A new method of analysis of chopper fed DC series motor using Walsh/block pulse function

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A new method of analysis of chopper fed DC series motor is presented using Walsh/block pulse-domain operation transfer function technique. Output response of average speed and average current is obtained having no restriction on the variations of width, frequency and amplitude of the input wave.

Orthogonal properties of the familiar sine-cosine functions have been known for over two centuries. In many areas of electrical engineering the basis for any analysis is such a system of sine-cosine functions. But with the growth of digital techniques of control, other general complete systems of non-sinusoidal binary valued orthogonal functions have drawn the attention of researchers. The suitability of these functions in the analysis of systems consisting of two-state digital logic is quite apparent.

The formal search for piecewise constant basis functions (PCBF) started with the proposition of Haar functions\(^1\). The most striking feature of the function set was its piecewise constant nature, which was very much unlike the popular sine-cosine functions. Researchers took notice of this unusual kind of function set and eventually the Rademacher function, Walsh function, block pulse function, slant function etc. were gradually introduced\(^2\).

Of all these functions, Walsh, block pulse functions were particularly utilized for the analysis of power electronic circuits and control system analysis and synthesis\(^3\).\(^4\).

In this paper, analysis of a chopper-fed DC motor is presented using Walsh/block pulse function and a newly defined Walsh operational transfer function (WOTF) which is analogous to the conventional Laplace domain transfer function.

Brief Review of Walsh and Block Pulse Functions

In 1923, Walsh presented\(^10\) a set of normal orthogonal functions, later named Walsh functions. These functions possess some interesting properties to make it suitable for a variety of applications. Fig. 1 shows a set of Walsh functions \(\Phi\) arranged in dyadic order. Since dyadically ordered Walsh functions offer advantages for analytical and computational purposes, this ordering is chosen for possible use in analysing power converter circuits. In addition, there is a one-to-one relationship between Walsh and block pulse functions, a set of which is shown in Fig. 2.

Chen et al.\(^11\) have shown that the block pulse functions \(\Psi\) is related to the Walsh functions \(\Phi\) by the relation

\[
\Phi_{(m)} = W_{(m)} \Psi_{(m)}
\]

where, \(W_{(m)}\) is a square matrix of order \(m\) called the Walsh matrix\(^12\). \(\Phi_{(m)}\) and \(\Psi_{(m)}\) are Walsh and block pulse vectors respectively of compatible order. Walsh matrix has a property such that

![Fig. 1—A set of Walsh functions arranged in dyadic order](image-url)
$W_{(m 	imes n)} = mI_{(m)}$  \hspace{1cm} \text{(2)}$

$\text{where, } I_{(m)} \text{ is a unit matrix of order } m.$

**Philosophy of the Proposed Walsh Domain Operational Technique**

The input-output relationship of a linear time-invariant single input single output (SISO) system in Laplace domain is well-known and is given by

$$C(s) = G(s)(s)R(s)$$  \hspace{1cm} \text{(3)}

$\text{where, } G(s) \text{ is the transfer function of the system; } C(s) \text{ and } R(s) \text{ are the Laplace transformed output and input respectively.}$

The operational matrix for differentiation $D$ performs in the Walsh domain like the Laplace operator $s$ in the Laplace domain. So, we can convert $G(s)$ in (3) to Walsh operational transfer function (WOTF) by simply replacing $s$ by the operational matrix $D$.

That is,

$$G(s) \big{\mid}_{s=D} = G(D) = \text{WOTF}$$  \hspace{1cm} \text{(4)}

By using the transformation of (4) the system transfer function is converted to an operational transfer function defined in the Walsh domain$^{113}$. Thus,

$$C_{\phi} = R_{WOTF} \phi$$  \hspace{1cm} \text{(5)}

$C_{\phi}$ in the above equation gives the output time response in the form of a series combination of Walsh functions.

Any input block pulse vector, $R_{1_{(m)}} \psi$ say, can be formed from $r(t)$ simply by inspection of $r(t)$ is the output of a parametric converter. To convert $R_{1_{(m)}} \psi$ to $R_{\phi} \phi$ the transformation represented by Eq. (1) is used.

Therefore,

$$R_{1_{(m)}} \psi = R_{1_{(m)}} W_{1_{(m)}} \psi \phi$$

or, $R_{1_{(m)}} \psi = (1/1_{(m)}) W_{1_{(m)}} \phi$ where use is made of Eq. (2).

Now to obtain $C_{\phi}$, (5) is used. Thus

$$C_{\phi} = R_{1_{(m)}} W_{WOTF} \phi$$  \hspace{1cm} \text{(6)}

Since, easier reconstruction of output response $r(t)$ is possible if block pulse functions are used, $C_{\phi}$ is determined instead of $C_{\phi}$ by using Eq. (1). Thus we have

$$C_{\phi} = R_{1_{(m)}} W_{WOTF} W_{1_{(m)}} \phi$$  \hspace{1cm} \text{(7)}

By inspecting Eq. (7) and keeping in mind relation (5) it is seen that $(1/1_{(m)}) W_{WOTF} W_{1_{(m)}}$ is the operational transfer function in the block pulse domain and it is operative upon $\psi$ vector only. It may also be noted that the use of either Walsh function or block pulse function yields the same result.

**Analysis of a Continuously Pulse-Width Modulated (PWM) Chopper System$^n$**

A continuously pulse-width modulated (PWM) chopper system is now investigated and its output response is determined. In the present analysis, sixteen basis Walsh functions are used.

The pulse-width modulations of the chopper is effected by shifting the trailing edge of an input square pulse while keeping its leading edge fixed, with constant repetition rate. In Fig. 3a, a pulse input in a time interval of $T$ seconds is considered which is divided into $m$ equal sub-intervals of $(T/1_{(m)})$ seconds each, where $m$ is of the form $m = 2^n$, $n$ being a positive integer.

Let the repetition rate of the input square wave be $(1/2)$ seconds. Evidently only two pulses are considered initially, each of which can cover any portion of the repetition period $T/2$. However, each

![Fig. 2—A set of block pulse functions.](image-url)

![Fig. 3a—A pulse train input covering $m$ sub-intervals and a](image-url)
Com b inin g a ll su c ti o n s, th e fo ll o w in g ge n e r a l matri x e qu a ti o n is obtai n e d a s pulse s a r e co n s id e r e d in a p e ri o d o f

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( x7)lm

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sys te m i n c orpora tin g t h e

m1 2)

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th e a n a lys is o f a c lose d lo o p

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p e r t urb a ti o n t e c hni q u e. T hi s m o d e l

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DC se ri es m o to r , th e

Mathematical Op er a ti o n s

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I n o rd e r t o ob t a in th e o ut p ut

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W a ls h s t e p do m a in b y u s in g E q .

(1 111)

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In E q .

E q . (9) is te r me d as th e

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Th e m a tr i x

Th e m a tr i x

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P u l s e

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P W M M 

( PW M M )

( PW M M )

R .Ψ=PWMM.Ψ( m)

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The matrix PWMM on the right hand side of

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Eq (9) is termed as the Pulse Width Modulation Matrix (PWMM) of the chopper which takes care of any possible pulse-width modulation of the input waveform. Choosing proper values of \( m , T_0 , \gamma \), and \( T_c \), the output characteristics of the system for any duty interval as well as repetition rate may be computed.

Mathematical Operations

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Mathematical Operation.

Walsh Operational Transfer Function of DC Series Motor

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has to be obtained. To overcome the problem associated with the non-linearities in the equations describing the performance of the series motor, a linear model and the corresponding transfer function for the series motor are used by applying small-signal perturbation technique. This model can be applied in the analysis of a closed loop system incorporating the

Fig. 3b—Area preserving transformation of the pulse train of Fig. 3a.

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series motor and the analysis is valid for small perturbations around a given steady state operating point. The differential equations governing the performance of the DC series motor is given by Ramamoorty and Thangoo as

\[ e_a = R_i a + p L_i a + \omega M J \]

\[ M_j i_j^2 = Bn + p Jn + T_l \]

where the variables have usual significances.

In terms of average values, above equations are modified as,

\[ E_{av} = R_{av} i_{av} + R_{av} N_{av} i_{av} \]

\[ M_j K^2 i_{av}^2 = B N_{av} + p J N_{av} + T_{av} \]

where \( R_b = \frac{2\pi}{60} M_j \) and \( K = \frac{1}{i_{av}} \).

For small-signal perturbation about the operating point, the DC series motor may be represented by the block diagram of Fig. 4 and its input-output relationship in Laplace domain is

\[ \Delta N_{av}(s) = \frac{2 M_{av} K_{av}}{Z_{av}} \Delta V(s) \]

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\[ Z_{av} = D_{av} + s J_{av} \]

This WOTF is used for computation of output response of the system.

**Illustrative Example**

As an illustration of the proposed theory, a converter-fed DC series motor is considered. This may seem to pose an immediate problem, because, earlier in this paper only the method for a PWM system was presented. But to stress that the proposed method works as well for converter-fed systems, the following relation of equivalence is derived.

For a converter-fed system the average voltage is given by

\[ E_{av}(\theta) = \frac{E_m}{\pi} (1 + \cos \theta) \]

where \( \theta \) is the triggering angle of the thyristors.

But for a PWM system having a DC value \( E_m \), the average value of the voltage is given by

\[ E_{av} = K_d E_m \]

where \( K_d \) is the duty cycle of the chopper.

Comparing (18) and (19), we have

\[ K_d = \frac{1 + \cos \theta}{\pi} \]

In case of a converter-fed system the average voltage \( E_m \) can have a maximum value of \( 2/\pi E_m \), while in case of a PWM chopper-fed system the average value is \( E_{av} \). Representing the equivalent \( K_0 \) of a converter-fed system by \( K_{10} \), it is noted that

\[ 0 \leq K_{10} \leq 2/\pi \], for \( 180^\circ \leq \theta \leq 0^\circ \)

While for \( K_d \) the relation is, \( 0 \leq K_d \leq 1 \).

This correspondence is elaborated in Fig. 5 which shows conversion of converter waveform to that of an equivalent PWM waveform.

**Fig. 4**—A simplified block diagram of the DC series motor for small signal perturbation

**Fig. 5**—Transformation of converter output voltage to equivalent PWM chopper voltage
The computed results are found to be in close agreement with the conventional solution.

**Conclusion**

The major advantage of the proposed method is that a generalization with regard to amplitude, frequency and nature of waveform of the input voltage is maintained and accommodated at every stage of analysis, particularly with the establishment of the area-preserving technique. In the analysis presented, though 8 basis Walsh functions have been used, the accuracy of the method can be improved by using more basis functions in a set. However, the manipulations of matrix equations with more than 32 basis Walsh functions is likely to offset the advantages of accuracy.

**References**