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Investigation of stress homogenization near inner cavity in the polymer structure

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The stress concentrates near inner cavities in deformed structures. Sometimes, it is useful to obtain stress magnitude on cavity surface as uniform and low as possible. In this study, the attempt is made to determine magnitude of stress concentration near spherical cavity in polymeric structure loaded under unidirectional tension or compression and to find possibility to decrease and homogenize this stress with internal pressure.

Contemporary polymeric products and materials contain inner cavities are ubiquitous in miscellaneous applications[1-5]. The shape, size, distribution and nature of such cavities vary in the wide range. Some cavities come into being as defects during structure manufacturing or exploitation. The other ones are designed in purpose to decrease structure weight, to increase energy absorption, improve properties of thermal conductivity, etc. However, the stress concentrates near cavities under the loading. The location and value of concentrated stress depend upon the large number of factors, i.e., the load magnitude, mechanical properties of material, size and shape of cavity, etc. In all cases, the magnitude of stress concentration must be minimized.

The stress magnitude is the same on all surfaces of cavity under specific conditions only. Generally, the magnitude differs in different zones. It is useful to uniform such unbalance, i.e., to obtain stress magnitude on cavity surface as uniform and lower as possible.

Although the stress distribution near the cavity of loaded structures has been widely investigated[5-12]. The ways of stress reduction and homogenization near the cavity in the polymeric materials are a subject that has been poorly covered in the literature.

One way to obtain uniform stress on cavity surface is to use internal pressure. It is certainly true that the pressure magnitude will depend upon the value of factors influencing on the stress concentration magnitude. The pressure magnitude required to uniform stress in the case of the highest degree of structure deformation could be set as a boundary value.

The aim of this study is to determine magnitude of stress concentration near spherical cavity in polymeric structure loaded under unidirectional tension or compression and to find possibility to decrease and homogenize this stress with internal pressure.

Problem Formulation and Solution

The most practical structure for verification of above-mentioned idea is unidirectionally deformed volumetric element with inner spherical cavity (Fig. 1). In such element, the stress concentrates in six points, i.e., by two on every axis x, y and z (points A, B and C are on visible element side). In this case of deformation, stress values at points A and B are of equal value, and at point C, they differ.

Spherical coordinates are convenient to describe the local stress situation around the cavity because of the problem of the spherical geometry. The geometry and coordinate system are shown in Fig. 2. For the element under unidirectional deformation in the z-direction, the maximum stress at the surface of cavity appears at the pole ($\theta = 0$). For the calculation of stress around the spherical cavity, the method based on Lame's equations can be used, which summarize the equilibrium of stresses, stress-strain relation and

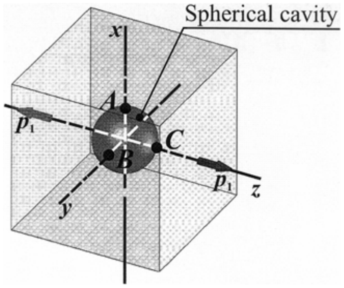

Fig. 1 — Scheme of volumetric element with inner spherical cavity

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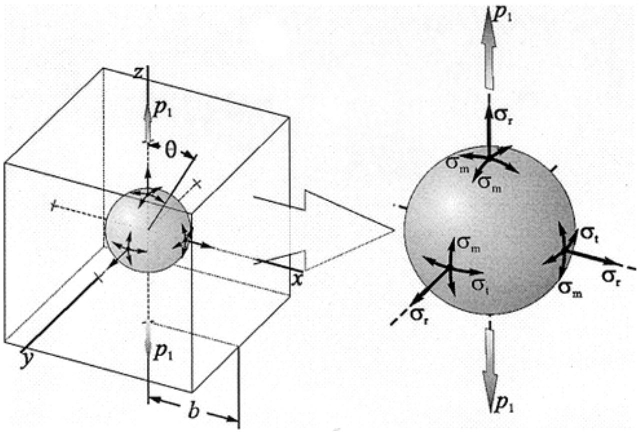

Fig. 2 — Geometry and coordinate system of a spherical cavity within rectangular element

relation between deformations and displacements and provide differential equations for the displacements (*u*, *v*, *w*)[6]. Each stress and strain tensor can be subdivided into the volumetric and deviatoric parts. It is possible to calculate stresses σ_r, σ_t and σ_m[8] as follows:

$$\sigma_r = \frac{p_1}{2}(1+\cos 2\Theta) + p_1\left[-\frac{13-10\nu}{2(7-5\nu)}\left(\frac{r}{a}\right)^3 - \frac{5\nu}{2(7-5\nu)}\left(\frac{r}{a}\right)^3 + \frac{3}{7-5\nu}\left(\frac{r}{a}\right)^5 + \left\{\frac{5(5-\nu)}{2(7-5\nu)}\left(\frac{r}{a}\right)^3 + \frac{9}{7-5\nu}\left(\frac{r}{a}\right)^5\right\}\cos 2\Theta\right], \quad \ldots(1)$$

$$\sigma_t = \frac{p_1}{2}(1-\cos 2\Theta) + p_1\left[\frac{13-10\nu}{4(7-5\nu)}\left(\frac{r}{a}\right)^3 - \frac{5}{2(7-5\nu)}\left(\frac{r}{a}\right)^3 - \frac{3}{4(7-5\nu)}\left(\frac{r}{a}\right)^5 + \left\{\frac{5(1-2\nu)}{4(7-5\nu)}\left(\frac{r}{a}\right)^3 - \frac{21}{4(7-5\nu)}\left(\frac{r}{a}\right)^5\right\}\cos 2\Theta\right] \quad \ldots(2)$$

where: p_1 is the external pressure; *r* is the radius of spherical cavity; ν is the Poisson's ratio; *a* is the distance from the centre of cavity; Θ is the angle.

In the case of the structural element loading under internal pressure p_2 acting in cavity, it is possible to use methods applicable for thick-walled spherical vessels:

$$\sigma_r = -p_2 \frac{r^2}{b^2-r^2}\left(\frac{b^2}{a^2}-1\right),$$

$$\sigma_t = p_2 \frac{r^2}{b^2-r^2}\left(\frac{b^2}{2a^2}-1\right), \quad \ldots(3)$$

where *b* is the length, width and height of the volumetric element; *r* is the radius of spherical cavity and *a* is the distance from the centre of cavity.

As the volumetric element is under complex loading, i.e., it is under the external load p_1 and internal pressure in the cavity p_2, according to principle of superposition, the stress could be expressed as a sum:

$$\sigma_i = \sigma_{i,p1} + \sigma_{i,p2}, \quad \ldots(4)$$

where *i* is the index of stress: *r*, *t* or *m*.

According to Eqs. (1), (2) and (3) stresses in the point *C* (θ = 0, *a* = *r*) can be calculated as follows:

$$\sigma_r = p_2 \frac{r^2-b^2}{b^2-r^2}, \; \sigma_t = p_2 \frac{b^2-2r^2}{2(b^2-r^2)} - p_1 \frac{5\nu+4}{7-5\nu},$$

$$\sigma_m = \sigma_t, \quad \ldots(5)$$

and at the points *A* and *B* (θ = 90°, *a*=*r*)

$$\sigma_r = p_2 \frac{r^2-b^2}{b^2-r^2}, \; \sigma_t = p_1\left(1+\frac{4}{7-5\nu}\right) + p_2 \frac{b^2-2r^2}{b^2-r^2},$$

$$\sigma_m = p_2 \frac{b^2-2r^2}{b^2-r^2} \quad \ldots(6)$$

Von-Misses stress is calculated as follows:

$$\sigma_e = \sqrt{\frac{1}{2}\left((\sigma_m-\sigma_t)^2 + (\sigma_t-\sigma_r)^2 + (\sigma_r-\sigma_m)^2\right)} \quad \ldots(7)$$

Thus, after mathematical operations, Eq. (7) can be written for the point *C* as follows:

$$\sigma_e = \sqrt{\left(p_1 \frac{4+5\nu}{7-5\nu} + p_2 \frac{4r^2-3b^2}{2(b^2-r^2)}\right)^2}, \quad \ldots(8)$$

and for points *A* or *B*, respectively,

$$\sigma_e = \sqrt{\frac{1}{2}\left[\left\{p_2\frac{4r^2-3b^2}{2(b^2-r^2)} - p_1\left(1+\frac{4}{7-5\nu}+\right)\right\}^2 + \left\{p_1\left(1+\frac{4}{7-5\nu}+\right)\right\}^2 + \left(p_2\frac{3b^2-4r^2}{2(b^2-r^2)}\right)^2\right]} \quad \ldots(9)$$

Thus, if equalization of von-Misses stresses at points *A*, *B* and *C* is done, it is possible to calculate pressure, which is needed in the spherical cavity. For this purpose, it is necessary to equalize Eqs (8) and (9) and to express the quantity that is needed, i.e., p_1 or p_2. It is practical to use Mathcad® to determine the quantities.

Technologically it is not difficult to produce such structure with the desirable magnitude of pressure within spherical cavity.

Methodics

The finite element analysis (FEA) was used to identify influence of internal pressure in spherical cavity on the stress state of structure. The analysis was performed by finite element code ANSYS. The 3D models were made to utilize symmetry. 3D 20-node structural solid elements were used. The exact number of element of each model depends on cavity size. Five volumetric models, which differ from each other in cavity size, were used to estimate influence of cavity size on stress magnitude at points *A*, *B* and *C*.

The geometrical parameters and material properties are presented in Table 1.

The dependence of von-Misses stress on the external load pressure p_1 as well as on the internal pressure in cavity p_2 was determined in points *A, B* and *C* when the diameter of cavity was varied. TableCurve 2000® was used for mathematical description of the dependencies. The cross-points of the obtained dependencies for different points (*A*, *B* and *C*) were determined by Parabola® v 1.0.

Table 1 — Geometrical parameters and material properties of model

Dimension	Value	Variation size
Length, mm	200	–
Width, mm	200	–
Height, mm	200	–
Young's modulus *E*, MPa	4	–
Poisson's ratio ν	0.499	–
Diameter of cavity *d*, mm	2 ÷ 10	2
External load pressure p_1, MPa	–2 ÷ 2	0.25
Internal pressure in cavity p_2, MPa	–2 ÷ 2	0.25

Results and Discussion

The stress concentration maximum point due to internal pressure migrates from points *A* and *B* to point *C*. The distribution of stresses calculated numerically and analytically (generalized stress and stress components) along OZ axis is presented in Fig. 3. The results obtained by FEM as well as obtained analytically according to Eqs (8) and (9) show the zone near cavity whereat stress decreases in the case of the model loading under external load only. This zone disappears as the internal load (pressure in cavity) acts.

The analogical results were obtained for all models investigated. The stress magnitude was varied, only. The dependences of von-Misses stress on external and internal loads are presented in Fig. 4. As it can be seen, these dependencies have extremes, i.e., minima. This lets make illation that there is such relation between p_1 and p_2 when stress magnitude at the points

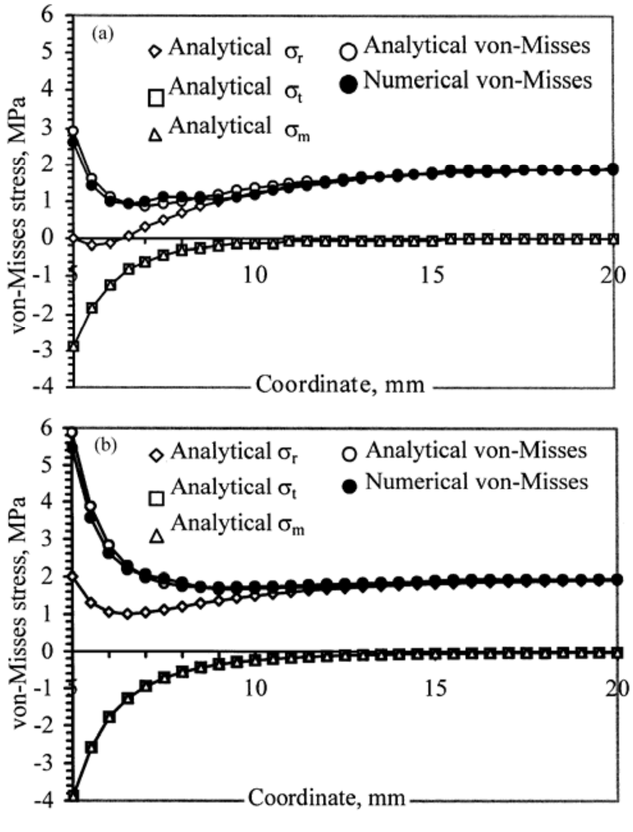

Fig. 3 — The distribution of von-Misses stress on OZ axis when *d*=10 mm, p_1=2 MPa (tension), and (a) p_2=0 MPa and (b) p_2=-2 MPa (vacuum).

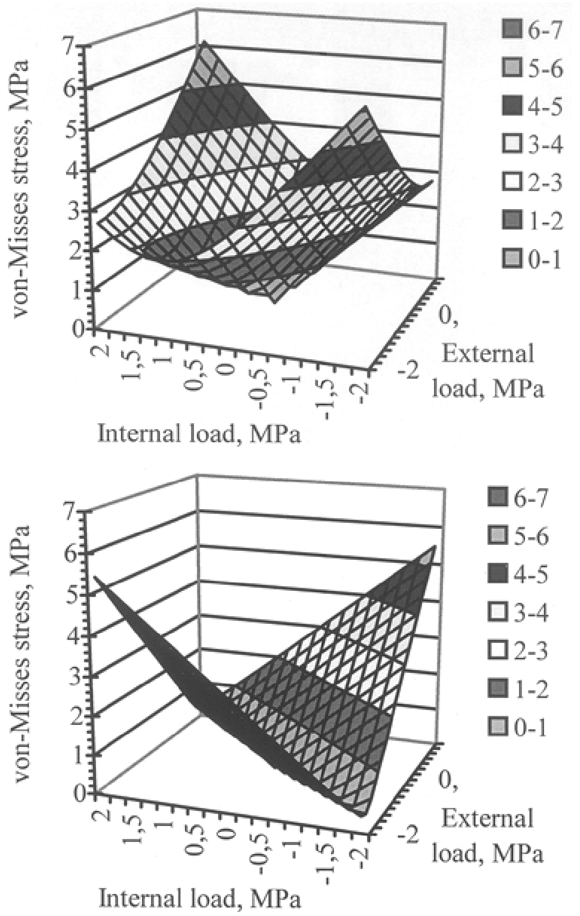

Fig. 4 — The dependencies of von-Misses stress upon p_1 and p_2 when d=10 mm(a) in points *A* and *B* and (b) point *C*

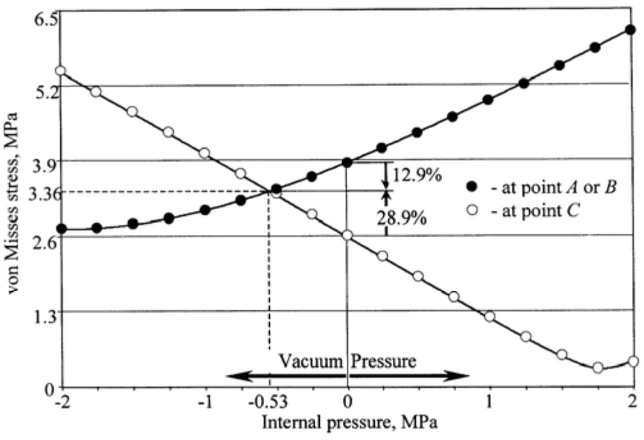

Fig. 5 — Von-Misses stress at points *A* (or *B*) and *C* dependence on internal pressure p_2 when p_1=2 MPa (tension) and d=10 mm

A, *B* and *C* is the same. For the better understanding of this thesis, let's discuss the change of stress in model loaded under tension (Fig. 5).

As the model is loaded under tension only, the maximum stress appears in cross-section XOY, and the minimum this in cross-section XOZ (Fig. 1), i.e., at the points *A*, *B* and *C*, respectively. Due to the model symmetry and spherical cavity, von-Misses stress is the same through all perimeter of cavity on cross-section XOY. Therefore, the stress magnitude at the point *A* is the same as at the point *B*. Thus, further only point *A* can be taken under discussion.

The model under unidirectional tension lengthens in direction of external load and shortens in other two perpendicular ones. If the positive internal pressure into the cavity is added, the principle of superposition is valid in the model and the cavity becomes wider. The stress at the point *C* decreases and this at the point *A* increases. That is so due to the directions of loads p_1 and p_2, which are the same at point *C*, and this is opposite at point *A*.

However, as the ratio p_1/p_2 reaches 0.90, the von-Misses stress starts to increase at the point *C*. It can be explained by the domination of material compression around the point under tension.

In the case of the cavity loading under negative pressure (vacuum), the directions of loads p_1 and p_2 are the same at the point *A*, and opposite at the point *C*. Therefore, the von-Misses stress increases at the point *C*, and this decreases at the point *A*. The stress magnitude at points *A* and *C* becomes of equal value as the pressure p_2 reaches on the average 28% of pressure p_1. The stress magnitudes are lower in 13 % at points *A* and *B* than these in the case of p_2=0 MPa. However, the stress magnitude increases by 29 % at the point *C*. From the viewpoint of position of smooth stress distribution, the redistribution of the stress concentration is important. On the other hand, from the standpoint of failure, the increase of stress by 29 % might provide unfavourable results.

The analogical, but "mirror-image" stress change is observed as the model is loaded under unidirectional compression.

The dependence of Von-Misses stress on both the load pressure and the internal pressure in cavity with high precision can be described by following equations:

$$\sigma_e = \sqrt{a_1 + b_1 p_1 + c_1 p_1^2} \qquad \ldots(10)$$

$$\sigma_e = \sqrt{a_2 + b_2 p_2 + c_2 p_2^2} \qquad \ldots(11)$$

where: $a_1 = f(p_2)$, $b_1 = f(p_2)$, $c_1 = f(p_2)$ and $a_2 = f(p_1)$, $b_2 = f(p_1)$, $c_2 = f(p_1)$ are the constants

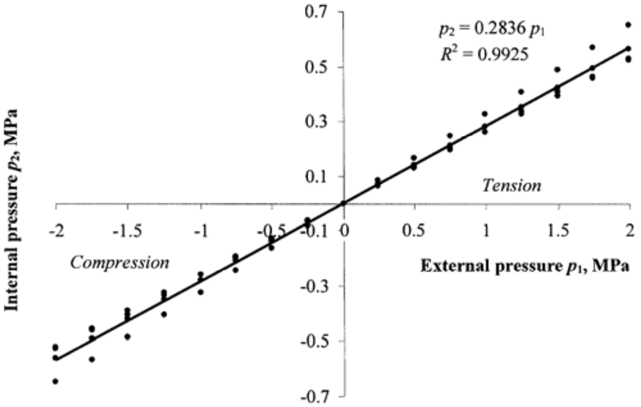

Fig. 6 — Changes of the load in the cavity p_2 as the stress in all investigated points is of the similar value in the dependence upon the element load p_1

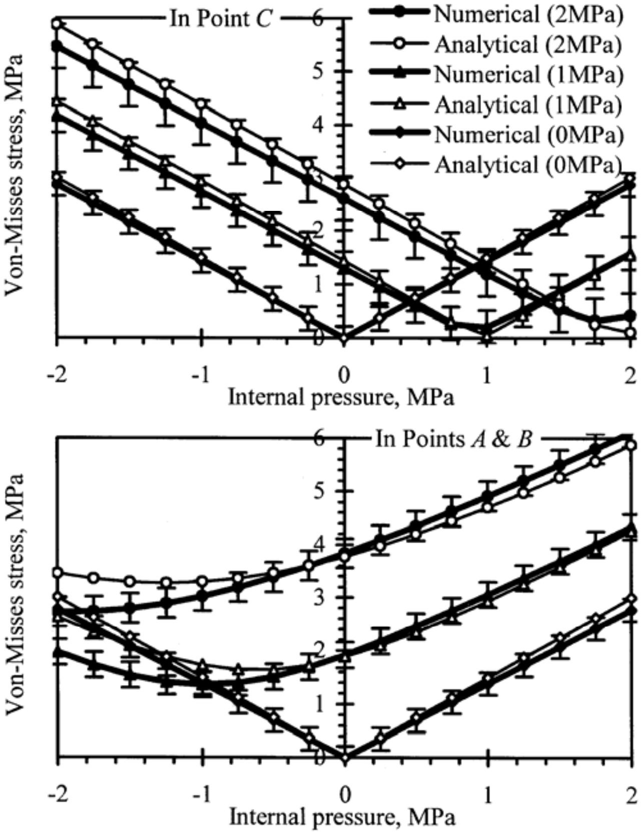

Fig. 7 — The relationship of analytical and numerical results obtained for structure with cavity (d = 10 mm) when external load is equal 2 MPa, 1 MPa and 0 MPa

depended on geometry of structure and on load history.

The dependence of the pressure in the cavity p_2, when the stress in all points investigated is of the similar value, upon the element load p_1 is presented in Fig. 6. It is seen that this dependence can be described by linear equation as the high determination factor R^2 is obtained. From this relationship, it is clearly seen that the ratio of the load in the cavity to the load of element is equal to 0.28. It means that in order to obtain the homogeneous distribution of stress near the cavity, the load in the cavity must be equal to 28% of general load. It can be noted, if the loading is compression, the homogenous stress distribution near cavity can be reached with vacuum in the cavity, and if the tension is applied, the pressure must act in the cavity.

The relations between analytical and numerical calculations for model with cavity diameter d = 10 mm are presented in Fig. 7. As it can be seen, the higher loads are applied, the higher difference between analytical and numerical calculations is observed.

Conclusions

The structures with inner spherical cavity loaded under external and internal loads, the magnitudes of which vary, were investigated. The dependencies of von-Misses stress upon the loads were determined by analytical, numerical and experimental methods. Equations were developed for magnitude determination of external or internal load when the magnitude of one-in-two load is known. The results obtained by the analytical method were proved by experimental and numerical methods. The investigation proved that it is possible to redistribute stress in structures loaded under tension or compression so the stress concentration magnitudes would be smoother near cavity. The internal pressure is a means to an end.

Nomenclature

σ_r, σ_i, σ_m = the deviatoric parts of stress
σ_e = the generalized von-Misses stress
p_1 = the external load (pressure)
p_2 = the internal load (pressure)
r = the radius of spherical cavity
ν = the Poisson's ratio
a = the distance from the centre of cavity
b = the length, width and height of the volumetric element.

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