Long-term deflections of reinforced concrete beams

Terezia Nürnbergerová, Martin Krížma & Ján Hájek

Institute of Construction and Architecture, Slovak Academy of Science,
Děbravská cesta 9, 842 20 Bratislava, Slovak Republic

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The effects of different long-term loading levels beyond cracking limit on the deflections of the reinforced concrete beams are investigated. Six identical beams were subjected to a sustained load at three levels over a period of about 200 days. The arrangement of measurements enabled the separation of the effect of bending moments from that of shear forces. The results show that the deflection curves at the two arbitrary time intervals are similar in spite of intensive crack development. The linear relationship between the initial and time-dependent deflections can be assumed. This linear relationship also applies for the shear-induced deflections.

The serviceability of concrete structures has been one of a major concern of the designers for a long time. Its importance is increasing with the availability of high quality materials (concrete and reinforcement). The higher quality and the more precise calculations as well, result in the design of more slender and therefore more economical structures. However, the reduction of the stiffness of a structure directly influences its deformations. From this point of view, the practical importance of the correct prediction of the deflections is indisputable. As the long term contribution to the total deflections exceeds the short-term contribution, the accuracy of the prediction of the deflection is primarily influenced by the accuracy of the determination of the long-term and/or cycle dependent deflections.

Verification of the methods of calculation requires a sufficient amount of experimental data. Therefore, attention has been paid to the investigations on time-dependent deformations under long-term sustained load and the deformations under repeated load as well. Investigations on these deformations are reported by Corley and Sozen, Dilger and Abele, Jaccoud and Favre, Pitorňák, Ding Dajun et al.

An increase in the deflection of an element subjected to the sustained load is influenced mainly by: the creep of concrete in compression; time-dependent changes in tension stiffening; deterioration of bond between the concrete and reinforcement; the shrinkage of concrete; increasing crack widths and formation of new primary cracks. The analytical models taking into account all of these processes are, as a rule, very complicated and inconvenient for practical calculations. These models often require detailed information as input data which might not be known at the time of the design of a structure, and give results which sometimes differ from those obtained in tests.

According to Corley and Sozen, the total deflection of a reinforced concrete beam may be divided into three components, viz.: deflection resulting from instantaneous strains, deflection resulting from creep strains, and deflection resulting from shrinkage strains. This can be expressed by Eq. (1):

\[ \delta_{\text{tot}} = (1 + \beta_t)\delta_{\mu} + \delta_{sh} \]

where \( \delta_{\text{tot}} \) is the total deflection in a distance \( x \) from the left support of the beam, \( \delta_{\mu} \) is the initial deflection in the same point at the time of load application, \( \delta_{sh} \) is the deflection due to shrinkage, \( \beta_t \) is the multiplier of the initial deflection giving the deflection increment due to creep. If coefficient \( \beta_t \) does not depend on the load level at a given instant, then Eq. (1) represents a straight line. In this case, the hypothesis of linearity of deflections can be assumed. To verify this hypothesis, investigations on the long-term loaded slabs are reported from the Institute of Construction and Architecture in Bratislava, confirming the assumed linearity of deflections of slabs. But the question about the long-term behaviour of the beams with flanged cross sections has remained unresolved.

Therefore, the tests on beams subjected to sustained load with different load levels have been carried out. The dimensions of the beams are devised in such a way that the shear-induced deflections can
not be neglected. A method of measuring the deformations enabling to separate the bending- and shear-induced deformations from the total deformations has been used.

Experimental
A series of six identical reinforced concrete beams was subjected to the long-term load. The only variable parameter was the intensity of the sustained load. The cross-section of the beams is shown in Fig. 1 and the elevation in Fig. 2. The overall length of the beams was 4.15 m and their span was 3.6 m. The aggregate of the river alluvia was used for the concrete. The particle size distribution for 0-4 mm, 4-8 mm, and 8-16 mm was 720, 300, and 725 kg, respectively, that is 41, 17, and 42% by weight. Portland cement PC I 42.5 in the amount of 470 kg/m³ was applied and the water to cement ratio was 0.4. The average values of concrete properties at the time of testing are given in Table 1. The beams were reinforced in the tension zone with 11 high-bond deformed bars (Ø = 16 mm). The stirrups were made of high-bond wire (Ø = 8 mm, spacing 180 mm). The average properties of both reinforcements were: yield stress \( f_y = 491.7 \) MPa, tensile strength \( f_u = 582 \) MPa, modulus of elasticity \( E = 199 \) GPa.

The beams were placed in pairs during the long-term tests. The concentrated force in the middle of the span of a beam was produced by the hydraulic jack with the incessant check on the value of the loading force. Three levels of sustained load were chosen on the basis of the preliminary short-term test, namely 0.35 (beams A), 0.50 (beams B), and 0.65 (beams C) of the short-term failure limit which was 500 kN. There were two beams loaded on each level. All the beams were loaded gradually using deflection rate control until the required loading level was reached except the beam B1 where the simulated moving load was applied.

The deflections in five points over the span and strains of the continuously linked-up measurement bases at the compressed and tensioned edges, as well as at the crossing diagonals were registered and cracking was observed during long-term loading at regular time intervals. The bases shown in Fig. 2 form a "truss" consisting of "struts" and "ties" enabling the calculation of the deflections by using a method based on Williot-Mohr translocation polygons. This method offered the possibility of separating the deflection due to shear from those due to bending.

Results and Discussion
The relationship between the short-time deflections and the loading force obtained during the gradual loading to the required load-term loading level is shown in Fig. 3. In this figure the results of all tested beams except the beam B1 (a moving load has been

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Fig. 1 — Cross-section of the beams

Fig. 2 — Side-view of the beams; a — network of bases for mechanical measurement of deformations
LONG-TERM DEFLECTIONS OF REINFORCED CONCRETE BEAMS

Fig. 3—Short-time deflection vs loading force relationship

Fig. 4—Deformations along the compressive (dashed lines) and tensioned (full lines) zones of the beam B2

Fig. 5—Deformations in the crossing diagonals of the beam B2: descending diagonals from the left end of the beam (dashed lines); ascending diagonals (full lines)

Fig. 6—Relationship of the total time-dependent deflection $a_{\text{tot}}$ vs total initial deflection $a_{\text{ini}}$

Fig. 7—Relationship of the shear-induced time-dependent deflection $a_{\text{sh,ini}}$ vs shear-induced initial deflection $a_{\text{sh,ini}}$

Table 1—Average values of mechanical properties

<table>
<thead>
<tr>
<th>Beam</th>
<th>Age (days)</th>
<th>Cube strength $f_{\text{cu}}$ (MPa)</th>
<th>Prism strength $f_{\text{pm}}$ (MPa)</th>
<th>Modulus of elasticity $E_s$ (GPa)</th>
<th>Tensile strength $f_{\text{st}}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>30</td>
<td>55.94</td>
<td>47.56</td>
<td>44.92</td>
<td>2.43</td>
</tr>
<tr>
<td>A2</td>
<td>40</td>
<td>58.04</td>
<td>39.70</td>
<td>39.31</td>
<td>2.48</td>
</tr>
<tr>
<td>B1</td>
<td>32</td>
<td>54.34</td>
<td>47.60</td>
<td>35.83</td>
<td>2.40</td>
</tr>
<tr>
<td>B2</td>
<td>48</td>
<td>61.89</td>
<td>54.42</td>
<td>38.36</td>
<td>2.56</td>
</tr>
<tr>
<td>C1</td>
<td>42</td>
<td>61.84</td>
<td>49.67</td>
<td>38.78</td>
<td>2.56</td>
</tr>
<tr>
<td>C2</td>
<td>53</td>
<td>60.06</td>
<td>50.94</td>
<td>38.38</td>
<td>2.52</td>
</tr>
</tbody>
</table>
applied in this beam) are plotted. It can be seen that the shear-induced deflections (full triangles) by higher forces starting at the loading force \( F = 150 \text{kN} \) are approximately one third of the total deflections (crosses). The dashed lines are the deflections calculated according to the Slovak Standard\(^{10}\), the full line represents the bending-induced deflections calculated according to the ENV\(^{11}\). The deflections have been calculated using the average values of materials properties.

The deformations at the compressive (dashed lines) and tensioned (solid lines) zones of the beam B2 are plotted in Fig. 4. The figure shows similarity (proportionality) between the deformations measured at the time of the sustained load application and those after some period of the duration of the load (28, 207, and 498 days in the figure). Fig. 5 depicts the deformations obtained by using the Williot-Mohr translocation polygons. Also, the shear-induced deformations show the similarity between their initial and time-dependent values.

Fig. 6 shows the relationship between the time-dependent total deflection \( \Delta_{\text{total}} \) (after 28 and 207 days of loading) and the initial total deflection \( \Delta_{\text{init}} \) at the application of sustained load. Every point on the graph represents a pair of values (\( \Delta_{\text{init}}, \Delta_{\text{total}} \)) of all the deflections (in tenths of span) resulting from the calculation using Williot-Mohr translocation polygons for all the beams. It can be seen that all the points lie practically on a straight line no matter to what spot and loading level the points belong. Similar relationships for the shear-induced deflections are plotted in Fig. 7 and could be drawn also for bending-induced deflections.

It follows from the above mentioned evaluations that the arbitrary values of the initial deflection \( \Delta_{\text{init}} \) depend linearly on the values of \( \Delta_{\text{init}} \), i.e. the long-term deflection belonging to them. This means that the initial deflection curve and the deflection curve after time \( t \) of sustained loading are affine-similar. Since this affinity exists between the initial deflection curve and the deflection curve at the arbitrary time interval, it holds between the two arbitrary time intervals, too.

It should be emphasised that this affinity exists despite of the time-dependent development of cracks. The variations of coefficient \( b_i \) of the straight line regression is given in Table 2. It is evident that the coefficients of the straight line regression of the total (bending- and shear-induced) deflections do not differ substantially.

Fig. 8 shows the variation over time of the increment of the coefficient \( b_i \) of the straight line regression (the creep factor). The symbols represent the experimental value of total (crosses), bending-(full circles), and shear-induced (triangles) deflections, and lines express the regression functions given by the Eq. (2):

\[
\beta_i = c \left[ 1 - \exp \left( -\left( \frac{t - t_i}{t_r} \right)^2 \right) \right]
\]

where \( t \) is the age of the concrete at the sustained load application, \( t_i \) the age of the concrete in the time considered, and \( t_r \) is the retardation time\(^{12}\). The Eq. (2), with the retardation time of \( t_r = 200 \text{ days} \) is the basic formula for calculating the strain increase due to creep according to the Slovak Standard\(^{10}\). The same retardation time has been applied for the regression curves and seems to approximate the test results quite satisfactorily. The parameter \( c \) for all the deflections is given in Table 3. These results confirm the hypothesis that the deflections at the arbitrary time

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**Table 2—Parameter \( b_i \) of straight line regression of the time-dependent deflections vs initial deflection**

<table>
<thead>
<tr>
<th>Deflection</th>
<th>Time of loading (days)</th>
<th>( b_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total deflection</td>
<td>28</td>
<td>1.2504</td>
</tr>
<tr>
<td></td>
<td>207</td>
<td>1.4634</td>
</tr>
<tr>
<td>Bending-induced deflection</td>
<td>28</td>
<td>1.2415</td>
</tr>
<tr>
<td></td>
<td>207</td>
<td>1.4432</td>
</tr>
<tr>
<td>Shear-induced deflection</td>
<td>28</td>
<td>1.2682</td>
</tr>
<tr>
<td></td>
<td>207</td>
<td>1.5082</td>
</tr>
</tbody>
</table>

**Table 3—Parameter \( c \) of the time function of the coefficient \( \beta \)**

<table>
<thead>
<tr>
<th>Deflection</th>
<th>Parameter ( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{\text{init}} )</td>
<td>0.746</td>
</tr>
<tr>
<td>( \Delta_{\text{init}} )</td>
<td>0.712</td>
</tr>
<tr>
<td>( \Delta_{\text{init}} )</td>
<td>0.821</td>
</tr>
</tbody>
</table>

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![Fig. 8—Time development of the increment of the creep coefficient \( \beta \)](image-url)
interval of the sustained load depend linearly on the initial deflections.

As the parameter $c$ of the Eq. (2) in Table 3 does not differ significantly for the individual deflections, it implies that the ratio of the shear-induced deflection to the total deflection does not depend on the time of duration of the sustained load. This can be seen in Fig. 9, where the time development of the quotients $a_{0}/a_{tot}$ and $a_{d}/a_{tot}$ is shown. In this figure, the average values of the quotients of the bending- and shear-induced deflections to the total deflections are plotted, respectively. It is evident, that these quotients do not change with time and are not influenced by the load level (all the load levels, viz. 175 kN, 250 kN and 325 kN were above cracking limit).

Another question connected with the long-term behaviour of reinforced concrete elements is the position of the neutral axis. It is assumed that the neutral axis lowers with time. Our observations during the long-term tests show that cracks are not closed with time; on the contrary, their widths grow and new cracks occur, implying thereby that the neutral axis does not drop. This phenomenon can be explained by the assumption that creep takes place not only in the compressed but in the tensioned fibres, as well as in bond between the steel bars and concrete. The measurement of the deformations of the continuously linked-up measurement bases at the compressed and tensioned edges enables the calculation of the depth to the neutral axis. The time development of the relative depth to the neutral axis is shown in Fig. 10. The average value of the relative depth to the neutral axis of the two beams with the same level of sustained load at the middle of the span is plotted in the Fig. 10. It should be noted that the measured deformations contain also deformations due to shrinkage. The shrinkage induced deformations increase the deformations on the compressive edge, but decrease those on the tensioned edge. To this phenomenon the slight drop of the relative depth to the neutral axis until approximately 200 days of loading observed in Fig. 10 could be prescribed.

Conclusions

The reported investigations show that the initial and long-term deflections of reinforced concrete beams can be assumed to be dependent. The deflection curves of beams at the arbitrary time interval are affine similar. The ratio of the shear-induced deflections to the total deflections does not depend on time. This leads to the conclusion that a simplified method for calculation of deflections applying the assumption of the linearity of deflections is possible.

Acknowledgement

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Nomenclature

- $a_{0b}$ = initial deflection due to sustained load
- $a_{0l}$ = long-term deflection due to sustained load
- $a_{d}$ = bending-induced deflection
- $a_{d}$ = shear-induced deflection
- $a_{tot}$ = total deflection
- $b_{f}$ = coefficient of the straight line regression (creep factor $\beta$)
- $c$ = parameter of the creep factor $\beta$
- $E_{c}$ = modulus of elasticity of concrete
- $E_{s}$ = modulus of elasticity of steel
- $F$ = loading force
- $f_{cu}$ = cubic strength of concrete
- $f_{ct}$ = tensile strength of concrete
- $f_{pm}$ = prism strength of concrete
- $f_{y}$ = yield stress of steel
- $f_{st}$ = tensile strength of steel
- $t$ = time
- $t_{s}$ = age of concrete at the application of the sustained load
- $t_{c}$ = age of concrete in the time considered
- $t_{0}$ = assumed retardation time (200 days)
- $x_{o}$ = relative depth to the neutral axis
- $\beta$ = multiplier of the initial deflection, creep factor
References
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