Stability behaviour of angle ply plates subjected to various types of inplane loadings

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Investigations on the stability characteristics of rectangular angle ply plates subjected to various types of inplane loading are reported here. Variationally consistent governing equations and boundary conditions are derived for this purpose. The possibility of bifurcation instability in angle ply plates subjected to various types of inplane loading is examined. Employing a four node rectangular finite element having ten degrees of freedom per node solves the governing equations. The prediction accuracy of the element is demonstrated through series of numerical examples. The effect of lay-up, aspect ratio, type of loading (uniaxial/biaxial/positive and negative shear/tension-compression/compression-tension/inplane edge moment) and boundary conditions are studied in detail. Some interesting observations on the stability behaviour of angle ply plates are made.

Laminated composite plates are finding increasing applications in several fields such as aerospace, mechanical, civil, etc. A designer/analyst tailors these materials to suit each application. While fibre-reinforced composites are attractive due to their high specific strength and stiffness, the flexibility of tailoring these properties by designing and analysing the composite structures is complex as it involves choice of several parameters ranging from fibres, resin, lay-up sequence, fibre orientation, ply thickness, etc. This is particularly true for the composite plate buckling response, because presence of bending-extension coupling results in transverse deflections even under the action of pure inplane loads. Therefore, the question arises as to whether bifurcation phenomenon takes place or not; is the plate response similar to a plate with eccentric loading or initial imperfections?

A comprehensive summary of the state-of-art review in buckling and post-buckling compiled by Leissa\textsuperscript{2}, Chia\textsuperscript{3}, Kapania\textsuperscript{4} and Singh\textsuperscript{5} indicates that there is considerable confusion regarding this aspect. It is found that majority of the investigators tacitly assume the existence of bifurcation and searched for eigenvalues, while others have traced the load deflection curves for such plates. Harris\textsuperscript{6} and Prabhakara\textsuperscript{7} have studied the post-buckling response of antisymmetric angle and cross-ply plates respectively subjected to edge compression. Harris\textsuperscript{6} has stated that bifurcation exists for regular antisymmetric angle ply plates under uniaxial or biaxial compression but does not exist for generally unsymmetric angle ply plates. Prabhakara\textsuperscript{7} has admitted that non-trivial solutions exist, even in the pre-buckling regime, due to the presence of bending extension coupling. However, he still has solved the equilibrium equations as an eigenvalue problem, disregarding the coupling because its effect diminishes as the number of layers increase. Chia\textsuperscript{3} and Dave and Lam\textsuperscript{8} state that due to the presence of material coupling, eigenvalue type buckling will not usually occur since out-of-plane displacements develop from the commencement of inplane loading. In such circumstances, there is no post-buckling behaviour as such but it is reasonable to refer to behaviour at relatively large loads as pseudo post-buckling behaviour. Later, Singh \textit{et al.}\textsuperscript{9,10} have shown that the above statement is not really true. They have reported that the post-buckling curves are quite far from the true load-deflection curves even at large loads. The difference in the two curves is attributed to the non-consideration of bending moments (generated by inplane edge loads) in the post-buckling response curves.

Leissa\textsuperscript{11} has worked out the conditions for the unsymmetrically laminated plates to remain flat during pre-buckling regime when subjected to inplane edge loads. Singh \textit{et al.}\textsuperscript{9,10} have investigated the
existence of bifurcation in unsymmetrical laminated plates and report that the plate need not remain flat during pre-buckling regime for bifurcation to occur. The numerical results in this study are limited to simple loading. Most of the studies cited so far deal with the simple loading such as uniaxial, biaxial or shear. However in practical situations, plates are subjected to more complex loading. Nair et al.\textsuperscript{12} and Narita and Leissa\textsuperscript{13} have presented the bifurcation loads of symmetrically laminated rectangular plates subjected to uniaxial, biaxial, inplane shear, compression-tension, tension-compression edge loading using analytical and finite element method respectively. Singh et al.\textsuperscript{14} have studied the buckling behaviour of laminated plates subjected to inplane edge moments constraining out-of-displacements to be zero in the pre-buckling regime.

The bifurcation behaviour of angle ply plates subjected to the type of loading mentioned above has not been investigated so far and hence forms the subject of the present study. Equilibrium equations and corresponding boundary conditions are derived and presented here. The existence of bifurcation in antisymmetrically laminated plates under various edge loading considered herein is investigated and conditions for bifurcation to take place are derived. A high precision four node rectangular finite element is developed to compute the bifurcation loads for various plate configurations. A series of numerical examples are solved to study effect of various parameters such as lay-up sequence, number of layers, sign of shear loading, boundary conditions, etc. on the bifurcation loads.

**Formulation**

A thin plate of length $a$, width $b$ and total thickness $t$, made up of perfectly bonded layers having thickness $t_1, t_2, \ldots, t_n$, has been considered as shown in Fig. 1. Each lamina is considered to be homogeneous and orthotropic and made of unidirectional fibre-reinforced material. The orthotropic axis of symmetry in each lamina is oriented at an angle $+\theta$ or $-\theta$ with the plate $x$-axis.

The classical lamination theory is applicable as the angle ply plates under investigation are assumed to be thin. The displacement field of the theory that assumes mid-plane normals to remain normal even after deformation may be expressed as Eq. (1):

$$u(x, y, z) = u_0(x, y) - z w_0(x, y)$$
$$v(x, y, z) = v_0(x, y) - z w_0(x, y)$$
$$w(x, y, z) = w_0(x, y)$$

where $u$, $v$, $w$ and $u_0$, $v_0$, $w_0$ are the displacement components in $x$, $y$, $z$ directions anywhere in the plate and reference surface (mid-plane assumed here) of the plate respectively. $(\cdot)'$ and $(\cdot)''$ denote differentiation with respect to $x$ and $y$ respectively. $x$, $y$, $z$ are the Cartesian co-ordinates.

The von-Korman type non-linear strain-displacement relationships for the plate can be written as Eq. (2):

$$\varepsilon_x = u''_0 - z w''_0 + \frac{1}{2}(w')^2$$
$$\varepsilon_y = v''_0 - z w''_0 + \frac{1}{2}(w')^2$$
$$\gamma_{xy} = u'v'_0 + v'_0 - 2z w''_0 + w'w''_0$$

The constitutive relations of the classical plate theory can be expressed as Eq. (3)

$$\begin{bmatrix} N_i \\ M_i \end{bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} \\ B_{ij} & D_{ij} \end{bmatrix} \begin{bmatrix} \varepsilon_j \\ \kappa_j \end{bmatrix}$$

where

$N_i = [N_x, N_y, N_{xy}]$

$M_i = [M_x, M_y, M_{xy}]$

$\varepsilon_j = [\varepsilon_{xx}, \varepsilon_{yy}, \gamma_{xy}]$

![Fig. 1—Geometry of a layered composite plate](image-url)
\[\kappa_j = \begin{bmatrix} \kappa_{xx} & \kappa_{xy} & \kappa_{yy} \end{bmatrix}\]

\(A_{ij}, B_{ij}\) and \(D_{ij}\) are the usual extensional, bending-extension and bending coefficients with:

\[\begin{align*}
\varepsilon_{xx} &= u_x' + \frac{1}{2}(w'')^2 \\
\varepsilon_{yy} &= v_y' + \frac{1}{2}(w'')^2 \\
\gamma_{xy} &= u_y' + v_x' + w'w'' \\
\kappa_x &= -w'', \quad \kappa_y = -w''', \quad \kappa_{xy} = -2 w''
\end{align*}\]

As the present study is limited to symmetric and regular antisymmetric angle ply plates, the stiffness coefficients \(A_{16}, A_{26}, B_{16}, B_{26}\) and \(B_{22}\) are zero. In case of symmetrically laminated plates, coefficients \(B_{16}\) and \(B_{26}\) also vanish. Similarly coefficients \(D_{16}\) and \(D_{26}\) vanish in case of antisymmetric angle ply plates.

The strain energy of the plate can be written as Eq. (4):

\[U = \frac{1}{2} \int_A \left\{ A_{11} \varepsilon_{xx}^2 + 2 A_{12} \varepsilon_{xx} \varepsilon_{yy} + A_{22} \varepsilon_{yy}^2 + A_{66} \gamma_{xy}^2 + 2 B_{11} \left( \varepsilon_{xx} \kappa_{xy} + \gamma_{xy} \kappa_{xx} \right) \right. \]

\[\left. + 2 B_{21} \left( \varepsilon_{xy} \kappa_{xy} + \gamma_{xy} \kappa_{yy} \right) + D_{12} \kappa_{xx}^2 \right\} dA\]

The potential of the applied inplane edge loads \(P_x, P_y\) and \(P_{xy}\) is given by Eq. (5):

\[W = -\int_A \left\{ P_x \left( \int_u u' \, dx \right) + P_{xy} \left( \int_v v' \, dx \right) \right\} dy \]

\[\left. - \int_u \left( P_x \left( \int_v v' \, dy \right) + P_{xy} \left( \int_u u' \, dy \right) \right) dx \right\}

The variation of the total potential \(\Pi = U + W\) leads to the following equilibrium equations and boundary conditions [Eqs (6) to (17)].

\[N_x' + N_{xy}'' = 0 \quad \ldots (6)\]

\[N_{xy}' + N_y'' = 0 \quad \ldots (7)\]

\[M_{xx}'' + 2 M_{xy}'' + M_{yy}'' + N_x w''
\]

\[+ 2 N_{xy} w''' + N_y w'''' = 0 \quad \ldots (8)\]

At edges \(x = 0, a\)

\[\delta u = 0 \quad \text{or} \quad N_x = P_x \quad \ldots (9)\]

\[\delta v = 0 \quad \text{or} \quad N_{xy} = P_{xy} \quad \ldots (10)\]

\[\delta w = 0 \quad \text{or} \quad P_x w' + P_{xy} w'' + M_{xx}' + 2 M_{xy}'' = 0 \quad \ldots (11)\]

At edges \(y = 0, b\)

\[\delta u = 0 \quad \text{or} \quad N_{xy} = P_{xy} \quad \ldots (12)\]

\[\delta v = 0 \quad \text{or} \quad N_y = P_y \quad \ldots (13)\]

\[\delta w = 0 \quad \text{or} \quad M_{xx} = 0 \quad \ldots (14)\]

At corners

\[\delta w = 0 \quad \text{or} \quad 2 M_{xy} = 0 \quad \ldots (15)\]

\[\delta w = 0 \quad \text{or} \quad 2 M_{xy} = 0 \quad \ldots (16)\]

\[\delta w = 0 \quad \text{or} \quad M_{xy} = 0 \quad \ldots (17)\]

The equilibrium equations are coupled partial differential equations in terms of \(u, v\) and \(w\). These apparently simple equations do not offer any clue whether the bifurcation type instability in antisymmetric angle ply plates will take place or not. In order to examine the possibility of bifurcation instability, Eqs (6-17) are expressed in the following form by making use of the partially inverted constitutive Eqs (3).
Eq. (18) needs to be solved together with the compatibility Eq. (19):

\[ w^{ii} - (w^{ii})^2 = 0 \quad \ldots (19) \]

and edge conditions

At edges \( x=0, a \)

\[ w = 0 \text{ or } P_x w_t + P_{xx} w_t + \frac{B_{16}}{A_{66}} P_{xy} \]

\[ + 2 \left( \frac{B_{16} A_{22} - B_{26} A_{12}}{A_{11} A_{22} - A_{12}^2} \right) P_x - \left( \frac{D_{11} - B_{16}^2}{A_{66}} \right) w^{ii} \]

\[ + \left( \frac{D_{12} - B_{16} B_{26} + 4D_{66}}{A_{66}} \right) w^{m00} - \frac{4B_{16} A_{22} + 2B_{26} A_{11} - 2B_{16} B_{26} A_{12}}{A_{11} A_{22} - A_{12}^2} \]

\[ - 2B_{16} A_{11} A_{22} - A_{12}^2 \]

\[ w = 0 \quad \text{or} \quad \frac{B_{26}}{A_{66}} P_{xy} - \left( \frac{D_{11} - B_{16} B_{26}}{A_{66}} \right) w^{m0} \]

\[ - \left( \frac{D_{12} - B_{16} B_{26}}{A_{66}} \right) w^{m0} - 2D_{26} w^{m00} = 0 \quad \ldots (21) \]

At edges \( y=0, b \)

\[ w = 0 \text{ or } P_{xy} w_t + P_y w_t + \frac{B_{26}}{A_{66}} P_{xy} \]

\[ + 2 \frac{B_{26} A_{11} A_{22} - B_{16} A_{12} A_{12}}{A_{11} A_{22} - A_{12}^2} P_y - 2D_{16} w^{ii} - 4D_{26} w^{m0} \]

\[ - \left( \frac{D_{12}^2 - B_{16} B_{26}}{A_{66}} \right) w^{m00} - \left( \frac{D_{12} - B_{16} B_{26}}{A_{66}} \right) w^{m0} + 4D_{66} \]

\[ - \frac{4B_{16} A_{22} + B_{26} A_{11} - 2B_{16} B_{26} A_{12}}{A_{11} A_{22} - A_{12}^2} \]

\[ w = 0 \quad \text{or} \quad \frac{B_{26}}{A_{66}} P_{xy} - \left( \frac{D_{11} - B_{16} B_{26}}{A_{66}} \right) w^{m0} \]

\[ - \left( \frac{D_{12}^2 - B_{16} B_{26}}{A_{66}} \right) w^{m0} - 2D_{26} w^{m00} = 0 \quad \ldots \]

**Conditions for Bifurcation Instability**

An examination of Eqs (18-23) indicates that these highly non-linear equations do not guarantee an unconditional bifurcation. These equations are invariably linearised to estimate the linear buckling loads. The linearisation ensures the identical satisfaction of compatibility Eq. (19). In order to compute the linear buckling loads, pre-buckling stress state needs to be evaluated. In fact, this pre-buckling stress state will determine whether bifurcation will take place or not.

In the case of symmetrically laminated angle ply plates, Eq. (18) reduces to standard eigen-value problem (owing to the vanishing of bending-extension coupling) thus ensuring the existence of bifurcation instability. The edge conditions, Eqs (20-23) in this case, become similar to that of an isotropic plate. However, it may be noted that in the present formulation only normal and tangential edge loads are considered. As is well known, the problem will no longer belong to the class of eigen-value problems, when symmetrically laminated plate is subjected to edge moments.

An examination of the equilibrium Eq. (18) reveals that bifurcation instability is not guaranteed in antisymmetrically laminated plate due to the presence...
of terms containing derivatives of stress resultants. However, bifurcation instability will occur when the pre-buckling stress-state is constant or linearly varying. The stress developed need not be constant/linearly varying even when the applied edge load is constant/linearly varying. It is mainly due to the additional external loading in terms of transverse edge shear and moments, developed in case of antisymmetrically laminated plates.

Bifurcation in antisymmetrically laminated plates is guaranteed if applied edge loads are constant or linearly varying. Also, (i). An opposite transverse edge shear force and edge moment is applied in case of simply supported edge. The magnitude of the force and moment is equal to the double underlined term; (ii). An opposite edge moment is applied in case of free edge. The magnitude of the moment is equal to the double underlined term; (iii). Edges are clamped.

It may be noted that antisymmetrically laminated angle ply plates will exhibit bifurcation when subjected to constant/linearly varying edge compression. No additional transverse edge force and moment is needed in such a case.

**Brief Description of the Finite Element**

A four node $C^1$ continuous rectangular plate element developed by Singh et al.\textsuperscript{10} is employed in the present investigation. The plate is assumed in $X$-$Y$ plane. The field variables are expressed in terms of nodal variables as given by Eqs (24.1) to (24.3):

\begin{align*}
\delta &= \sum_{i=1}^{4} N_i \delta_i + \sum_{i=3}^{9} N_i \delta_i' + \sum_{i=9}^{12} N_i \delta_i'' \quad \ldots \quad (24.1) \\
\nu &= \sum_{i=1}^{4} N_i \nu_i + \sum_{i=5}^{9} N_i \nu_i' + \sum_{i=9}^{12} N_i \nu_i'' \quad \ldots \quad (24.2) \\
\omega &= \sum_{i=1}^{4} N_i \omega_i + \sum_{i=5}^{9} N_i \omega_i' + \sum_{i=9}^{12} N_i \omega_i'' + \sum_{i=14}^{16} N_i \omega_i''' \quad \ldots \quad (24.3)
\end{align*}

where $N_i$ are Hermite interpolation functions and $\delta_i$, $\delta_i'$, $\delta_i''$, $\nu_i$, $\nu_i'$, $\nu_i''$, $\omega_i$, $\omega_i'$, $\omega_i''$, $\omega_i'''$ are nodal degrees of freedom. Substitution of Eqs (24) into the total potential and minimising leads to the finite element equations.

\[ [K_o][\delta] + \lambda [K_G][\delta] = \{F\} \quad \ldots \quad (25) \]

where $[K_o]$ is a $40 \times 40$ linear stiffness matrix; $[K_G]$ is a $40 \times 40$ geometric stiffness matrix; $\{F\}$ is the load vector; $\lambda$ is the load factor; $\delta$ is displacement/eigen vector.

The pre-buckling stress state is obtained by solving the finite element Eq. (26):

\[ [K_o][\delta] = \{F\} \quad \ldots \quad (26) \]

The computed pre-buckling stress state is used to construct the geometric stiffness matrix $[K_G]$. The bifurcation loads are computed by solving the eigen value problem given by Eq. (27).

\[ [K_o][\delta] + \lambda [K_G][\delta] = 0 \quad \ldots \quad (27) \]

**Results and Discussion**

In this section, a number of numerical examples are solved and bifurcation loads are computed for simply-supported/clamped, symmetrically/antisymmetrically laminated rectangular plates. Various types of inplane loading, e.g., uniaxial/biaxial, shear, compression, tension-compression and linearly varying edge compression are considered. Additionally, external edge loading is applied, wherever necessary, to ensure same stress-state as on the edges.

Following mechanical properties have been used for numerical investigation unless otherwise stated:

\[ E_L/E_T = 25, \quad G_{LT}/E_T = G_{LT}/E_T = 0.5; \quad G_{TZ}/E_T = 0.2; \]

\[ \nu_{LT} = 0.25 \]

where $E_L$, $E_T$, $G_{LT}$, $G_{TZ}$ and $\nu_{LT}$ are the Young's moduli along and across the fibre, inplane shear modulus, transverse shear moduli and major Poisson's ratio respectively.

The finite element developed in the preceding section is tested for many standard problems and found to yield very accurate results. The results presented in this section are obtained, employing $4 \times 4$ mesh over the plate. This idealisation is chosen on the basis of a convergence study. The finite element developed herein is conforming, therefore converges monotonically. The details of this convergence study and detailed comparisons are omitted for the sake of
brevity. However, to demonstrate the prediction accuracy of the element, bifurcation loads for simply-supported, square isotropic and symmetrically laminated three layer angle ply plates subjected to different edge loading are compared with those available in the literature. Table 1 shows that this element not only converges monotonically but yields very accurate results even for coarse mesh sizes. The comparison of bifurcation loads, presented in Table 2, once again indicates the prediction accuracy of the element for layered composite plates.

The boundary conditions employed in the analysis are:

i) Simply-supported edges (SSSS):
At edges \( x = 0, \) \( a : w = w' = 0 \)
At edges \( y = 0, \) \( b : w = w' = 0 \)
ii) Clamped - clamped edges (CCCC):
At edges \( x = 0, \) \( a : w = w' = w'' = w''' = 0 \)
At edges \( y = 0, \) \( b : w = w' = w'' = w''' = 0 \)

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Table 1—Stability parameters for simply supported square isotropic \((v = 0.3)\) plate subjected to different edge loadings

**Table 2—Stability parameters for simply supported square angle \([\theta^o / -\theta^o / \theta]^n\) plates.**

\[
E_L = 60.7; \quad E_T = 24.8; \quad G_{LT} = 12.0; \quad v_{LT} = 0.23
\]

\[
\lambda_u = \lambda_b = \lambda_{14} = \lambda_c = \frac{N_1 b^2}{D_o}; \quad \lambda_{13} = \lambda_{NT} = \frac{N_3 b^2}{D_o}; \quad D_o = \frac{E_L t^3}{12(1 - v_{LT}^2)}
\]

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<th>( \lambda_b )</th>
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iii) Simply-supported - clamped edges (SCSC):
At edges $x = 0$, $a$: $w = w'' = 0$
At edges $y = 0$, $b$: $w = w'' = w'' = 0$

iv) Clamped- simply supported edges (CSCS):
At edges $x = 0$, $a$: $w = w'' = w'' = 0$
At edges $y = 0$, $b$: $w = w'' = w'' = 0$

The bifurcation load parameters for different load conditions (shown in Fig. 2) are defined as:

$\lambda_{ax}$ (uniaxial) = $\lambda_{bx}$ (biaxial) = $\frac{P_x b^2}{E_t t^3}$

$\lambda_{ps}$ (+ve shear) = $\lambda_{ns}$ (-ve shear) = $\frac{P_{xy} b^2}{E_t t^3}$

$\lambda_{et}$ (compression – tension)
= $\lambda_{nt}$ (tension – compression) = $\frac{P_x b^2}{E_t t^3}$

$\lambda_{la}$ (linearly varying)
= $\frac{P_{xm} b^2}{E_t t^3}$ ($P_{xm}$ = Maximum value. Refer Fig. 2)

The effect of fibre orientation, sign of inplane shear loading and boundary conditions on the bifurcation load parameter is investigated in Figs 3-6. As discussed in the preceding section, when an antisymmetrically laminated plate with simply-supported edges is subjected to pure inplane shear $P_{xy}$, edge moments $B_{16}$ and $B_{26}$ are

Fig. 3—Variation of critical load with fiber orientation for a square simply-supported (SSSS) plate subjected to positive shear (PS) or negative shear (NS)

Fig. 4—Variation of critical load with fiber orientation for a square clamped (CCCC) plate subjected to positive shear (PS) or negative shear (NS)

Fig. 5—Variation of critical load with fiber orientation for a square plate with two opposite edges clamped and two opposite edges simply supported (CSCS) plate subjected to positive shear (PS) and negative shear (NS)

Fig. 6—Variation of critical load with fiber orientation for a square plate with two opposite edges clamped and other two edges simply supported (SCSC) plate subjected to positive shear (PS) and negative shear (NS)
generated at edges $x = 0$, $a$ and $y = 0$, $b$ respectively. These edge moments do not allow the bifurcation phenomenon to occur. Therefore, in such cases, additional edge moments equal in magnitude and opposite in sign are applied externally to counter balance these moments. This ensures pure inplane shear stress state and also ensures the existence of bifurcation phenomenon. Following observations can be made from this study.

The bifurcation load for regular antisymmetrically laminated angle ply plates does not depend on the sign of shear loading. It is because a plate with $[\theta / -\theta]_n$ lay-up and subjected to positive shear is just a turnover of the plate with $[-\theta / \theta]_n$ lay-up but subjected to negative shear. In case of symmetrically laminated plates, however, sign of shear loading strongly influences the bifurcation loads. It is observed that plates with $[\theta / -\theta]_n$ lay-up yield very high bifurcation loads when subjected to negative shear compared to positive shear. This is very useful information for the designers, because one can suitably choose the lay-up depending on the sign of shear loading.

The bifurcation load parameter increases with the increase in number of layers. Interestingly, buckling resistance of symmetrically laminated plates subjected to positive shear is as low as that of a two-layer angle ply plate.

The bifurcation load parameter of four-layered antisymmetric and symmetric angle ply plates subjected to negative shear increases with increase in fibre angle.

Fig. 7—Variation of critical load with fiber orientation for a square clamped (CCCC) plate subjected to tension-compression loading

Fig. 8—Variation of critical load with fiber orientation for a square clamped (CCCC) plate subjected to compression-tension loading

Fig. 9—Variation of critical load with fiber orientation for a square simply-supported (SSSS) plate subjected to in-plane moment loading

Fig. 10—Variation of critical load with fiber orientation for a square clamped (CCCC) plate subjected to in-plane moment loading
The plates with clamped-clamped edge conditions obviously yield highest bifurcation resistance.

Figs 7 and 8 show the variation of bifurcation load parameter with fibre orientation for anti-symmetrically/symmetrically laminated plates with clamped-clamped edge conditions and subjected to tension-compression and compression-tension edge loading respectively. It may be observed that bifurcation loads increase with the increase in fibre angle when plate is subjected to tension-compression edge loading. However, reverse is true for compression-tension edge loading. Therefore, plates with ±45° type of lay-up may be chosen when loading is tension-compression type and with 0° lay-up when loading is compression-tension type.

The variation of bifurcation load parameter with fibre orientation for anti-symmetrically/symmetrically laminated simply-supported and clamped-clamped plates subjected to linearly varying edge compression is presented in Figs 9 and 10 respectively. It may be noted that in this case once again, bifurcation exists unconditionally. Results indicate that four layered antisymmetrically laminated angle ply yields higher bifurcation loads compared to four-layered symmetric angle ply plates. In the case of four layered plates, [±30°]s and [±30°]c, lay-ups turn out to be nearly optimal for this loading. The plates with clamped edges are obviously stiffer, resulting in higher bifurcation loads compared to the ones with simply supported edges.

Conclusions

Present investigations on the bifurcation phenomenon in angle ply rectangular plates show that such plates do not always exhibit bifurcation unconditionally. The cases, for which bifurcation exists unconditionally/conditionally, are worked out. Conditions have been derived which if satisfied, bifurcation is guaranteed. Numerical results are presented for many plate configurations and loadings. The antisymmetrically laminated plate with simply-supported edges, under pure inplane shear loading is supplemented with additional edge moments, i.e., \( B_{36} \) \( P_{XY}/A_{66} \) and \( B_{36} \) \( P_{XY}/A_{66} \) at edges \( X=0 \), \( a \) and \( Y=0 \), \( b \) respectively to ensure bifurcation. It is found that symmetrically laminated angle ply plates are sensitive to the sign of inplane shear loading. Such plates have higher resistance to buckling under negative shear loading compared to positive shear loading. In general antisymmetrically laminated plates are found to be stiffer than symmetrically laminated plates. The boundary conditions greatly influence the bifurcation behaviour of angle ply plates for all the loading considered here.

References