	10.9370	988.4	-	8.1247	10.9370	989.6	-
O	C	8.1244	10.9370	973.0	-	8.1245	10.9370	974.0	-
	D	8.1244	10.9370	957.2	-	8.1244	10.9370	958.0	-

LMBO = Linear model based optimization, QMBO = Quadratic model based optimization, ANNMBO = ANN model based optimization & EMBO = Exact process model based optimization; % D-EMBO = % deterioration compared to EMBO; A = -4% X_0, -4%S_0 & -4%P_0, B = +4% X_0, +4%S_0 & +4%P_0, C = +4% X_0, -4%S_0 & +4%P_0, and D = -4% X_0, +4%S_0 & -4%P_0

$$\text{subject to} \quad F_2^{\min} \le F_2(t) \le F_2^{\max},$$

$$\int_0^{t_f} F_2(t)\,dt \le (V(t_f) - V_0) \text{ and } \mathbf{Y} = \Psi[\mathbf{x_0}, \mathbf{U}, \mathbf{p}] \qquad \ldots(34)$$

where $\mathbf{x}_0 = \{X_0, S_{20}, P_0\}$, $\mathbf{p} = \{S_1, S_{2f}\}$,

$\mathbf{U} = \mathbf{u} \equiv [a_1\, a_2 \ldots\ldots a_7]^T \cong \{u(t) : t_0 \le t \le t_f\}$,

$u(t) = F_2(t) = \sum_{j=1}^{7} a_j \phi_j(t)$,

$\mathbf{Y} = \mathbf{y} \equiv [c_1\, c_2 \ldots\ldots c_6]^T \cong \{y(t) : t_0 \le t \le t_f\}$ and

$y(t_f) = P(t_f)V(t_f) = \sum_{j=1}^{6} c_j \phi_j(t_f)$. Further,

$S_1^{\min} \le S_1 \le S_1^{\max}$, $S_{2f}^{\min} \le S_{2f} \le S_{2f}^{\max}$ and

$\sum_{j=1}^{6} c_j \phi_j(t_0) = P(t_0)V(t_0)$.

At nominal initial conditions for single rate case, optimal trajectories for three PDD model based optimization (linear – LMBO, quadratic – QMBO and neural network – ANNMBO) approaches along with exact process model based optimization (EMBO) approach illustrate that optimal input trajectories obtained using ANN and quadratic model are very similar and are closer to input trajectories obtained using exact model, whereas linear model based optimal input trajectory is found to differ substantially from others during first 2 h (Fig. 5a). This difference is also reflected in productivity profiles of output trajectories (Fig. 5b), where actual process response obtained by open-loop implementation of generated optimal flow rate profiles using different models are plotted. In nominal case, deterioration in final objective function compared to EMBO is found for ANNMBO (0.7%), QMBO (1%) and LMBO (1.6%). Effect of variations in initial conditions is investigated by introducing w:±4% variations in all three initial conditions. For all 4 approaches, two optimal parameters (first substrate concentration in initial batch, S_1 and second substrate concentration in feed, S_{2f}) and productivity at final time were obtained by optimization for each case (Table 3). Initial conditions have not effected optimum process parameters (even in EMBO), whereas final productivity achieved is different in each case. ANNMBO has exhibitted performance nearly as good as EMBO, followed closely by QMBO with worst performance by LMBO. However, even with LMBO, the worst case deterioration is found to be about 2%, which illustrates proximity of the objective function value to that obtained using EMBO. In multi-rate case, problem formulation remains essentially the same as in single-rate case except for input representation. In Eq.(34), value of **U** is represented as

$$\mathbf{U} = \mathbf{u} \equiv [a_1 .....a_{18}]^T = [u_1,..., u_{18}] \cong \{u(t) : t_0 \le t \le t_f\}$$

where

$$u(t) = u_j;\ \ (j-1)\Delta t_i \le t < j\Delta t_i\ ,\quad j = 1,...,18\,.$$

Optimal input profiles (Figs.6a – d) indicate that some similarities exist among QMBO, ANNMBO and EMBO, whereas LMBO optimal input profiles have shown maximum deviations from the profiles of EMBO. Productivity profiles (Fig. 6e) illustrate that in multi-rate case, difference in final objective function value of EMBO compared to all the data-driven model based approaches is larger than in single-rate case. Order of best performance, however, remains the same: ANNMBO> QMBO> LMBO.

Deterioration in final productivity values for all 3 data-driven model based approaches compared to EMBO is considerably larger than in the single-rate for all the cases (Table 3). Minimum deterioration (2%) in multi-rate case, exhibitted by ANNMBO, is comparable to maximum deterioration obtained in single-rate case using LMBO. In multi-rate case, deterioration in performance using QMBO is 2.8%, while for LMBO it is 5.5% compared to EMBO.

Conclusions

ANN, quadratic and linear models were used in a PDD modeling approach in order to formulate an optimal control problem and to compute open loop optimal input policies (ANNMBO, QMBO and LMBO, respectively) for semi-batch processes. Proposed approach is capable of capturing nonlinear as well as time-varying behavior inherent in fed-batch systems fairly accurately in the region of interest. In addition, it is also able to capture batch-to-batch variations in initial conditions and other process parameters. Also, PDD can be used for capturing dynamics of single rate systems as well as multi-rate systems with equal ease. All three PDD model based approaches have exhibitted better performance in single-rate case compared to corresponding multi-rate cases. Order of best performance among three PDD model based approaches is found to be: ANNMBO> QMBO> LMBO. Comparing complexity of three models, quadratic model, with reasonable complexity, has exhibitted performance very close to the best data-driven model based approach (ANNMBO), and can therefore be considered as a better alternative in the absence of sufficient data for generating an ANN model. Thus, simulation studies demonstrate that proposed data driven modeling technique can be effectively used for solving operating trajectory optimization problem associated with industrial semi-batch processes where operating data is available in abundance.

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