Shovel-truck optimization study in an opencast mine—A queueing approach

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The shovel-truck combination is the versatile work horse in small as well as large opencast mining projects. The optimization of shovel-truck combination is imperative in order to increase productivity from this system. The optimization studies aim at eliminating the excess shovel or excess truck capacity in order to minimise the shovel and/or truck idling period. The present case study is the approach of queueing theory to the optimization of shovel-truck combination in an Indian open cast mine. The formulation of appropriate queueing model has been undertaken and the $(M/M/C; (FFO/K/K))$ model has been adjudged to be the best model to suit the shovel-truck optimization problem. Relationships between the length of queues, waiting time in queue, utilisation of shovel, approximate output from the shovel-truck combination and the costs involved in the system are critically analysed with respect to variation in the number of trucks. The optimum total cost and the optimum number of trucks in the case-study are deduced from the proposed queueing model.

The development of large open cast projects has necessitated developments in the state of mechanization also. Thus, a rapid growth to suit the increasing size and larger outputs has been given an immense headway to huge sized machineries which incorporate sophisticated technology. Shovel-truck combination appears to be the main work horse of opencast mining projects. Due to the technological upgradations, the rope shovels as well as trucks are now available in large capacities. The rope shovels are as large as 20 cu.m. capacity and rear dump trucks as large as 240 ton capacity are currently being manufactured and used. It is of interest to optimize the shovel-truck combination in order that neither shovel nor truck idles even for a few seconds, while in operation.

Operation research and critical analysis of shovel-truck combination can be done by two approaches—the former one being dependent on the cycle time analysis and the latter one is based on the “queueing-concept”. The present paper deals with the queueing theory and its application in solving the ticklish problem of shovel-truck optimisation. The theory provides a basis for decision making in relation to the size and number of trucks.

Application of Queueing Concept to Shovel-Truck Optimisation

The queues are formed because of higher demand for service beyond the capacity of the server. In shovel-truck combination shovel is designated as SERVER meeting the demands of service by loading the trucks which are CUSTOMERS. A typical example of queueing problem in underground mines has also been cited by Huang and Kumar by considering the loading by Load Haul Dump (L.H.D.) machines.

A typical schematic of a queueing system with a number of parallel servers is illustrated in Fig. 1. The simulation of this queueing concept to the shovel-truck combination, where one server (one shovel) provides service to a number of trucks by loading them, is proposed here and is illustrated in Fig. 2.
Formulation of Queueing Model for Shovel-Truck Combination

Formulation of an appropriate queueing model necessitates some stochastic characteristics and related assumptions. The model is formulated in accordance to truck arrival pattern, service pattern, size of truck-fleet, service discipline, ultimate queue size and cost of idling to arrival and service units are to be made realistic.

Various stochastic characteristics and assumptions made in the formulation of model are discussed as—

1. Truck arrivals, being a random function, can be best described by Poisson's distribution, according to this, the probability mass function is

\[ P(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}, \quad x = 0, 1, 2, \ldots \]

Here the number of trucks per unit time at the server is called truck arrival rate, being designated as \( \lambda \).

The assumptions for the adoption of Poisson's distribution are given below:

(a) An arrival may occur at random at any time.
(b) The occurrence of an arrival in a given time interval is independent of arrivals in other non-overlapping intervals.
(c) The probability of an arrival in a small interval \((t, t+h)\) is proportional to interval length \(h\) and equal to \(\lambda h\).
(d) The probability of more than one arrival in this small interval is negligible.

For similar assumptions on the arrivals the exponential distribution gives information on the time till the first arrivals. Hence,

\[ P(\text{arrivals occur at time } t) = P[x = 0, \text{ time period } t] = e^{-\lambda t} \]

here \(t\) is inter arrival time as arrivals are independent of each other.

2. Shovel's service times may be described similar to interarrival times. For a service rate \(\mu\) (number of services per unit time that the server is busy) of \(\mu\), the density function for the time \(t\) to serve a unit is often exponentially distributed as:

\[ P_s(t) = \mu e^{-\mu t} \]

That is, successive serving times are assumed independent. For a small interval of time \((t, t+h)\), the probability of completing a service in this interval is proportional to interval length \(h\) and equals to \(\mu h\) (ref. 4).

3. Service discipline is FIFO (first in first out).

4. Availability of all trucks and shovels is homogeneous.

It is easy to notice that if availability of all trucks is homogeneous then it doesn’t matter which truck is directed by dispatching rules to given shovel or dumping point in given moment. All shovels are equivalent in statistical sense taking this property (availability) into account. Similarly, all trucks are equivalent in the same sense and this regularity permits decomposition of large shovel-trucks system into the elementary one shovel-trucks system.

5. All the trucks used are of same type and of same capacity.

6. The service mechanism is single channel and single phase for simplifying the problem, the queuing time at dumping point is ignored.

7. The number of trucks corresponding to one shovel is finite and size of this fleet of trucks is equal to \(k\) and the queue capacity is also equal to or greater than \(k\). Hence this parameter doesn’t influence the calculation unless it is less than \(k\).

Thus, the whole system can be illustrated by the following queueing model—

\[(M/M/C) : (FIFO/K/K)\]

here, \(M/M\) represents the Poisson arrivals and exponential service distributions. \(C\) is the number of parallel server. First in first out, FIFO, is the service discipline and \(K/K\), the size of population of trucks and ultimate queueing capacity.

Evaluation of Important Parameters from the Proposed Model

For this finite source queueing model, some performance parameters have to be evaluated which are as follows:

![Diagram of proposed queueing system of shovel-truck combination](image-url)
Traffic Intensity ($e$) —
\[ e = \frac{\lambda}{\mu} \] (for the single shovel case)

here, $e < 1$, otherwise queue will be going on increasing.

Probability of 'n' trucks in the system ($P_n$) —
\[ P_n = P_0 \left[ \frac{K! \left( \frac{\lambda}{\mu} \right)^n}{(K-n)!n! \left( 1 + \frac{K!}{(K-c)!c! e^{\lambda-\mu}} \left( \frac{\lambda}{\mu} \right)^n \right)^1} \right]^{(K-n)!c!c^n} \]

(c) Expected number of trucks in the queue, i.e. length of queue ($L_q$) —
\[ L_q = \sum_{n=0}^{k} (n-c) P_n \]

(d) Expected number of trucks in the system ($L_s$) —
\[ L_s = L_q + \text{number of trucks being loaded} \]

or, \[ L_s = \sum_{n=0}^{k} nP_n \]

(e) Most expected value of truck arrival rate ($\lambda^*$) —
\[ \lambda^* = \sum_{n=0}^{K} \lambda(K-n)P_n = \lambda(K - L_s) \]

(f) Expected time that a truck spends in the queue ($W_q$) —
\[ W_q = L_q / \lambda \]

(g) Expected time that a truck spends in the system ($W_s$) —
\[ W_s = L_s / \lambda \]

(h) Shovel utilization ($\eta_s$) —
\[ \eta_s = 1 - P_0 \]

(i) Truck utilisation, ($\eta_t$) —
\[ \eta_t = 1 - \frac{W_q}{t_l + t_i + t_d} \]

where, $t_l$ = loading time, $t_i$ is the total travelling time (loaded truck travel time + empty truck travel time), $t_d$ is the dumping time and $W_q$ is the waiting time of truck in queue.

(j) Hourly production of the shovel ($Q_n$) —
\[ Q_n = n \cdot \mu \cdot q_n \text{ tons} \]

where, $q_n$ is the capacity of truck in ton.

(k) Total cost of the operation $C_T$ per ton of material moved —
\[ C_T = \left( \frac{C_m + K C_{tr}}{Q_n} \right) \text{ money units/ton} \]

where, $C_m$ is the owning and operating cost of the shovel per hour, money units/h and $C_{tr}$ is the owning and operating cost of each truck per hour, money units/h (ref. 8).

Case Study —
After formulating the appropriate queuing model, it was found appropriate to testify the model in actual field setting in an open cast mine deploying 4.6 cu.m. capacity shovel loading on the 50 tons Rear Discharge Trucks. The queueing theory aims to minimising the waiting and idling time of each equipment.

On the basis of field observations, the performance evaluation of the existing shovel-trucks system has been undertoken. The interarrival times of trucks and interdeparture times after the loading of the truck were observed from a number of such field observations; mean interarrival time and mean interdeparture time were estimated and these were used as basic field data for evaluation of the system. The field observations are given in Table 1.

Results and Discussion —
Based on the values mentioned in Table 1, various evaluation parameters are estimated for the case under study. The effect of variation in truck fleet size on the queue length, waiting time, shovel utilization, approximate production and on the total operating cost has been evaluated. With the variation in the truck fleet size, arrival rate and service rate may also vary but for convenience this small variation has been ignored and the results are shown in Table 2.

Table 1 — The field observations in shovel-trucks combination

<table>
<thead>
<tr>
<th>Items and notations</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrival rate ($\lambda$)</td>
<td>8 trucks/h</td>
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<tr>
<td>Service rate ($\mu$)</td>
<td>16 trucks/h</td>
</tr>
<tr>
<td>Traffic Intensity ($e = \lambda/\mu$)</td>
<td>0.5</td>
</tr>
<tr>
<td>Number of shovels ($c$)</td>
<td>1</td>
</tr>
<tr>
<td>Owning and operating cost of shovel $C_m$</td>
<td>2500 money – unit/h</td>
</tr>
<tr>
<td>Owning and operating cost of truck $C_{tr}$</td>
<td>1000 money – unit/h</td>
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</table>
The results obtained from Table 2 are initially analysed to establish the relationship between (i) length of queue and number of trucks \( L_q \) vs \( k \), (ii) the waiting time in queue and number of trucks, i.e., \( W_q \) vs \( k \), (iii) shovel utilization and number of trucks, i.e., \( \eta_t \) vs \( k \), (iv) shovel output and number of trucks i.e., \( Q_o \) vs \( k \), and (v) cost elements and number of trucks, i.e., cost vs \( k \).

\( L_q \) vs \( k \) and \( W_q \) vs \( k \) relationship

The graphs given in Figs 3 and 4 exhibit a linear relationship between \( L_q \) and \( k \) which obviously implies that as the population of trucks increases, the length of queue as well as the waiting time in queue also increases.

\( \eta_t \) vs \( k \) relationship

The typical curve is shown in Fig. 5. The nature of curve is such that as the population of trucks increases initially the utilisation of shovel also increases but only up to a certain point. After this point the curve becomes steady and the additional number of trucks beyond that point would simply mean increase in queue lengths and waiting time of trucks in the queue.

\( Q_o \) vs \( k \) relationship

This relationship between \( Q_o \) and \( k \) is represented by the curve shown in Fig. 6. The nature of curve is similar to the earlier curve which means that initially an increase in number of trucks increases the output of shovel. But, up to a specified number of trucks only. Beyond which the output of shovel becomes steady and excessive number of trucks with the shovel result in the formation of long queue and excessive idling time of the trucks.

Costs vs \( k \) relationship

The cost of loading per ton is falling down with increase in number of trucks as shown in Fig. 7 (curve a). However, with the increase in number of trucks, hauling cost, per ton, is rising as shown in curve b.

The curve c is obtained by summing up the curves a and b. The summation curve reveals the total oper-

<table>
<thead>
<tr>
<th>Truck fleet size</th>
<th>Queue length ( L_q ) (Trucks)</th>
<th>Waiting time ( W_q ) (min.)</th>
<th>Production ( Q_o ) ton/h</th>
<th>Total cost of operation money unit / ton</th>
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<tr>
<td></td>
<td>( L_q ) (Trucks)</td>
<td>( W_q ) (Trucks)</td>
<td>( W_t ) (min.)</td>
<td>( % )</td>
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<td>20.01</td>
<td>24.01</td>
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</table>

Fig. 3—Relationship between length of queue and number of trucks

Fig. 4—Relationship between waiting time in queue and number of trucks
TRIVEDI et al.: SHOVEL-TRUCK OPTIMIZATION STUDY IN AN OPENCAST MINE

Fig. 5—Relationship between shovel utilization and number of trucks

Fig. 6—Relationship between shovel output and number of trucks

Fig. 7—Relationship between cost element and number of trucks

The null point 'o' is the point where the total costs of the shovel-truck combination is minimum. The number of trucks corresponding to this null point are the matching number of trucks of 50 ton capacity with 4.6 cu.m. shovel and that are 4 in this case.

Conclusion
1. The Queueing model (M/M/C) : (FIPO/K/K) appears to be very useful tool for solving the optimisation problems of shovel-truck combination in open-cast mines. The stochastic characteristics and assumptions in the formulation of the model must be distinct and logical.
2. Length as well as waiting time in a queue is linearly related to number of trucks in shovel-truck combination.
3. The shovel utilisation and shovel output increase with the increase in the number of truck up to a specified number of trucks beyond which it becomes steady and this point at which \( \eta_t \) and \( Q_{st} \) become steady, is notably important.
4. The total costs of shovel-truck combination system exhibit a null or optimum cost point. This null point must be obtained in the shovel-truck combination to find the appropriate number of trucks with specified capacity of shovel.

Queueing model can be used in shovel-truck operations as a tool in selecting the size and required number of equipment scheduling them, evaluating loading procedures, haul route design parameters such as haul road length, grades and rolling resistance as well as other relevant economic parameters.

References