A 3-D model study for 2/1 twill and 3/1 twill weaves

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In this study, the 3-D geometry of 2/1 twill and 3/1 twill weaves have been studied in order to obtain realistic twill weave models by using certain structural parameters of the fabric such as settings and yarn linear density. The inherent skewness of twill fabrics in the floating region has been taken into consideration for modeling. The path of the floating and intersecting regions of the yarn center axis within fabric is first modeled as linear inside a rectangular prism while forming the 3-D geometry of twill weaves. Then B-spline method is used to define the 3-D path of the yarn with a smooth curve. The theoretical crimp factor and mass per unit area of the 2/1 twill and 3/1 twill fabrics have calculated from the obtained B-spline curves of 3-D fabric and they are compared with the experiment results. Besides, the visual 3-D simulations of twill weaves are also formed. The experimental results verify the results obtained from the 3-D model. Depending on structural parameters and inherent skewness property, reliable twill weave models are achieved which would be helpful in computer applications.

Keywords: 3-D fabric, B-spline curve, Geometrical modeling, Skewness, Twill weave

1 Introduction
The number of three-dimensional (3-D) model studies has increased in recent years together with developed computer-based design systems. The obtained 3-D fabric models could help designers to simulate the fabric appearance more accurately. On the other hand, it is important to have the fabric model as identical as possible to the real fabric especially in order to predetermine the performance properties of the product during the design stage by integrating 3-D models with special software.

The modeling studies of fabrics could be investigated into two groups, namely mechanical model and geometrical model. Mechanical models incorporate the energy balance while in geometrical models the parameters of the fabric are used in order to model the fabric¹. The reasons to prefer the geometrical models can be explained by the fact that their mathematical analysis is easier compared to mechanical models² and it helps to analyze the mechanical models when combined with mechanical models; and especially, it allows the designers to establish the geometrical structure, monitor it, and modify it when necessary, especially in computer-aided designs. The 2-D (two-dimensional) geometrical model, which is used most commonly and taken as the basic one while defining woven fabrics, is the ideal fabric geometry model recommended by Peirce³. According to Peirce geometry, yarn movement is formed of circular arcs and linear lines. This model is later improved by assuming the yarn cross-section in the shape of a race track⁴ and by applying it to the weaves other than the plain weave⁵. Geometrical models are established by accepting the yarn path as circular arcs⁶, linear lines⁷, and sinus curves⁸ in order to define the geometry of the fabric. The first study, conducted in an effort to define the three-dimensional structure of fabric, was made by Hoskins⁹. Later, other researchers modeled the fabric geometry as 3-D by using solid modeling techniques¹⁰ and differential calculations¹¹. One of the methods, which is preferred while modeling the geometrical structure of woven and knitting fabrics as 3-D, is B-spline curves, used especially in computer-aided vessel, plane, and automobile designs. 3-D fabric models were obtained using the B-spline method in modeling woven and knitting fabrics¹²-¹⁷.

The number of 3-D model studies increased in recent years together with developed computer-based design systems (CAD, CAM). The obtained 3-D fabric models were integrated with the mechanical or fluid model using software, developed to predetermine the end-use properties of fabrics such as mechanical properties and permeability properties¹⁸,¹⁹. In this study, 3-D geometrical models of relaxed twill fabrics

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with different weft settings and different weave types are investigated. Modeling of twill fabrics is different than other weave types because warp and weft floats are not perpendicular to each other. Twill weaves have an inherent skewness. The skewness in the floating region is taken into consideration for modeling twill fabrics in this study. The path of the yarn center axis modeled as linear at the beginning is obtained as a smooth curve by using the B-spline curves method. The centerline yarn curve, obtained from the model, defines the yarn geometry based on fabric parameters. The 3-D yarn geometry within the fabric is defined by rendering the yarn diameter, assumed as being ellipse to the obtained yarn center curve. The cross-sectional shape and size of the yarn are assumed to be constant along the yarn path. The change in the yarn cross-sectional shape due to the lateral contact between the yarns is neglected. Later, 3-D simulations are obtained based on the structural parameters by combining weft and warp yarns to form a weave unit. The model is then checked by comparing the crimp factors and mass per unit area of the fabric, calculated using the B-spline curve, with experimental results.

2 Materials and Methods

In this study, two groups of twill weaves (2/1 twill and 3/1 twill weave) were used in order to obtain the 3-D geometry of fabrics. The structural properties of cotton fabrics, produced in three different weft settings for every weave type according to the setting theory and production conditions and used to test the developed model, are given in Table 1. Cotton yarn (29.53 tex) was used in both warp and weft directions. Some structural properties of fabrics such as settings, yarn count, fabric thickness and skewness angle were used as input values of the model. On the other hand, the measured mass per unit area and measured crimp factors were chosen as control parameters which were compared with the model results. Mass per unit area of the fabrics was measured according to ISO 3801\textsuperscript{20}. The thickness of the fabrics was measured according to ISO 5084\textsuperscript{21}. Minitab 14, a statistical software program, was used to evaluate the experimental and theoretical results. The experimental crimp factors of warp and weft yarns ($k_m$) of each weave types were calculated from the ratio of measured un-crimped yarn length ($\ell'_m$) to the crimped yarn length ($\ell_m$), as shown in following equation:

$$k_m = \frac{\ell'_m}{\ell_m}$$ ...

The skewness angles of the floating regions of weft and warp yarns of relaxed twill fabrics were determined using the measuring tool within Adobe Photoshop CS3 10.0. To carry out this process, first the face and back surfaces of the fabrics were scanned in order to obtain the skewness angle of floating yarns both in the warp and weft directions. Afterwards, as shown in Fig. 1, two lines were drawn on the floating region of the yarn by using guide lines and measuring tool. The first line was a straight line between guide lines of the floating movement and the other line was an inclined line which followed the movement of the float. The inclined line was started at the beginning of the floating, tracking the movement of the float and ended at the finishing point of floating segment. The angle between these two lines was defined as skewness angle. The average skewness angle was calculated by making 20 measurements on different regions of scanned fabric images.

2.1 B-spline Method

In computer graphics, the formation of an object could be considered as a combination of lines, curves, and surfaces. A curve or a surface could be defined with a group of points with small distances between them\textsuperscript{22}. The curves used in computer graphics were complex shapes formed by adding numerous curve sections end-to-end\textsuperscript{9}. The B-spline method is used as a more effective method in comparison to other

<table>
<thead>
<tr>
<th>Fabric code</th>
<th>Weave type</th>
<th>Measured mass per unit area ($w_m$), g/m²</th>
<th>Calculated mass per unit area ($w_c$), g/m²</th>
<th>Fabric thickness ($t$), cm</th>
<th>Warp-weft setting ($S_1$-$S_2$), cm(^{-1})</th>
<th>Warp-weft crimp factor ($k_{m1}$-$k_{m2}$)</th>
<th>Warp-weft skewness angle ($\alpha_1$-$\alpha_2$), deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>2/1 Twill</td>
<td>158</td>
<td>167</td>
<td>0.038</td>
<td>36-18</td>
<td>1.036-1.067</td>
<td>4-3</td>
</tr>
<tr>
<td>A2</td>
<td>2/1 Twill</td>
<td>171</td>
<td>180</td>
<td>0.038</td>
<td>36-22</td>
<td>1.034-1.075</td>
<td>3-2</td>
</tr>
<tr>
<td>A3</td>
<td>2/1 Twill</td>
<td>186</td>
<td>199</td>
<td>0.039</td>
<td>36-26</td>
<td>1.076-1.099</td>
<td>3-1</td>
</tr>
<tr>
<td>B1</td>
<td>3/1 Twill</td>
<td>161</td>
<td>165</td>
<td>0.040</td>
<td>36-18</td>
<td>1.027-1.062</td>
<td>3-4</td>
</tr>
<tr>
<td>B2</td>
<td>3/1 Twill</td>
<td>177</td>
<td>179</td>
<td>0.041</td>
<td>36-22</td>
<td>1.022-1.080</td>
<td>2-2</td>
</tr>
<tr>
<td>B3</td>
<td>3/1 Twill</td>
<td>190</td>
<td>195</td>
<td>0.041</td>
<td>36-26</td>
<td>1.040-1.098</td>
<td>2-2</td>
</tr>
</tbody>
</table>
mathematical methods because of the advantages of the control properties, such as local control property and continuity in connection points.

Control points, the order of the curve, and the knot sequence should be known in order to define B-spline curves. Control points were the corner points given to define B-spline curves and surfaces. These control points formed a control polygon by coming together (Fig. 2). The curve did not pass through these control points; however, control points enabled the curve to follow the shape of the control polygon, and made sure that it was a smooth and characteristic curve within these borders. One of the basic properties sought in B-spline curves, composed of one or more polygons, was the continuity in connection points; this condition was achieved by using second-order continuity \( C^2 \). The point at which spline segments got connected was called the knot. The biggest advantage of spline curves, used in computer designs, was that the curves were not global curves compared to other curve forming methods. That is, a change made to one point of spline curves affected only the curve in that specific region. This local control property was desired in design and model studies.

Ideal mathematical definition of the B-spline function was given by following Eqs (2) and (3)\(^\text{24,25}\). In the equations, \( k \) was the order of the curve, \( N_{i,k} \) was the \( i \)th normalized B-spline basis function with a \( k \) order, \( t \) was the curve parameter that changed between \( t_{\text{min}} \) and \( t_{\text{max}} \) along the B-spline curve \( P(t) \), \( x_i \) was the elements of the knot vector, \( n+1 \) was the number of control points, and \( B \) was the coordinates of the \( i^\text{th} \) control point. The degree of the polygon was one less than its order \( (k-1) \), and \( 0/0 = 0 \) assumption was made for this mathematical definition. Equations (2) and (3) are given below:

\[
N_{i,k}(t) = \begin{cases} 
1 & x_i \leq t \leq x_{i+1} \\
0 & \text{else} 
\end{cases}
\]

\[
N_{i,k}(t) = \frac{(t-x_i)}{(x_{i+k-1}-x_i)}N_{i,k-1}(t) + \frac{(x_{i+k}-t)}{(x_{i+k}-x_{i+1})}N_{i+1,k-1}(t) & 1 \leq i \leq n+1
\]

\[
P(t) = \sum_{i=1}^{n+1} B_i \times N_{i,k}(t) & t_{\text{min}} \leq t \leq t_{\text{max}}, \ 2 \leq k \leq n + 1
\]

Knot vectors, the type of the B-spline basis function, the order of the basis function, the number and location of control points, the repeating control points, and the repeating parameter values were the properties that controlled the shape of the B-spline curve. The B-spline method is used as a more effective method in comparison to other mathematical methods because of the advantages of these control properties. In this study, the yarn centre curve was established with using open non-uniform cubic B-spline curves by determining the coordinates of control points according to the weave-type model.

2.2 Theoretical Model for Twill Weave

The definition of fabrics as 2-D or 3-D arises by defining the yarn geometry within the fabric, based on the structural parameters such as the fabric construction, settings, weave type, yarn linear density, and yarn cross-section. The cross-section shape of the yarn is one of the factors that results in errors in modeling studies. Numerous studies modeled the yarn cross-section as circular, race track, elliptical, and lenticular shaped. The shape and size of the yarns in the fabric were formed by the forces during the weaving and by the structural parameters. An elliptical yarn cross-section shape was preferred because in the fabric, yarns were compressed at the contact points. The cross-section shape of the yarn changed within the fabric. However, in this study, analysis was made by considering the cross-section shape of the yarn, which changed within the weave, to be an approximate ellipse and constant along
the yarn path. The circular yarn diameter ($d$) was first calculated using the Ashenhurts approach to determine the cross-sectional size of the yarn; then, the cross-sectional size of the yarn was calculated by assuming the circle and ellipse circumferences as equal. Calculated circular yarn diameter according to Ashenhurts formula is given in following Eq. (4). Here $N$ was the yarn count in metric system and $K$ was a constant about the yarn spinning system and defined as 8.3 for cotton yarn. The relationship between circular yarn diameter ($d$) and the major ($a$) and minor ($b$) diameters of elliptical yarn cross sections is given in following Eq. (5). The major ($a$) and the minor ($b$) diameters of the ellipse were determined by considering the flattening rate ($e=b/a$). In the calculations, the value of flattening ratio was assumed as 0.7. In this study, the fabric thickness parameter, used as input in the model, was assumed to be formed by warp yarns, and the thickness and amplitude of weft yarns were calculated assuming that warp and weft yarns are in contact with each other. Equations (4) and (5) are given below:

$$d = \frac{1}{K\sqrt{N}} \quad \ldots \quad (4)$$

$$d = \frac{a + b}{2} \quad \ldots \quad (5)$$

The characteristic property of twill weaves is that the weft and warp floatings are not perpendicular, but form an angle. This state is named as skewness. While the yarn skews during long floating regions due to the force applied by other yarns, it skews in the opposite direction during the intersecting motion, and returns to its former position (Fig. 3). Thus, the balanced fabric geometry occurs in twill weaves.

The control polygon of fabrics like 2/2 twill weave was defined by the floating and intersecting segments in weaves, which created symmetric floatings on front and back surfaces of fabrics. In this study, 2/1 twill and 3/1 twill weaves were used. These were twill fabrics formed of asymmetric floatings. Therefore, the yarn path of these twills could not be modeled like 2/2 twill weaves. Because in twill fabrics (2/2 or 3/3 twill) the length of floating sections was same in both surfaces, and hence the forces were assumed to be equilibrium in both intersecting regions. But in twill weaves (2/1 and 3/1 twill), length of floating sections were not equal on both surfaces and therefore equilibrium condition was different from symmetric floatings. For such fabrics, the path of the yarn center axis was modeled linearly in the rectangular prism (Fig. 4), and a control polygon was established for the weave unit. It is easy to imagine the linear yarn path in a rectangular prism. Accordingly, the yarn made an angle by skewing in the floating region, and then it made a new angle in the intersecting region to the opposite direction until it reached to the next floating position. According to the model as shown in Fig. 4, the linear floating and intersecting lines in twill weaves were located in different planes. The region that made the return movement in the model was termed as the intersection region. The projection of the linear lines of the intersection region, formed of two inter-sectings ($s_i$) and one short floating line ($s_f$), on the xy plane were assumed to be equal.

![Fig. 3—Forces affecting yarns and the formation of skewness [(a) 3/1 twill weave and (b) 2/1 twill weave]](image-url)
They-axis was rotated around the z-axis as much as the skewness angle \((\alpha)\) in order to analyze the geometry of the 2-D yarn centre axis based on fabric parameters; as a result, a new x’y’z’ space was formed and the projection of the control polygon on the y’z’ plane was analyzed (Fig. 4). Figure 5 shows the 2-D fabric geometry of the 2/1 twill and 3/1 twill weave on the y’z’ plane. Following Eq. (6) was used to calculate the major diameter \((a’)\) of the weft yarn, in the opposite position, on the new projection plane.

The weave unit length, defined as \(p_1\) for the warp yarn, was \(pr_1\cos\alpha_1\) for the new plane. The unit weave length \((pr)\) of 2/1 and 3/1 twill weaves was defined in following Eqs (7) and (8) as related to floating length \((\ell)\), the projection length of the intersection region on the x-y plane \((\ell_f)\), the skewness angle \((\alpha)\), the return angle \((\gamma)\), and the warp and weft settings of the fabric \((S_1, S_2)\) respectively. In the equations, subscripts 1 and 2 refer to parameters in the warp and weft directions respectively, as shown below:

\[
a'_1 = a_1 \sec(\alpha_1 + \alpha_2) \quad \text{... (6)}
\]

\[
pr_1 = \frac{3}{S_2} = \ell_{f1} \cos \alpha_1 + \ell_{s1} \cos \gamma_1 \quad \text{... (7)}
\]

\[
pr_1 = \frac{4}{S_2} = \ell_{f1} \cos \alpha_1 + \ell_{s1} \cos \gamma_1 \quad \text{... (8)}
\]

All of the relationships between the parameters of linear models such as lengths and angles were written by using equations for different planes. Then, the value of optimum floating length \((\ell)\) was aimed to define in order to solve the linear 2-D geometry of 2/1 and 3/1 twill models mathematically (Fig. 5). The optimum floating length \((\ell)\) was calculated based on the unit length of the weave \((p_r)\). The tangent condition of the linear yarn segment in the intersecting region of the control polygon with ellipse-modeled yarn cross-section was analyzed in order to calculate the optimum floating length. A code was written using Visual Basic 6.0 program in order to obtain the tangent position. In the program, the floating length was changed within certain limiting values \((a \leq \ell f \leq pr \cos \alpha)\) and the position of the intersecting segment was checked. The optimum floating length \((\ell)\) was determined for the point at which the linear line tangent to the ellipse, and the other parameters of linear yarn path model \((y, \ell_t)\) were calculated using this length. Consequently, the control polygon of B-spline curves was defined.

The control points to be used in calculating the B-spline curve were defined on the control polygon that models the 2/1 and 3/1 twill weave units (Fig. 6). Seven control points were defined on the linear model of each weave unit. They are the corner points of the control polygon and the center points of the floating and intersecting segments. The x, y, z coordinates of control points were determined for the weave, whose fabric thickness \((t)\), yarn counts \((N_1, N_2)\), settings \((S_1, S_2)\), and approximate skewness angles \((\alpha_1, \alpha_2)\) of weft and warp yarns were known, by calculating the optimum floating length \((\ell_f)\), based on the unit length of the weave \((pr)\). After defining the coordinates of control points, the B-spline algorithm was applied and the new points which created a smooth B-spline curve were achieved.

Total length of the weave unit was calculated from the new generated B-spline curve. Using this unit length theoretical crimp factor \((k)\) was calculated from the ratio of the length of B-spline curve \((\ell)\) to the unit weave length \((pr)\) for both warp and weft directions (Eq. 9). In addition, theoretical mass per unit area \((w)\) was determined from the theoretical calculated crimp factors \((k_{s1}, k_{s2})\), settings \((S_1, S_2)\) and metric system yarn counts \((N_{m1}, N_{m2})\) as given in following Eq. (10). In conclusion, theoretical and experimental results were compared with each other in order to test the accuracy of the model. Equations (9) and (10) are given below:

\[
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\]

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Fig. 4—(a) New position of linear yarn path on x’y’z’ space and (b) projection of floating and intersecting segments on xy plane.
Fig. 5—2-D geometrical model of 2/1 twill (a), and 3/1 twill (b), weave, on y’z’ plane

Fig. 6—Control points of B-spline curve on 2-D models of weave units
\[ k_t = \frac{\ell_t}{p_r} \quad \cdots (9) \]

\[ w_t = \frac{100k_{t1}S_1}{N_{m1}} + \frac{100k_{t2}S_2}{N_{m2}} \quad \cdots (10) \]

### 3 Results and Discussion

In this study, while the path of the yarn center is modeled linearly, the program inputs used are the fabric thickness, the density and number of weft and warp yarn, and the skewness angles in the direction of weft and warp yarns for twill weaves, measured experimentally. The B-spline curve, obtained from the model, defines the yarn geometry within the fabric. Crimp factors and mass per unit area of the fabric, calculated using the B-spline curve, are compared with experimental results in order to control the curve defined based on parameters of the fabric (Table 2).

The correlation coefficient between the theoretical mass per unit area of the fabric \( (w_t) \) and the experimental mass per unit area of the fabric \( (w_m) \) is 0.96; the percentage difference is within the 7% limit.

The correlation coefficient between the calculated mass per unit area of the fabric \( (w_c) \), calculated using experimentally measured crimp factors, tightness, and yarn counts, and the theoretical mass per unit area of the fabric \( (w_t) \), is 0.993; the difference between the two is 2%. The warp-direction correlation coefficient is 0.828, and the weft-direction correlation coefficient is 0.447, as per the result of comparing experimental crimp factors \( (k_{m1}, k_{m2}) \) to theoretical results \( (k_{t1}, k_{t2}) \). The difference is within the 3% limit for both weft and warp results. These statistical results show that the structural parameters of the real twill fabrics could be predicted by using the outputs of this geometrical model.

The 3-D simulations of 2/1 and 3/1 twill fabrics (Fig. 7) are obtained using the student version of 3ds Max 2011. First, B-spline curve points are used to draw the centerline axis of the yarn. Then, the yarn cross-section defined as ellipse is rendered to this yarn path and the 3-D yarn geometry within the fabric is achieved. After the 3-D geometries of the weft and warp threads were obtained, yarns are gathered according to their weave unit in order to establish fabric simulations. By this model, the 3-D geometry of 2/1 twill and 3/1 twill weaves is defined by taking into account the skewness property. When actual and model images are compared it is found that the open

<table>
<thead>
<tr>
<th>Fabric code</th>
<th>Theoretical (B-spline) mass per unit area ((w_t)), g/m²</th>
<th>Theoretical crimp factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
<td>Warp ((k_{t1}))</td>
</tr>
<tr>
<td>A1</td>
<td>169</td>
<td>1.037</td>
</tr>
<tr>
<td>A2</td>
<td>183</td>
<td>1.050</td>
</tr>
<tr>
<td>A3</td>
<td>198</td>
<td>1.066</td>
</tr>
<tr>
<td>B1</td>
<td>166</td>
<td>1.021</td>
</tr>
<tr>
<td>B2</td>
<td>180</td>
<td>1.028</td>
</tr>
<tr>
<td>B3</td>
<td>194</td>
<td>1.039</td>
</tr>
</tbody>
</table>

Fig. 7—3-D simulations of 3/1 twill weave (a), and 2/1 twill weave (b)
areas of simulations are greater. Because, in the model, the cross-section shape of the yarn is assumed to be solid and constant along the yarn path. On the other hand, in actual fabrics the shape of the yarn changes according to yarn geometry. By this model, a realistic model is achieved for yarn path of 2/1 and 3/1 twill weaves. With the help of these models the view of the fabrics could be observed. Besides, these models could be used to predict mechanical properties of the fabric, depending on defined fabric geometry.

In this study, the leading reasons behind errors are based on modeling the yarn as a uniform-structured monofilament, and modeling the cross-section as an ellipse with a certain flattening rate. The crimp rates, determined experimentally, also incorporate measurement errors. It is very natural that the modeled fabric displays certain differences from the real fabric, as a model is based on a homogeneous structure, whereas structural parameters in real fabric change throughout the fabric. However, experimental data obtained in this study are very close in terms of control parameters used in this study. The simulations formed in this study also support this result. Especially, the skewness property of twill weaves used in the models makes it possible to constitute 3-D fabric simulations in the way that yarns do not cut each other.

4 Conclusion

This study includes the structural parameters and the skewness property of 2/1 and 3/1 twill weaves into consideration while modeling the 3-D geometry of these fabrics. In order to test the model, 2/1 and 3/1 twill fabrics having different settings have been used and the outputs of the model are found close to the experimental results. The differences between the theoretical crimp factors of warp and weft yarns, calculated from B-spline curve, and measured crimp factors are within limit. Besides, good simulations are generated for different settings of 2/1 and 3/1 twill weaves, which verify the accuracy of models. Consequently, this type of reliable 3-D geometrical models which are defined by using structural properties of fabrics can be integrated with special softwares in order to predict end-use performance of the fabric more integrity, such as mechanical properties and permeability properties that are affected form geometrical properties of the fabric. In further studies, it is aimed to take in to account the change of the yarn cross-section in the fabric geometry in order to ensure more reliable models.

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