Nonlinear wave propagation in ferroelectric/ferrite interface

Michael Augustine, Roji Pius, Sunny Mathew & Vincent Mathew*
Post Graduate and Research Department of Physics, St. Thomas College, Palai 686 574, India

Received 2 November 2006; accepted 28 February 2008

The propagation of electromagnetic wave through the interface of magnetized ferrite and nonlinear ferroelectric half-spaces has been theoretically studied. A dispersion relation for the wave propagation corresponding to the TE mode has been derived and numerically solved. It is observed that the propagation is non-reciprocal. The dependence of power on frequency has also been evaluated.

Propagation of electromagnetic waves in a layered waveguide structure containing magnetized ferrite media has attracted great interest due to its applications in nonreciprocal devices such as isolators, circulators. Similarly, various nonlinear materials have also been investigated for possible applications in dielectric waveguides as well as optical signal processing.

Recently, ferroelectric nonlinear materials like barium strontium titanate which has low loss and field dependent permittivity has gained much importance. In this paper, the combined effect of magnetized ferrite and ferroelectric (nonlinear) layered media in guiding electromagnetic wave has been studied. Starting from an analytical solution of the nonlinear wave equation in the nonlinear layer, a dispersion relation for magnetostatic surface wave, which is the TE mode of propagation, has been derived and numerically solved. The dependence of power on frequency is also evaluated. It is found that the magnetostatic wave propagation in the structure is highly nonreciprocal.

The derivation of the dispersion relation and the calculation of power propagation have been briefly outlined in the following section. Numerical results for the dispersion of forward and backward propagation as well as the propagation of power have been presented.

Theory

The structure being analyzed is given in Fig. 1. The substrate \((y < 0)\) is linear ferrite and the cladding \((y > 0)\) is nonlinear ferroelectric. The ferrite material is subjected to biasing magnetic field \(B\) in \(x\)-direction. With respect to the geometry of the structure, transverse electric (TE) modes, corresponding to magnetostatic condition, can propagate in \(z\)-direction. For TE waves, the nonzero field components are \(E_x\), \(H_y\) and \(H_z\). The electromagnetic behaviour of the ferrite medium \((y<0)\) is characterized by the following permeability tensor:

\[
\mu = \mu_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & \eta & j \kappa \\ 0 & -j \kappa & \eta \end{bmatrix}
\]

where,

\[
\eta = 1 + \frac{\omega \mu_0 \omega_n}{\omega_0^2 - \omega^2}, \quad \kappa = -\frac{\omega \mu_0 \omega}{\omega_0^2 - \omega^2},
\]

\[
\omega_0 = \gamma \mu_0 H_0, \quad \omega_n = \gamma \mu_0 M_s
\]

\(\gamma\) is the gyromagnetic ratio, \(\mu_0\) is the permeability of free space, \(H_0\) is the intensity of the applied magnetic field.
field, $M_s$ is the saturation magnetization, $\omega$ is the signal frequency and $j=\sqrt{-1}$. The ferroelectric medium (barium strontium titanate) has a field dependent permittivity given by

$$\varepsilon(\omega,E_s)=\varepsilon_0 a_0 + \varepsilon_1 a_1 E_s + \varepsilon_2 a_2 E_s^2,$$  

(2)

where, $a_1$ and $a_2$ are nonlinearity coefficients and $\varepsilon_0$ is the permittivity of free space (For a Kerr type nonlinearity $a_1=0$).

Now from Maxwell’s equations, the following wave equations can be constructed in the two regions:

$$\frac{\partial^2 E_x}{\partial y^2} + \left(k_x^2 \varepsilon_0 a_0 - k^2\right) E_x + a_1 k_x^2 \varepsilon_0 E_x^2 + a_2 k_x^2 \varepsilon_0 E_x^3 = 0; \quad y > 0$$

(3)

$$\frac{\partial^2 E_x}{\partial y^2} - k_x^2 E_x = 0; \quad y < 0$$

(4)

where,

$$k_x = (k^2 - \omega^2 \varepsilon_0 \mu_0 \varepsilon_f) \frac{1}{\eta^2 - k^2}$$

$$\mu_y = \frac{\eta^2 - k^2}{\eta}$$

$\varepsilon_f$ is the relative permittivity of ferrite and $k$ is the propagation constant. Eq. (3) can be analytically integrated by making use of tabulated integrals. The result of this procedure is

$$E_x = C 2k_x^2 e^{i(ar-kz)}$$

(5)

where $D$ is a constant.

The magnetic field components are derived using Maxwell’s equation $\text{curl } H = -j\omega \mu E$. Now equating the tangential field components $E_x$ and $H_z$ at the interface, the following dispersion relation can be derived:

$$q r s - k_x m r + k_y q t = 0,$$

(7)

where,

$$q = -b_1 \text{sech}^2(\Lambda_1) \pm \text{sech}(\Lambda_1) \tanh(\Lambda_1) \sqrt{4k_x^2 c_1 - b_1^2}$$

$$r = b_2 - 4k_x^2 c_1 \tanh^2(\Lambda_1)$$

$$s = \left( \frac{k}{\mu} - \frac{k_f}{\mu_f} \right)$$

$$m = 2b_1 \text{sech}^2(\Lambda_1) \tanh(\Lambda_1)$$

$$\pm \left( \text{sech}^2(\Lambda_1) - \text{sech}(\Lambda_1) \tanh^2(\Lambda_1) \right) \sqrt{4k_x^2 c_1 - b_1^2}$$

$$t = -8k_x^2 c_1 \text{sech}^2(\Lambda_1) \tanh(\Lambda_1)$$

$$\Lambda_1 = -k_2 y_0$$

$$\mu_j = \frac{\eta^2 - k^2}{\kappa}.$$

The propagation of power can be easily computed using Poynting’s theorem. The expression for power is given by,

$$P = -\frac{1}{2} \Re \int_{-\infty}^{\infty} E_x H'_y dy$$

(8)

where * indicates complex conjugation.

The dispersion relation can also be constructed in a numerical scheme in which the nonlinear layer is subdivided into a number of layers and assuming in each layer, the field being constant. Power computation can also been done in this way. However, in the numerical results presented here, the analytical scheme is only used, as it is not based on approximations.
Numerical Results

The dispersion relation (Eq. (7)) has been solved numerically. In computation, $\mu_0 H_0$ is set to be equal to 0.05 Tesla and $\mu_0 M_0$ equal to 0.175 Tesla. The gyromagnetic ratio $\gamma = 1.76 \times 10^4$ C/kg. The nonlinear parameters $a_1$ and $a_2$ are set equal to $3/2$ and $2/3$ respectively. For magnetostatic wave propagation, $\mu < 0$ where the frequency falls in the region $(\sqrt{\omega_1(\omega_1+\omega_m)} < \omega < (\omega_1+\omega_m))$. However, in computation the other situation, that is, $\mu > 0$ is also considered.

The variation of propagation constant with frequency is presented in Fig. 2. The curves 1-4 correspond to four values of the nonlinear term $\tanh(\Lambda_1)$, viz., 0.3, 0.5, 0.7 and 0.9. As expected, the propagation in the structure is highly nonreciprocal. In order to explicitly present this effect, dispersion for forward and backward waves are plotted for one value of $\tanh(\Lambda_1)$ in Fig. 3. This nonreciprocal property of the structure makes it suitable for signal processing.
applications. If we put nonlinearity coefficient \( a_1 = 0 \) in Eq. (1), then the dispersion relation becomes that for a Kerr-like medium and the corresponding dispersion curves are displayed in Fig. 4. Here also the curves 1-4 correspond to four values of the nonlinear term \( \tanh(\Lambda_1) = 0.3, 0.5, 0.7 \) and 0.9. For higher frequencies, the propagation constant of the Kerr-like structure is larger than that of the non Kerr-like structure.

The power flow in the structure is also studied and the variation of power with frequency is plotted in Fig. 5. The power depends on frequency. Since the nonlinearity coefficient values assumed are comparatively high, the power is only of the order of milliwatts/m.

**Conclusion**

The propagation of electromagnetic wave in a structure consisting a nonlinear ferroelectric cladding and linear ferrite substrate is studied. A general dispersion relation is derived and the propagation in the structure is found to be nonreciprocal. The dependence of power propagation in the structure on frequency is also studied.

**Acknowledgements**

Michael Augustine thanks University Grants Commission, India for support through a Faculty Improvement Programme. Vincent Mathew is thankful to Kerala State Council for Science Technology and Environment for financial assistance through a research scheme (#351/DIR/04-05).

**References**