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Symmetry in planetary gear trains

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Along with the generation of gear trains it will be useful for the designer to know other structural characteristics. Symmetry is one such characteristic. Structural symmetry will make the balancing easy. This will also reduce the generation effort. In this paper, an attempt is made to compare all the planetary gear trains with up to four gear pairs (or six elements) based on symmetry.

Gear trains are typically used to transmit specified motion and/or torque between two or more shafts. The design of gear trains to transmit motion at a desired velocity ratio has received its impetus from the age of clock making. Innumerable other applications exist like in automobile gear boxes, helicopter mechanisms, differentials, gas turbine engines, machine tool gear boxes, robot actuator mechanisms etc.

A gear train is referred to as an ordinary gear train if the axes of the gear shafts are fixed in position. In gear trains called planetary gear trains (PGT), planetary gears revolve around the axis of sun gear producing epicyclic motion, because of which these gears trains are also known as epicyclic gear trains (EGT). Fig. 1a shows a simple epicyclic gear train with one central sun wheel and planetary gear which rotates freely on bearings in the crank-arm and revolves along with the crank-arm around the sun wheel. Fig. 1b shows the Levai notation[1] and functional diagram of the EGT is shown in Fig. 1a.

Synthesis is creation of a mechanism to satisfy a desired functional requirement and is traditionally accomplished by the designer's intuition and experience. Synthesis of planetary gear trains has been the subject of study for the last two decades and recently there has been a lot of work done in the generation of PGTs in a systematic manner.

Kinematic structures of the same type, i.e., same dof, number of links and nature of desired motion are enumerated systematically with the aid of graph theory. The application of graph theory to the systematic synthesis of epicyclic gear trains was first investigated by Buchsbaum and Freudenstein[2] and later by various researchers[3-8].

All these enumerated studies for PGTs have the same task of synthesis of PGTs with major differences in two areas. The first difference lies in the method adopted for the generation of graphs, which should contain all the PGTs. The second difference is in the procedure used to test for isomorphism in the chains generated. Reported in the literature are three different methods for generation of PGTs: (i) non-recursive method, (ii) recursive method and (iii) acyclic graphs method. Using the above three methods one and two dof geared kinematic chains with up to eight links have been studied. Test for isomorphism is essential part of the synthesis of PGTs.

Several methods have been adopted by different authors[4-8] to test for isomorphism in PGTs, each method having its own merits and demerits. All these methods are limited to isomorphism. Hamming number technique[9] besides checking for isomorphism gives other characteristics like parallelism and symmetry of the chain. In this paper, an attempt is made to determine the symmetry of different non-isomorphic graphs and rate the graphs by their symmetry.

Symmetry

As shown in Fig. 1 an epicyclic gear train consists of a central sun gear 'A' and planetary gear 'B' which produce epicyclic motion by rolling around the periphery of the sun gear. Crank arm 'L' contains bearings for the planetary gear to maintain the mesh between the wheels.

For better force balance, two more planet gears are added, because adding more planet gears increases the number of forces. But additional planets do not con-

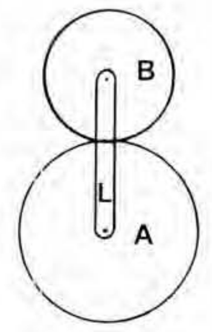

Fig. 1a—A simple epicyclic gear train (EGT)

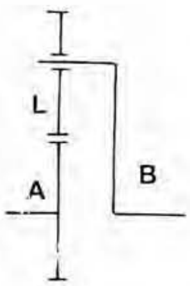

Fig. 1b—The Levai notation

tribute to kinematic performance of gear trains. This is explained through Figs 1 and 2. In Fig. 2, two more planet gears are added to the PGT shown in Fig. 1. Due to symmetrically placed planet gears B, C and D it is better balanced compared to the gear train in Fig. 1.

Structural symmetry[10] is important from the viewpoint of aesthetics and generation of structures. The former aspect contributes to architectural beauty while the later aspect simplifies the generation process which otherwise is tedious and time-consuming, i.e., by noting the symmetry of a graph one can reduce the number of isomorphic graphs/structures enumerated. An algorithm to identify the symmetric vertices in a graph is needed so that enumeration process can be automated using a digital computer.

Pugh[11] in his discussion of Tsai[4] paper on topological synthesis of gear trains gives an example in which he explains how one can reduce the number of isomorphic graphs enumerated with the standard generation techniques, if symmetry of a graph is considered. While agreeing to this point, Tsai[4] stresses the need for an algorithm to identify the symmetric vertices in a graph, so that the enumeration process can be completely automated by a digital computer. Obviously there was no algorithm at that time.

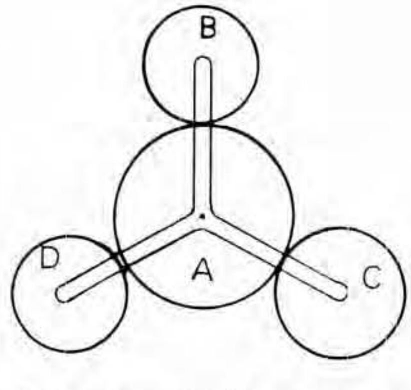

Fig. 2—Epicyclic gear train with two more planet gears

Symmetry refers to the symmetrical placement of members in the structure, i.e., identical location of identical members with reference to another member indicates symmetry about the member. Symmetry, if any, and the number of symmetric member pairs, with reference to every member can be detected by the hamming number matrix[9] of the chain/graph.

Adjacency matrix or connectivity matrix gives whether or not each link in a structure/graph is connected directly to any other link in the chain. Depending on the type of connectivity the elements of adjacency matrix are 1, 2 or 0. Adjacency matrix is a square and symmetric matrix of size 'n', where n is the number of links in the chain. If a link 'i' is directly connected to another link 'j' by gear pair the element in the i^{th} row and j^{th} column is 2, if they are connected by a turning pair its value is 1. If the links are not connected directly then the element value is zero.

To write the adjacency matrix the following rules are given [9]:

The element $a_{ij} = 2$, if link i is connected to link j by gear pair

$=1$, if link i is connected to link j by turning pair

$=0$, if links are not connected directly

$a_{ii} = 0$, since no link can connect to itself.

Example

For the PGT shown in Fig. 3, adjacency matrix is given below:

$$A = \begin{vmatrix} 0 & 2 & 2 & 2 \\ 2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 \end{vmatrix}$$

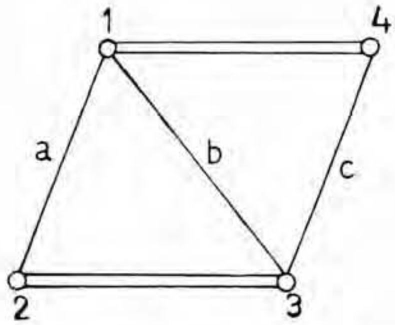

Fig. 3 — Graph of planetary gear train

Every row in adjacency matrix represents a particular member and every element in that row indicates whether or not, any other member is directly connected to that member.

With the help of hamming number matrix explained in the following paragraphs, symmetry can be revealed by identifying the identical members and their locations with respect to any reference member. For a given graph/chain, link adjacency matrix is defined from which hamming number matrix is written.

Hamming matrix is written for every graph/structure from the adjacency matrix, based on the definition of hamming number[9]. In this work Hamming number between any two members is obtained by considering the elements in the two corresponding rows of the adjacency matrix. Elements in the two rows compared may or may not be identical at all the digital places.

Hamming number or an element of hamming matrix (h_{ij}) is obtained by adding the elements of i^{th} row to the elements of j^{th} row in an adjacency matrix, if the corresponding elements are not equal.

Rules for writing the elements of hamming matrix are[9]:

(i) $h_{ij} = \sum_{k=1}^{n} a_{ik} + a_{jk}$ if $a_{ik} \neq a_{jk}$.

(ii) $h_{ij} = 0$ if $a_{ik} = a_{jk}$. and

(iii) $h_{ii} = 0$

For example consider the rows 1 and 2 of the adjacency matrix for Fig. 3.

Row 1: 0 2 2 2
Row 2: 2 0 1 1

The elements of these two rows differ at first, second, third and fourth digital places. Hence, the hamming number $h_{12} = (0 + 2) + (2 + 0) + (2 + 1) + (2 + 1) = 10$.

With this understanding for the chain shown in Fig. 3, hamming number matrix is:

$$H = \begin{vmatrix} 0 & 10 & 9 & 9 \\ 10 & 0 & 3 & 3 \\ 9 & 3 & 0 & 0 \\ 9 & 3 & 0 & 0 \end{vmatrix}$$

Elements of each row of Hamming matrix H are summed up and the same is called hamming value of the member corresponding to the row. For example, in Fig. 3 hamming value of member 1 is 28 and for member 2 it is 16 and for members 3 and 4 hamming values are same and equal to 12.

Each member can be represented by a numerical string consisting of its hamming value followed by the elements of its row in descending order. For example, for member-1 the hamming string is 28|10, 9, 9, 0| and for member-2 it is 16|10, 3, 3, 0|.

Two members are considered identical if their strings are identical, i.e., one-to-one correspondence, otherwise they are non-identical. For the Fig. 3, links 3 and 4 are identical.

Symmetry about a member is identical location of identical members. To know symmetry about a link, search for identical location of identical links other than the members identical to link about which symmetry is sought. In this example links 3 and 4, having identical strings, are identical. They are considered symmetrically placed about links 1 and 2, since their hamming values with respect to the members 1 and 2 are same (as evident from the first and second rows of the hamming matrix) i.e., links 3 and 4 have identical hamming elements (9, 9) in row-1. Therefore with respect to link 1, links 3 and 4 are symmetric. Similarly with respect to link 2, links 3 and 4 are symmetric. It is evident from the graph in Fig. 3, that links 3 and 4 (gears 3 and 4) are symmetrically placed with respect to link 1 (crank arm 1) and link 2 (gear wheel 2).

The number of symmetric pairs of members about every member of the graph/chain can be determined. Sum of all such number of pairs of members of a chain is an indication of symmetry that exists in the structure. Higher the sum, greater is the symmetry. As another example consider Fig. 4, for which adjacency and hamming matrices are:

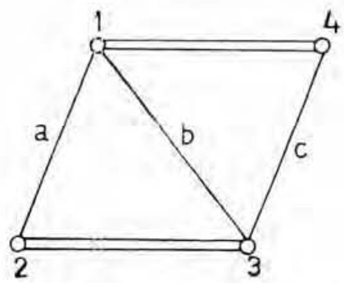

Fig. 4 — Graph of planetary gear train for grater symmetry

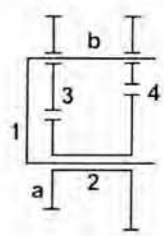

Fig. 5a — Functional diagram of the graph in Fig. 3

$$A = \begin{vmatrix} 0 & 2 & 2 & 1 \\ 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{vmatrix} \qquad H = \begin{vmatrix} 0 & 8 & 10 & 4 \\ 8 & 0 & 4 & 6 \\ 10 & 4 & 0 & 8 \\ 4 & 6 & 8 & 0 \end{vmatrix}$$

Hamming strings for the four links in the Fig. 4 are, for links 1 and 3, 22[10, 8, 4, 0] and for links 2 and 4, 18[8, 6, 4, 0]. Hence, links 1 and 3 are identical. It is true with the links 2 and 4.

However, no pair of members is symmetric about any other member in the chain, as the identical links have no equal hamming values in the rows of Hamming matrix. For example links 2 and 4 do not have the same Hamming values about link 1, i.e., in the row 1. Even though there are two pairs of identical members in the chain, symmetry in the graph/chain is zero, which is also clearly evident from the graph in Fig. 4 and its functional diagram in Fig. 5b.

Comparing the two graphs with two gear elements, PGT in Fig. 3 is more symmetric compared to PGT in Fig. 4. The functional diagrams of the graphs in Figs 3 and 4 are given in Figs 5a and 5b, which supports the above statement.

Conclusions

1. Structural symmetry is important both from architectural beauty of the structure and generation of structures.

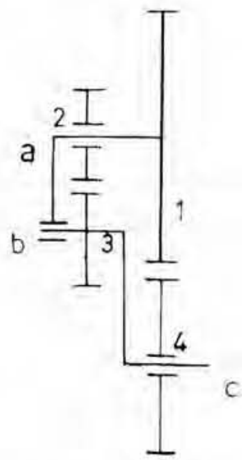

Fig. 5b— Functional diagram of the graph in Fig. 4

2. Generation process is simplified if symmetry aspect is incorporated into synthesis process.
3. Better balance can easily be achieved by having more symmetry in the PGT.
4. Isomorphism among links in a PGT is identified using hamming strings of links. Then identical pairs of links are determined and a search is carried for the identical location of the identical pairs of links about every link. Numbers of symmetric pairs are identified. Sum of such symmetric pairs (about all links in the chain) is the index of symmetry.
5. The above method is highly useful to automate the generation of graphs.

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