PID control of ball and beam system — A real time experimentation

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Ball and beam system is a nonlinear, open loop unstable and second order integrating process which is widely used as a test bed for evaluating various control strategies. The stabilization of ball and beam system by Proportional, Integral and Derivative (PID) controller is investigated. Simple internal model control (SIMC) based PID controller and H infinity controller are deliberately employed for stabilizing the ball at a desired position on the beam. The simulation and experimental results which demonstrate the dynamic behaviour of ball and beam system, reveals the effectiveness of controller in industries for any second order integrating processes.

Keywords: Ball and Beam system, nonlinear, PID controller, SIMC and H infinity.

Introduction

A Proportional-Integral-Derivative controller (PID controller) is one of the common feedback loop components in industrial control that effectively meets most of the control objectives in the industries. The reason for wide use of PID controller still in industries is its mathematical simplicity, safety and performance. Based on the history and rate of change of the error signal, the PID controller produces process outputs and thus provides more accurate and stable control. In order to get the desired control response, the controller adjusts the parameters such as gain/proportional band, integral gain/reset and derivative gain/rate to the optimum values. Most of the PID tuning rules for various processes are summarised in1. There are various control techniques available in the literature2, 3, 4, 5 and most of the controls are intelligent controls. Intelligent controllers cannot be tuned easily without vast system knowledge and thus conventional PID controller is designed due to ease of use and simplicity of tuning rules. It is noted that most of the tuning rules that were proposed are related to stable, unstable and integrating process6, 7, 8, 9, whereas only a few literature is available for double integrating process. The design of PID controllers for double integrating system is given by Skogestad’s method10 and Tao Liu method11. SIMC tuning method is a model-based tuning technique in which the controller parameters are expressed as a function of the process model parameters.

In this paper, the simulation and experimental results based on SIMC method and H infinity for ball and beam system is presented. The set point tracking and square wave tracking of the controller is performed. The performance analysis is done based on time and frequency domains.

Ball and Beam System

Figure 1 shows the closed loop architecture of ball and beam system. Ball and beam system is experimental equipment specially designed for learning dynamic control principles, modern control engineering and electrical motor control. The ball is placed on a beam and is free to move over the beam. To one end of the beam, motor gear is attached while the other end is fixed. The actual position of the ball is sensed with a position sensor and it is compared with the desired position to calculate the error (e). The PID controller calculates the control signal based on the error and the controller parameters. The control command from Personal computer (PC) controls the ball position on the beam through the control box.

Dynamics of Ball and Beam System

When the ball is moved away from the horizontal position, the gravity causes the ball to roll along the beam. With the change in angle (θ) of servo gear, the beam angle (α) changes. This change in angle along with
gravity causes the ball to roll along the length of the beam (L). Here, it is assumed that the ball rolls without slipping and also the friction between beam and ball is negligible. The gravitational, inertial and centrifugal forces contribute to the dynamics of the ball. The simplified Lagrangian equation of motion for the ball is given by the differential equation as

\[
\alpha \left( \frac{J}{r} + m \right) \ddot{\alpha} + mg \sin \alpha - m r \alpha \dot{\alpha}^2 = 0 \quad \text{...(1)}
\]

On linearizing the equation (1) about the beam angle, the system is obtained as

\[
\left( \frac{J}{R^2} + m \right) \ddot{\theta} = -m g \alpha \quad \text{...(2)}
\]

Equating the arc distance, the equation which relates the beam angle to the angle of the gear is approximated as linear as in equation (3)

\[
\ddot{\theta} = \frac{d \theta}{L} \quad \text{...(3)}
\]

Substituting equation (3) in equation (2) and taking Laplace transform, the transfer function of the system is obtained as

\[
\frac{r(s)}{\theta(s)} = \frac{0.7}{s} \quad \text{...(4)}
\]

and with the parameters in Table 1, the transfer function is obtained as,

\[
\frac{r(s)}{\theta(s)} = \frac{0.7}{s^2} \quad \text{...(5)}
\]

The above plant transfer function is a double integrator as in equation (5). As such, it is unstable and it provides a challenging control problem. In real time experimentation, the sample time is 0.01 sec and this is considered as the delay ($\tau_m$) to calculate the parameters of the controller.

**PID Control**

A PID control algorithm is implemented in such a way that the desired position of the ball is stabilized on the beam by adjusting the beam angle. Skogestad tuning rule is based on modification of the widespread industrial accepted Internal Model Control (IMC) PID tuning rules of Rivera, Morari and Skogestad. In case of integrating processes, the disturbance reduction is achieved by modifying the integral term. The method shows the dependence of tuning on the process parameters.

The general form of double integrating process is given as $G_p$.

\[
G_p = \frac{K_m e^{-s \tau}}{s^2} \quad \text{...(6)}
\]

and the SIMC-PID parameters are given as
$K_c = \frac{0.0625}{K_m \bar{U}_m}$

$T_1 = 8 \bar{U}_m$

$T_d = 8 \bar{U}_m$

Where $K_c$, $T_1$, $T_d$ and $\bar{U}_m$ are the proportional, integral, derivative parameters and time delay respectively.

The PID controller parameters for Ball and Beam system designed by SIMC are tabulated in table 2. H infinity optimal criterion proposed by Tao Liu is an analytical design method of PID controller for industrial second order integrating processes\textsuperscript{11}. The tuning parameters of H infinity controller are computed and tabulated in table 3.

### Result and Discussions

#### Simulation Results

Control simulations are done to demonstrate the performance of SIMC based PID controller and H infinity controller. The non-linear system with the PID controller is simulated in MATLAB-Simulink. From the simulation results, it is predicted that the ball position would be controllable with SIMC-PID controller and H infinity controller effectively. Figure 2 shows the dynamic response with set point tracking. Simulation results of the same position, positive and negative step change results in same time responses as the system is approximated to a linear model. The system controlled with SIMC based PID controller and H infinity controller operates adequately in the desired range of beam with acceptable control actions. To examine the control performance over the entire range, the set point is changed from 10 cm to 20 cm, 30 cm, 20 cm and 10 cm at time 20s, 40s, 60s and 80s respectively. The performance analysis based on steady state error, overshoot, settling time, rise time, ISE and IAE are tabulated in Table 4.

#### Experimental Results

The set point is given as 0.1 m, 0.2 m, 0.3 m, 0.2 m and 0.1 m at regular interval of time. The tracking of the controller i.e. maintaining the ball in the given input position is performed by the controller successfully. From figure 3, it is clear that both SIMC-PID and H infinity;
the controller takes necessary control actions in order to maintain the desired ball position. The Dead band, discrete position sensing, Backlash introduced by the DC motor and belt pulley are the nonlinearities of the system. The behaviour of the system under control is affected by these nonlinearities.

**Conclusion**

In this paper, we have presented SIMC based PID control and H infinity control of ball and beam system. Analysis shows that both the control schemes can provide satisfactory performance for position control in terms of both time domain and frequency domain. Tracking cases shows that the performance of the controller is well established. Even when the position is changed rapidly, the controllers immediately take the system under control and are well suited for real time application of second order integrating processes in industries. Nyquist plot facilitates the analysis and convergence of SIMC based PID controller and H infinity controller. Its effectiveness is further supported by experimental results.

**References**