Mathematical modeling and prediction of the thrust force and torque in drilling of sisal/glass-vinyl ester hybrid composite using the RSM, MLPNN, RBFN and ENN methods

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Drilling a polymer composite material without defects is not an easy task; even so, these materials are largely used in aerospace and other industries, due to their excellent mechanical properties. Tool wear, delamination and fiber pull out are the major problems in drilling. Literature reveals that the thrust force (th) and torque (tq) have a significant effect on tool life and delamination. Therefore, to improve the performance of drilling, it is essential to study the thrust force and torque. This paper discusses the methodologies used in the prediction and evaluation of the thrust force and torque in the drilling of sisal and glass fiber reinforced vinyl ester resin hybrid composite materials. Experiments are conducted in the CNC machining center, and high speed steel (HSS) drill bits are used in the machining center for the drilling operations. Process parameters such as the drill diameter (d), spindle speed (s) and feed rate (f) are considered as the controlling factors for the study. The experimental data obtained are used for training and testing with RSM and ANN techniques such as MLPNN, RBFN and ENN. The predicted thrust force and torque, based on the MLPNN model, are found to be in very good agreement with the experimental values.

Keywords: RSM, MLPNN, RBFN, ENN, Thrust force, Torque

In recent years, fiber-reinforced materials have revolutionized the making of high performance structures in aerospace, military and transportation industries, since they possess significant advantages in the strength to weight ratio, stiffness to weight, corrosion resistance, fatigue strength, thermal resistance, and damping properties. The prediction of cutting forces in machining will be helpful to improve reliability, accuracy and productivity in conventional and CNC machining. Actually, it gives information about cutter deflection, machine tool chatter, tool wear, breakage and then tool life. The surface integrity can be improved by selecting the appropriate cutting conditions, tool path and tool geometry.

Drilling is a hole producing process, and is taken as one of the important machining processes, because it accounts for a major portion of the overall machining operations. In addition, the drilling processes may result in the production of unnecessary waste, because many drilling operations are usually among the final steps in fabricating a part.

A finite element approach was used by Shatla and Altan⁵ to determine the drilling torque and thrust force. Chandramohan et al.⁶ predicted the thrust force and torque for sisal and roselle, sisal and banana, and roselle and banana fiber reinforced composites, using the regression model, and concluded that sisal and roselle (hybrid) is the best, as it can be used for internal fixations compared to other materials. Gaitonde et al.⁷ studied the effects of the process parameters on delamination during the high-speed drilling of carbon fiber reinforced plastic (CFRP) composites. Durao et al.⁸ investigated the thrust force and delamination in the composite material, when drilling holes with five different drill geometries. Singh et al.⁹ investigated the thrust force and torque developed during the drilling of glass fiber reinforced
plastics (GFRP) composites with L27 orthogonal array experiments. A fuzzy rule based model had been developed for the prediction of thrust force and torque in the drilling of GFRP composites.

Rubio et al.\textsuperscript{10} investigated the performance drilling of glass fiber reinforced plastics (GFRP) in high speed machining (HSM), and reported that HSM is suitable for drilling GFRP for low damage levels. Abrao et al.\textsuperscript{11} investigated the effect of the cutting tool geometry and material, on the thrust force and delamination produced, when drilling a glass fiber reinforced epoxy composite, and found that the damaged area increased considerably with the feed rate, and moderately with the cutting speed.

Ramji et al.\textsuperscript{12} studied the thrust and torque of cryogenically treated and tempered carbide tipped drills at various levels of cutting speed, feed and tool conditions, and also reported that the cryogenically treated tools showed superior results in terms of reduced forces, tool wear and surface roughness of the drilled holes. Davim et al.\textsuperscript{13} investigated the delamination problems, and evaluated the cutting parameters (cutting velocity and feed) and the influence of the matrix under specific cutting force, delamination factors and surface roughness in two types of matrix (Viapal VUP 9731 and ATLAC 382-05).

Davim\textsuperscript{14} has elaborately dealt with delamination, damage reduction in the drilling of composite materials, and the influence of the machining parameters on delamination. Ganesh Babu et al.\textsuperscript{15} studied the machining properties of composite material by measuring the cutting forces, and used response surface methodology for the optimization. Davim et al.\textsuperscript{16} studied a novel technique to measure the adjusted delamination factor $F_{da}$ using the digital analysis.

Jayabal and Natarajan\textsuperscript{17} investigated the drilling of coir-glass fiber reinforced composites with the HSS drill bit, to evaluate the thrust force and torque under different cutting conditions, by the Regression Analysis and Neuro Fuzzy methods, and observed that the Neuro Fuzzy model performed better than the Regression model. Zhang et al.\textsuperscript{18} presented a general closed-form mechanical model, for predicting the critical thrust force for the drilling of composite material. Shetty\textsuperscript{19} studied the Taguchi optimization methodology, and applied it to optimize the cutting parameters.

Jayabal et al.\textsuperscript{20} investigated the drilling of composites and optimized the drilling parameters, using the optimal settings of the Box-Behnken design, Nelder-Mead and genetic algorithm methods. Sait\textsuperscript{21} investigated the machining parameters on turning glass-fiber reinforced plastic (GFRP) pipes, and optimized the parameters using the desirability function analysis. Garg et al.\textsuperscript{22} studied the performance of back propagation neural networks (BPNN) and radial basis function networks (RBFN), in the prediction of the flank wear of high speed steel drill bits for drilling holes.

Davim\textsuperscript{23} reported the fundamentals and recent advances in the machining of composite materials (polymers, metals and ceramics) for modern manufacturing engineering.

The literature review reveals that the study of machinability behavior and tool condition monitoring are important areas of research, and already a lot of works have been published in either the response surface methodology (RSM) or artificial neural networks (ANN) techniques individually. In this paper, a comparative study of the performance of RSM, multi layer perceptron neural network (MLPNN), RBFN and Elman neural network (ENN) techniques has been made.

The drilling parameters were selected at three levels (drill bit diameters of 6, 8 and 10 mm, spindle speeds of 600, 1200 and 1800 rpm and feed rates of 0.1, 0.2 and 0.3 mm/rev.), and the responses, i.e., the thrust force and torque were measured for each run, when the experiments were conducted. The assignment of levels to factors is given in Table 1.

**Experimental Procedure**

**Materials and methods**

In order to perform the experimental work, Chopped Sisal and E-Glass hybrid fiber reinforced vinyl ester resin composite laminates of size 180 mm × 160 mm with a thickness of 5 mm were produced by the cold process method. The DMA, cobalt octovate and MEKP were mixed with the vinyl ester resin, to initiate, accelerate and maintain the chemical reaction. Straight shank HSS twist drills of 6, 8 and 10 mm were used for this drilling investigation.

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Table 1 – Factors and their levels
Experimental set-up

The experiments were carried out in the three axes CNC Vertical Milling Machine (make: MTAB MAX MILL, motor capacity: 3.7 kW and max. tool diameter: 60 mm), using HSS twist drills. The thrust force and torque were measured with a 3-axes piezoelectric dynamometer (Kistler 9257B). It has a measuring range of 0-5 kN. The dynamometer was connected to a multi channel charge amplifier, and a personal computer was also connected, for data acquisition and processing. All the experiments were performed under dry drilling conditions. The drilling set up is shown in Fig. 1.

The work pieces were prepared by cutting the fabricated composite sheet into smaller sizes of 100 mm × 50 mm for holding it conveniently in the machine. Drilling tests were planned to be conducted at the CNC Machining center in the speed range of 600-1800 rpm, and the work piece was held tightly in the vice which was placed over the Kistler dynamometer. A straight shank drill was fixed in the chuck. Before carrying out the experiments, wiring connections between the dynamometer, the amplifier and the personnel computer (PC), and power connections to these instruments were established. With the design of experiments, runs were planned for three factors, each at three levels, as given in Table 2. With CNC programming, the spindle speeds and feed rates for the 27 runs were obtained. The measured thrust force and torque data are given in Table 2.

Response Surface Methodology (RSM)

Response surface methodology (RSM) is a collection of mathematical and statistical techniques that are useful for empirical model building and analyzing problems. The RSM technique was introduced by Box et al.\textsuperscript{24}, the main objective was to optimize and predict the response surface (dependent variable), which is influenced by several independent variables (input variables).

The first step in RSM is to find a suitable approximation to the true relationship. The most common forms are the lower order polynomials (first or second-order).

\[ Y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i=1}^{k} \sum_{j=1}^{k} \beta_{ij} x_i x_j + \varepsilon \quad \ldots (1) \]

where, \( Y \) is the response or output, \( k \) is the number of the factors, \( i \) and \( j \) are the index numbers for the factor, \( \beta_0 \) is the free or offset term called intercept term, \( x_1, x_2, \ldots, x_k \) are the independent variables, \( \beta_i \) is the first-order main effect, \( \beta_{ii} \) is the second order (quadratic) effect, \( \beta_{ij} \) is the interaction effect, and \( \varepsilon \) is the error or uncertainty between the predicted and the

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Fig. 1 – Photographic image of CNC marching center with drilling attachments
measured values. The RSM predicted thrust force and torque values are shown in Figs 2 and 3, respectively.

The RSM surface plot of the thrust force between $ds$, $df$ and $sf$ are shown in Figs 4, 5 and 6, respectively. Similarly, The RSM surface plot of the torque between $ds$, $df$ and $sf$ are shown in Figs 7, 8 and 9 respectively.

The data for the internal architecture of the response surface model were obtained from the experimental tests. After training the database, a model was created using the regression method.

To check the performance of the developed model, a validation test comprising experiments under different cutting conditions was planned. The developed RSM model was utilized to predict the responses for the given inputs, which are shown in Tables 3 and 4 for the thrust force and torque respectively.
Multi Layer Perceptron Neural Networks (MLPNN)

Multilayer perceptron networks (MLP) is a feed forward artificial neural networks model that maps sets of input data onto a set of proper output. They have a highly interconnected structure, similar to the brain cells of human neural networks, and consist of a large number of simple processing elements called neurons, which are arranged in different layers in the network. MLPs are used with the supervised learning technique, and it lead to a successful back propagation algorithm.

The MLPNN network diagram is shown in Fig. 10. The model has three types of layers an input layer, an output layer and one or more intermediary layers called the hidden layer(s). For the perceptrons in the input layer, linear transfer functions have been used, and for the perceptrons in the hidden layer, the output layer sigmoidal or squashed functions are used. The
input layers serve to distribute the values they receive to the next layer, and so on. In this supervised learning type, the input-output parameters are sequentially presented to the networks undergoing a training phase with error correction learning, which means that depending upon the deviation of the predicted output from the desired output; the various interconnections are adjusted using an average gradient information.

Input of \( i \)th neuron in the hidden layer \( I_{yi} \) was calculated by

\[
I_{yi} = \sum_{i=1}^{n} (w_{xyi}x_i + b_{xyi})
\]  

(2)

where \( n \) is the number of neurons in the input layer and \( w_{xyi} \) is numerical weight value of the connection between the input and the hidden layer neurons. \( x_i \) is the \( i \)th normalized output from the input layer. \( b_{xyi} \) is the bias weight between the input and the hidden layer neurons. \( y_i \) is the \( i \)th neuron in the hidden layer. \( w_{yzi} \) is the weight between the \( i \)th neuron in the hidden layer and the output layer. \( b_{yzi} \) is the bias weight between the hidden and the output layer. \( z_i \) is the output or response. \( f \) and \( g \) are the functions.

The mathematical representation is expressed as

\[
z_i = \sum_{i=1}^{n} g(w_{yzi}I_{yi} + b_{yzi})
\]  

(3)

The performance of the ANNs was statistically measured by the mean squared error (MSE) and obtained as follows:

\[
ErrSqd. = \sum_{i=1}^{n} (d_i - y_i)^2
\]  

(4)

\[
MSE = \frac{1}{n}\sum_{i=1}^{n} (d_i - y_i)^2
\]  

(5)

where \( d_i \) is the desired value and \( y_i \) is the output value. MSE is the mean square error. The output values obtained from the MLPNN are given in Tables 3 and 4 for the thrust force and torque, respectively.

**Radial Basis Function Networks (RBFN)**

The radial basis function neural network is a multilayer feed-forward neural networks model. It was introduced by Moody and Darken\(^25\). The RBFN networks diagram is shown in Fig. 11. These networks are three layered or with many hidden layers.

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Fig. 10 – MLPNN networks diagram
between the input and the output layer, and each hidden unit implements a radial activation function, and each output unit implements a weighted sum of hidden units. It uses the Gaussian potential functions.

The output values were obtained after processing in the RBFN engine, and the results are given in Tables 3 and 4 for the thrust force and torque, respectively.

Elman Neural Networks (ENN)

Elman neural is one of the types of recurrent artificial neural networks, first employed by Elman. The special type of networks diagram is shown in Fig. 12. These networks vary from the normal feed forward networks in the sense of loops in the network. It means that at any time there exists at least one feedback connection from the output of the hidden layer to the context units fixed with a weight of one. A three layer network is normally used, and the Elman network has sigmoid artificial neurons in its hidden layer, and linear artificial neurons in its output layer. Every time, the input is propagated in a standard feed forward fashion, and then a learning rule is applied. Thus, the network allows it to perform such tasks as sequence-prediction that is beyond the power of a standard multilayer perceptron.

The processed output values are given in Tables 3 and 4 for the thrust force and torque, respectively.

Development of Mathematical Model

The response surface methodology can be expressed as:

\[ Y = f(d, s, f) + \varepsilon \]  \hspace{1cm} … (6)

where, \( Y \) is the response or yield with the function of diameter, speed and feed. The first order polynomial (linear) equation is used to represent the response surface with the interactions of three factors.

The general linear equation with interactions is given by

\[ Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_{12} x_1 x_2 + \beta_{13} x_1 x_3 + \beta_{23} x_2 x_3 + \beta_{123} x_1 x_2 x_3 \]  \hspace{1cm} … (7)

where \( \beta_0 \) is the free term of the regression equation, the coefficients \( \beta_1, \beta_2, \beta_3 \) are the linear terms and \( \beta_{12}, \beta_{13}, \beta_{23}, \) and \( \beta_{123} \) are the interaction terms.

Mathematical model of thrust force

With the measured dependent variables from 27 runs, the corresponding independent variables were used, and a mathematical model (regression equation) was developed to represent the response surface \( Y \). Here \( Y \) is replaced by the thrust force. The measured dependent variables from 27 runs, the corresponding independent variables were used, and a mathematical model (regression equation) was developed to represent the response surface \( Y \). Here \( Y \) is replaced by the thrust force.

\[ th = -77.815 + 12d - 0.003s + 446.1f + 0.0009ds - 24.17df - 0.004sf \]  \hspace{1cm} … (8)

where, \( th \) is the process response or output (dependent variable) thrust force, \( d \) is the drill diameter, \( s \) is the spindle speed and \( f \) is the feed rate mm/rev.

\( R^2 \) is a statistic residue that will give information about the integrity of the fit of a model. \( R^2 \) of 1.0 indicates that the regression line perfectly fits the data. \( R^2 = 1 - (\text{residual sum of squares (SS)/corrected SS}) = 0.96 \) for this equation. A comparison chart of the RSM predicted and experimental values of the thrust force is shown in Fig. 2.
Mathematical model of torque

The mathematical relationship for correlating the response surface torque ($t_q$) using the process variables ($d$, $s$ and $f$) was obtained as:

$$
t_q = 0.249 - 0.0036 d + 0.0007 s + 1.91 f + 0.0001 d s - 1.52 d f - 0.0002 s f \quad \ldots (9)
$$

$$
R^2 = 1 - \frac{\text{residual SS/corrected SS}}{} \quad \ldots (9)
$$

After training, the coefficient values of the regression equations were found, and the $R^2$ value was obtained as 0.96 for the thrust force. 0.96 is the value of the relative deviation of the RSM predicted value to the measured value. Based on the $R^2$ value and $F$ value, the order of the polynomial regression equation was decided, and in this case, the linear equation with 2FI was selected. The effect of the parameters to the responses is shown in Figs 4-6 for the thrust force. The graph plot between the predicted values and the measured values of the thrust force is shown in Fig. 2, which clearly reveals that minor deviations exist in the thrust values corresponding to the input values when compared with the measured values and later this deviation was used for calculating error percentages.

Similarly, for the torque $R^2$ value was calculated as 0.95, and based on the $R^2$ and $F$ values, the linear equation with 2FI was chosen, since the 2FI equation has given a better fit with the measured values.

The effect of the input variables on the response is shown in Figs 7-9 for the thrust force. The graph plot between the predicted value of torque and the measured values is shown in Fig. 3, which clearly reveals that the torque value calculated by the RSM is very close to the measured values.

Results and Discussion

Drilling operations were conducted over a wide range of cutting parameters on a sisal/E-glass-vinyl ester composite sheet. In this work, the diameter of the drill, the spindle speed and feed rate were varied equally in between their bounds. HSS drill bits of diameters 6, 8 and 10 mm were used for drilling holes on the composite. Three process parameters in 27 different combinations were taken up for measuring the thrust force and torque. The measured values were tabulated for different combinations of input parameters, and each run was denoted by a unique identification number.

In general, the RSM plots (Figs 2-9) show the significance of each factor in finding its responses.

The results in these figures indicate that, the thrust force and torque were increased with increasing feed. This was due to increasing the cross-sectional area of the un-deformed chip. Also, the thrust and torque were increased with an increase in the diameter of the drill. A larger thrust force occurs with larger diameter drills. When the speed increased, the thrust force and torque produced only minor variations. One of the facts for the non active nature was that the high speed, tool and composite material were heated up due to friction, and the property of both the materials was changed.

In the RSM technique, the measured values were processed in the statistical software (Design Expert).
The predicted results for the thrust force and torque have been given in Tables 3 and 4, respectively.

Tables 3 and 4 indicate the values obtained from RSM, MLPNN, RBFN and ENN and it was found that the deviation was less than 8% in the RSM.

The comparisons of the prediction by the RSM and experimental thrust force and torque are shown in Figs 2 and 3, respectively. On the other hand, Tables 3 and 4 show the values of the thrust force and torque obtained in the use of MLPNN, RBFN and ENN which are below 7% and 8%, respectively. From, Tables 3 and 4, the MLPNN is found to be more precise than the RSM.

Confirmation tests

The cutting conditions randomly selected between the lower and upper bounds of parameters, and used in the confirmation tests are shown in Table 5. Experiments were conducted at the newly fixed parameter settings, and the measured values and the values calculated by the prediction models are given in Tables 6 and 7 for the thrust and torque, respectively.

Table 6 shows the thrust force values predicted by RSM 4.57% Av.abs. error for the set of experiments when compared with experimental values. MLPNN, RBFN and ENN showed error values of 2.22%, 3.05% and 6.49%, respectively.

Table 7 shows the torque values predicted by RSM 6.03% Av.abs. error for the set of experiments when compared with experimental values. MLPNN, RBFN and ENN showed error values of 2.94%, 7.52% and 7.35%, respectively.

Conclusions

An experimental approach for the evaluation of the thrust force and torque during the drilling the Sisal/E-glass-vinyl ester composite laminate was studied. Mathematical models for thrust force and torque were developed. Three process parameters, such as the drill diameter, speed and feed were considered for the model development.

A study has been carried out to assess the comparative performances of four important techniques, namely, RSM, MLPNN, RBFN and ENN in the prediction of the responses, such as the thrust force and torque for the drilling operation of sisal glass hybrid vinyl ester resin composites. 27 runs of the experiment had been planned to generate the data for the training and testing of the neural networks technique.

It was observed from Tables 6 and 7 that the conventional technique of RSM gives an average absolute error (4.57% for thrust force and 6.03% for torque). It is comparable with the error percentage of ANN techniques, such as MLPNN, RBFN and ENN, and also with in the ±5% limits.

When the MLPNN, RBFN and ENN techniques are compared, both ENN and RBFN were found to give less accuracy than MLPNN in predicting the thrust force (6.49% and 3.05% for ENN and RBFN respectively) and torque (7.35% and 7.52% for ENN and RBFN respectively).

The MLPNN technique is one of the very simple algorithms, and had given better results than all other
techniques taken here for comparison. It gives an error of 2.22% in predicting the thrust force and 2.94% in predicting the torque.

**Nomenclature**
MLPNN = multi level perceptron neural networks  
ANN = artificial neural networks  
RBFN = radial basis function networks  
ENN = Elman neural networks  
RSM = response surface methodology  
CNC = computer numerical control  
DOE = design of experiments  
DMA = di-methyl aniline  
MEKP = methyl ethyl ketone peroxide  
HSS = high speed steel  
HSM = high speed machining  
MSE = mean square error  
ErrSqd = square of error  
PC = personnel computer  
GFRP = glass fiber reinforced plastics  
CFRP = carbon fiber reinforced plastics  
Av. abs. Error = average absolute error  
Exp. = experimental  
th = thrust force (N)  
tq = torque (N-cm)  
d = drill bit diameter (mm)  
s = spindle speed (rpm)  
f = feed rate (mm/rev.)  
ε = error

**References**