Rayleigh-Benard convection in horizontal fluid layers is a problem of fundamental as well as practical importance. The flow pattern associated with this configuration shows a sequence of transitions from steady laminar to unsteady flow and ultimately to turbulence. This configuration has been studied by various researchers using computational techniques as well as by experiments to understand the transition phenomenon. The experimental techniques have been strengthened by the availability of optical methods to visualize the flow phenomena and computers for data storage, processing and analysis of the images. A large amount of literature is available in the field of Rayleigh-Benard convection. Computational studies are quite abundant. Most of the available literature relates to small aspect ratio enclosures. This is because numerical calculations are easier to perform over a small domain. Analytical studies have shown that in an infinite fluid layer, the first transition from the conduction state to the steady cellular occurs at Rayleigh number of 1707.8. This value is independent of the Prandtl number of the fluid, though it is a function of the overall geometry. When the Rayleigh number (Ra) just exceeds the critical value, hexagonal convection cells have been observed in experiments as well as in computational studies. All subsequent transitions are dependent on the Prandtl number and the geometry of the confining surfaces.

Grotzbach\(^1\) performed a direct numerical simulation of the Rayleigh-Benard convection problem under both laminar and turbulent conditions. The fluid considered was air. The cavity was an infinite parallel plates channel. At \(Ra=4000\), skewed-varicose instability was seen and at \(Ra=7000\), the velocity field was unsteady and three-dimensional.

Mukutmony and Yang\(^2\) have demonstrated the loss-of-rolls phenomenon as the Rayleigh number is increased in an intermediate aspect ratio box. They have simulated the experimental conditions of Kolodner \textit{et al.}\(^3\), and investigated the transition from 10 rolls to 6 rolls using Rayleigh number in the range of 10,000 to 24,000. Mukutmony and Yang\(^4\) have numerically simulated the experiment conducted by Gollub and Benson\(^7\). The process of flow reversal to a steady state at higher Rayleigh number was observed. The calculations were performed for low aspect ratio enclosure and for a fluid of \(Pr=5\). The Rayleigh numbers considered were in the range of 40,000 to 120000.

Loss of rolls has also been documented by other workers. Lin \textit{et al.}\(^8\) have numerically shown this phenomenon for an intermediate aspect ratio box and for a fluid of Prandtl number 3.5. Mishra \textit{et al.}\(^7\) have experimentally studied Rayleigh-Benard convection in intermediate aspect ratio enclosure with air as the

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\(^{1}\)Presented at the 28\textsuperscript{th} National Conference on Fluid Mechanics & Fluid Power, held at Punjab Engineering College, Chandigarh, during 13-15 December 2001
working fluid considering the Rayleigh numbers as 13900, 34800 and 40200. The authors concluded that at a Rayleigh number of 13900, the fringes were steady near the boundary walls but mild unsteadiness was present in the central horizontal layers. At higher Rayleigh numbers, the unsteadiness was more pronounced with flow switching between the two well-defined states. Apart from this study, the authors contributed notably in the field of tomography.

In the present study, numerical simulation has been carried out for two Rayleigh numbers namely 5861 and 12,124. Since these values are significantly lower than the critical Rayleigh number for turbulent flow (=50,000 for a square cavity of intermediate aspect ratio)\footnote{Rayleigh number for a square cavity of intermediate aspect ratio.}, the study is in the laminar regime. The governing differential equations for buoyancy-driven flow in an axisymmetric cavity have been numerically solved. The grid used is 601x81 in \( r \) and \( z \) directions, respectively. Experiments have been conducted in a fluid layer of 64 cm diameter and 2.3 cm vertical height. A Mach-Zehnder interferometer has been used to visualize the convection patterns in the fluid layer. The numerically generated roll patterns, streamlines and the heat transfer rates in terms of the Nusselt numbers at the top and the bottom walls have been reported and compared with the experimental results.

### Mathematical Model

The axisymmetric form of continuity, Navier-Stokes and energy equations have been numerically solved in the present study. The flow is taken to be incompressible, with buoyancy accounted for via the Boussinesq approximation. The convective terms of the momentum equations are discretized using the 2nd order upwind scheme while central difference discretization has been applied to the diffusive terms. The flow equations have been solved using the stream function-vorticity approach. The matrix inversion is carried out by the Gauss-Seidel method. The solutions for velocity and temperature have been obtained by marching in time with initial transitions neglected. Due to the axisymmetric nature of the fluid cavity, the governing equations have been solved in \( r-z \) coordinate system. The grid on which the results have been obtained has 601x81 cells in the \( r \) and \( z \) directions respectively. The radial data obtained from the numerical simulation have been transformed onto a rectangular grid, which is of the same dimensions as the dimensions of the window used in the experimental test cell. The rectangular grid has also been divided into 81x601 grid points.

### Governing Equations

The governing equations for the present numerical study are continuity equation, Navier-Stokes equations and the energy equation, which in general form are:

**Continuity**

\[
\frac{\partial u}{\partial r} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = 0
\]  

**r-Momentum**

\[
\rho \left( \frac{\partial u}{\partial r} + u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial r} + \mu \left( \frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} \right)
\]

**z-Momentum**

\[
\rho \left( \frac{\partial v}{\partial r} + u \frac{\partial v}{\partial r} + v \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left( \frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho \beta \left( T - T_0 \right)
\]

**Energy**

\[
\rho c_p \left( \frac{\partial T}{\partial r} + u \frac{\partial T}{\partial r} + v \frac{\partial T}{\partial z} \right) = k \left( \frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} \right) + \rho \beta \left( T - T_0 \right)
\]

### Non-dimensionalization

Various non-dimensional parameters used in the numerical simulation are:

**Reference temperature**—The temperature has been non-dimensionalised as \( \theta = \frac{T - T_c}{T_h - T_c} \).

The temperature is defined in such a way that it varies from zero at the cold wall to unity at the hot wall.

**Reference length**—The height of the cavity, \( h \), plays a major role in the flow as the temperature variations...
and thus density variations responsible for buoyancy force due to gravity exist in the vertical direction. Therefore, the height of the enclosure is considered as the reference length for non-dimensionalization.

Reference velocity—The non-dimensionalized velocity is taken as $V = \frac{\nu}{h}$.

Reference pressure — $P = \rho \left( \frac{V}{h} \right)^2$

The non-dimensional parameter responsible for the buoyancy induced flows in the cavity namely the Grashoff number is defined as:

$$Gr = Ra / Pr$$

where $Ra$ is defined as:

$$Ra = \frac{g \beta (T_h - T_b) h^3}{\nu \alpha}$$

$$Pr = \frac{\nu}{\alpha}$$

$q - \omega - T$ Formulation

The $u$ and $v$ velocities have been defined in terms of stream functions $\varphi$ as:

$$u = \frac{1}{r} \frac{\partial \varphi}{\partial z}$$

$$v = \frac{1}{r} \frac{\partial \varphi}{\partial r}$$

Accordingly, the governing equations can be transformed to the stream function, vorticity and energy equations as:

Stream Function Equation

$$\frac{\partial \omega}{\partial t} + \frac{u}{r} \frac{\partial \omega}{\partial r} + \frac{v}{r} \frac{\partial \omega}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \partial \varphi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial z^2}$$

Vorticity Equation

$$\frac{\partial^2 \omega}{\partial t^2} + \frac{u}{r} \frac{\partial \omega}{\partial r} + \frac{v}{r} \frac{\partial \omega}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{r^2 \partial \varphi}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial z^2} + \frac{Ra}{Pr} \frac{\partial T}{\partial r}$$

Boundary conditions

The boundary conditions for stream functions, vorticity and temperature for the circular cavity can be derived by using the fact that the velocity satisfies the no-slip condition, the horizontal surfaces are isothermal, and the side walls are insulated. These can be expressed mathematically as:

Top Wall — $T = 0; \psi = 0$; and, $\omega = -\frac{1}{r} \frac{\partial^2 \varphi}{\partial z^2}$

Bottom Wall — $T = 1; \psi = 0$; and, $\omega = -\frac{1}{r} \frac{\partial^2 \varphi}{\partial z^2}$

Side Walls — $\psi = 0; \frac{\partial T}{\partial r} = 0$; and $\omega = 0$

Initial conditions

The following initial conditions have been applied:

$\psi = 0; \omega = 0$; and, $T = \text{linear}$

Experimental

Laser interferometric technique has been employed to visualize the flow field when subjected to a temperature gradient between its top and the bottom surfaces. A Mach-Zehnder interferometer is used to collect the line-of-sight projections of the temperature inside a Rayleigh-Benard set-up. Since the cavity (as discussed earlier) is circular in plan, hence one is led to an axisymmetric configuration. Fluid employed is air with $Pr=0.71$. Experiments were conducted in a fluid layer of 64 cm diameter and 2.3 cm vertical height. The side walls were kept insulated with temperature gradient applied in the vertical direction. All experiments have been performed to record the long-time convection patterns in the test cavity. This has been referred to as the steady state thermal field in the fluid layer. The experiments are run for a period of 11-12 h to achieve this condition. Long time convection patterns are collected to ensure that steady state prevails in the fluid layer with the initial transients eliminated. At steady state, three to four interferograms have been recorded in a gap of 10-15
The experiments have been performed at two different angles namely $0^\circ$ and $45^\circ$ by rotating the test cell. These steady state interferograms are processed and analysed to extract the quantitative information. The thermal field in the fluid layer is three dimensional, despite the axisymmetric geometry. Hence the interferograms are to be interpreted as path integrals of the refractive index field.

**Results and Discussion**

**Nusselt number calculation**

The Nusselt number representing the dimensionless wall heat transfer rates is defined as:

$$Nu = -h \frac{\partial T}{\partial y} \bigg|_{y=0, h}$$  \hspace{1cm} (8)

In the present work, the local and average Nusselt numbers at the top and the bottom walls have been reported. The average Nusselt number for each of the plates has also been compared with the experimental correlation reported by Gebhart et al.\textsuperscript{10}. For air, the correlation is given by:

$$Nu = 1 + 1.44 \left[ 1 - \frac{1708}{Ra} \right] + \left[ \frac{Ra}{5830} \right]^{1/3} - 1$$  \hspace{1cm} (9)

**Numerical results**

The results discussed here pertain to the steady state solutions of the process. The numerically determined streamlines and isotherms in the fluid layer for the full cavity for $Ra=5861$ are shown in Figs 1 and 2.

A comparable plot for $Ra=12,124$ is shown in Figs 3 and 4. In both the figures, the formation of rolls is quite clear. The number of rolls is less than the aspect ratio of the fluid layer and decreases with an increase in the Rayleigh number. This loss-of-roll phenomenon is quite similar to that seen in a rectangular cavity (Kolodner et al.).

To compare with the experimental results, the numerically generated radial data have been transformed onto a rectangular grid. The grid has the dimensions of the window used in the experiments for capturing the interferometric projections. Figs 5 and 6 show the numerically generated contours from the projections calculated along the length of the cavity for $Ra=5861$ and $Ra=12,124$, respectively. For clarity, only a few rolls are shown. The formation of rolls similar to those interferograms captured experimentally (shown in Fig. 7) is clearly seen in the cavity. In an axisymmetric geometry, an equivalent result is not available in the published literature till date to the best of the knowledge of the authors.
Evidence reported in the present work shows that the axisymmetric rolls are possibly in the form of concentric rings, the schematic for which has been shown in Fig. 8. For $Ra=12,124$, the rolls are well-defined through a clear displacement of the contours. The contours in Fig. 6 have been shown along with a shading scheme, a darker background indicating higher gradients. As expected, the temperature gradients near the walls are highest. The shaded regions near the walls are relatively straight, similar to straight fringes in the experimental interferograms. These are also coincident with the sites of the highest convective fluid velocities.

Comparison with the experimental results

Interferograms collected experimentally were processed and analysed to get quantitative data in terms of temperature distribution in the fluid layer, heat transfer rates determined by local and average Nusselt numbers at the top and bottom walls.

Using the numerically determined projection data, the variation of the width-averaged temperature profile with respect to the vertical coordinate has been plotted in Figs 9 and 10. Experimentally obtained width-averaged temperature profile is also shown for the two view angles ($0^\circ$ and $45^\circ$) and for both the Rayleigh numbers in the same figure. A good agreement can be seen between the numerical simulation and the experimental results. A slight difference can be attributed to the mild unsteadiness and lateral movement of the fringes present in the fluid layer during the experiments that was not reflected in the simulation. The inverted S-shaped curve in both the cases reveals the buoyancy driven convective flow. The slopes of the individual curves in near the walls ($y=0,1$) are quite close to each other. It reflects the fact that the net heat transfer across the cavity at steady state from the hot surface to the cold does not depend on the view angle. In this respect, the convection in the fluid layer is axisymmetric. This also reflects the energy balance at the steady state.

The numerically determined local Nusselt numbers within a cell at the hot and the cold walls are presented in Figs 11 and 12 for both the Rayleigh numbers. The average Nusselt number for the full cavity at $Ra=5861$ and $Ra=12,124$ numbers have been calculated to be 2.061 and 2.531, respectively.

Experimentally, the local and average heat transfer rates at the bounding walls have also been calculated in terms of local Nusselt number defined by Eq. (8). Fig. 13 show the variation of the local Nusselt number at the top and the bottom walls of the cavity at $Ra=5861$ and $Ra=12,124$ for $0^\circ$ and $45^\circ$ projection.
angles. For $0^\circ$ projection angle, the individual wall-averaged Nusselt numbers are 2.07 and 1.996 at the cold and hot surfaces respectively. For the $45^\circ$ projection angle, these are 1.916 and 1.893 respectively. The average values of the Nusselt numbers, as calculated from both the projection angles, match well. This reveals the axisymmetric nature of the convection in the fluid layer at $Ra=5861$.

At $Ra=12,124$, for $0^\circ$ projection angle, the individual wall-averaged Nusselt numbers are 2.48 and 2.38 at the cold and the hot surfaces, respectively. The corresponding values for $45^\circ$ projections are 2.31 and 2.26 at the cold and the hot surfaces respectively. For $Ra=12,124$, the difference in the values of the heat transfer rates for the two projection angles may be attributed to the mild unsteadiness present in the flow field. This can be the reason for the loss of axisymmetry in strictly quantitative terms. These values of the heat transfer rates, determined experimentally and numerically, are in good agreement with the values given by experimental correlation of Gebhart et al.\textsuperscript{10} The average Nusselt number as determined from the correlation is 2.081 at $Ra=5861$ and 2.51 at $Ra=12,124$ with an uncertainty level of $\pm 20\%$.

The local Nusselt number in Fig. 13 has been plotted as a function of the dimensionless distance within a roll. Regions of high heat transfer on one wall correlate well with those of low heat transfer on the other. This trend brings out the skewness of the streamline pattern in the individual rolls. An examination of the adjacent cells would show that the rolls have opposing inclination.

Two factors that can additionally lead to a discrepancy between the experiments and the numerical simulation are: (a) the average Nusselt number in the experiment is calculated over only a single roll, and (b) unsteadiness in the experiments that is not reflected in the simulation.

The results obtained from the numerical simulation support the formation of the concentric rolls as predicted from the experimental observation. Experimentally, the fringe patterns as shown in Fig. 7 have been recorded in a direction parallel to the diameter of the test cell. Appearance of circulation rolls can be proposed for the two Rayleigh numbers. Thus, the viewing axis parallel to the light beam is at right angles to that of the circulation pattern within the rolls (Fig. 8). Owing to the component of the radial velocity normal to the viewing direction, additional cell-like structures can be expected in the experimentally obtained interferometric projections.

A typical interferogram of the circular fluid layer is shown in Fig. 7. The flow cell in the interferogram, distinct from those in Fig. 8 can be identified as follows. The cell boundaries will pass through the locations where the fringe slope is zero. This procedure is meaningful because one can visualize hot fluid rising along one boundary, on the right side in Fig. 7, and descend along the other. Hence the cell in Fig. 8 can be considered as primary and that in Fig. 7 as secondary.

Conclusions

The present investigations are focused on the study of Rayleigh-Benard convection for an axisymmetric
cavity both experimentally and numerically which has not been reported till now. The published literature on this topic shows that this study is limited to rectangular and square cavities only till date. This paper presents the numerical study of Rayleigh-Benard convection in a circular, differentially heated large aspect ratio fluid layer for two Rayleigh numbers, 5861 and 12,124 with air as the working fluid. The numerical results are compared with the experimentally obtained quantitative results. For both the values of the Rayleigh numbers, the numerically generated streamlines and the isotherms in the fluid layer for the full cavity show the clear formation of rolls. The loss-of-rolls phenomenon has also been observed with an increase in the Rayleigh number. The numerically generated contours from the projections calculated along the length of the cavity also show the formation of rolls similar to those in the interferograms obtained experimentally. Wall heat transfer rates calculated in terms of Nusselt numbers show good agreement between the numerical simulation and the experiments and compare well with the correlation proposed by Gebhart et al. An acceptable agreement in the width-averaged temperature profiles between the numerical simulation and the experimental values is also observed.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$G$</td>
<td>Acceleration due to gravity</td>
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<tr>
<td>$h$</td>
<td>Height of the cavity</td>
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<tr>
<td>$n$</td>
<td>Refractive index of the fluid</td>
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<tr>
<td>$Nu$</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>$Pr$</td>
<td>Prandtl number of the fluid</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Reference temperature</td>
</tr>
<tr>
<td>$r, z$</td>
<td>Cylindrical co-ordinates</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity of the fluid</td>
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<tr>
<td>$\beta$</td>
<td>Coefficient of volume expansion</td>
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<tr>
<td>$\lambda$</td>
<td>Wavelength of the laser</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Kinematic viscosity of the fluid</td>
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<tr>
<td>$\rho$</td>
<td>Density of fluid</td>
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</table>

References