Transient free convection flow of a viscous dissipative fluid with mass transfer past a semi-infinite vertical plate

Fathia Moh. Al Samman\textsuperscript{a}, M Y Gokhale\textsuperscript{b}, V M Soundalgekar\textsuperscript{c}

\textsuperscript{a}Dept. of Mathematics, University of Pune, Pune (411 007), India
\textsuperscript{b}Dept. of Mathematics, Maharashtra Institute of Technology, Kothrud, Pune 411 038, India
\textsuperscript{c}31A-12, Brindavan Society, Thane (400 601), India

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Transient free convection flow of a dissipative fluid past a semi-infinite vertical plate is studied by considering the presence of species concentration. The governing equations in non-dimensional form are solved by implicit finite-difference technique of Crank-Nicholson which is stable and convergent. Transient velocity, temperature and concentration profiles, local and average Skin-friction, Nusselt number and Sherwood number are shown graphically for air. The effects of viscous dissipative heat, Schmidt number, buoyancy ratio parameter on the transient state are discussed.

Steady free convection flow of a viscous fluid past a semi-infinite vertical plate was studied extensively in 50's and 60's as it has wide applications in nuclear reactors, heat exchanges, cooling appliances in electronic instruments. Similarity solutions under different boundary conditions were presented and these are discussed in Ch 3 of the book by Gebhart \textit{et al.}\textsuperscript{1}. In many papers at early stage, viscous dissipative heat was neglected. But Gebhart\textsuperscript{2} first showed the importance of viscous dissipative heat in free convection flow past a semi-infinite vertical plate and presented a similarity solution by defining a dissipation parameter $\epsilon = \beta L / \nu$ which is equal to the ratio of the kinematic energy of the flow to the heat transferred to the fluid.

All these studies are confined to steady-flows. Transient free convection flows also play an important role in cooling of nuclear reactors, etc. So, Callahan and Marner\textsuperscript{3} studied the transient free convection flow past a semi-infinite vertical plate without taking into account viscous dissipative heat, but considering the presence of a foreign mass. However, the considered fluid was having Prandtl number unity and the foreign mass was considered with Schmidt number as 0.2, 0.7, 7.0. The non-linear system of equations were solved by explicit finite-difference scheme and its convergence for stability was established for $P_D = 1.0$. So, Soundalgekar and Ganesan\textsuperscript{4} resolved Callahan and Marner's problem by implicit finite-difference scheme which is always stable and convergent and compared the results with those of Callahan and Marner and the agreement was excellent.

Again, Soundalgekar \textit{et al.}\textsuperscript{5}, also studied the effects of viscous dissipative heat on the transient free convection without mass transfer effects and solved the partial differential equations by implicit finite-difference scheme of Crank-Nicholson type and the effects of viscous dissipative heat on time to reach the steady state were studied.

How the viscous dissipative heat affects the transient free convection flow in the presence of foreign mass? It has not been studied at all. Results of the investigations of this paper can be used in the various applications to the related fields mentioned earlier.

\textbf{Mathematical Analysis}

We consider the flow of viscous incompressible fluid containing foreign mass like CO\textsubscript{2}, O\textsubscript{2}, etc. past a semi-infinite vertical plate. The plate, the fluid and the foreign mass are assumed to be at the same temperature $T_w$ and the concentration level $C_w$ initially. At time $t > 0$, the plate temperature and the concentration level are assumed to be raised to $T_v$ and $C_v$ respectively. The $x$-axis is taken along the plate in a vertically upward direction and the $y$-axis is taken normal to the plate. Then, under usual Boussinesq approximation, with low concentration level, the flow is governed by the equations:

\begin{equation}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}
\end{equation}
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} &= v \frac{\partial^2 u}{\partial y^2} + g \beta (T - T_m) + g \beta' (c - c_m) \\
\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= K \frac{\partial^2 T}{\partial y^2} + \mu \left( \frac{\partial u}{\partial y} \right)^2 \\
\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} &= D \frac{\partial^2 c}{\partial y^2}
\end{align*}
\] ... (2)  

with the following initial and boundary conditions:

\[
\begin{align*}
t &\leq 0, \quad u = 0, \quad v = 0, \quad T = T_m, \quad c = c_m, \quad \text{for all} \quad y \\
t > 0, \quad u = 0, \quad v = 0, \quad T = T_m, \quad c = c_m, \quad \text{at} \quad x = 0 \\
u = 0, \quad v = 0, \quad T = T_m, \quad c = c_m, \quad \text{at} \quad y = 0 \\
\lim_{y \to \infty} u = 0, \quad v = 0, \quad T = T_m, \quad c = c_m, \quad \text{as} \quad y \to \infty
\end{align*}
\] ... (5)

There \( u, \ v \) are the velocity components of fluid along \( x \) and \( y \) directions respectively, \( \rho \) is the density \( T \) the temperature of the fluid, \( T_m \) the temperature of the plate, \( \mu \) the coefficient of viscosity, \( \beta \) the coefficient of volume expansion, \( c_p \) the specific heat at constant pressure, \( \alpha = \frac{\beta}{\rho c_p} \) is the thermal diffusivity, \( c \) the species concentration in the fluid, \( c_m \) the species concentration in the fluid away from the plate and \( c_a \) the species concentration in the fluid near the plate, and all others are defined in the nomenclature.

We now introduce the following non-dimensional quantities:

\[
\begin{align*}
\frac{u}{L} &= \frac{u L}{v G r^{1/2}}, \quad \frac{v}{L} = \frac{v L}{v G r^{1/2}}, \quad X = x/L \\
\frac{Y}{L} &= \frac{y L}{G r^{1/2}}, \quad \frac{T}{T_m} = \frac{T - T_m}{T_m - T_a}, \quad C = \frac{c - c_m}{c_m - c_m} \\
\frac{G r}{v^2} &= \frac{g \beta L^2 (T_m - T_a)}{v^2}, \quad \frac{G r_c}{v^2} = \frac{g \beta' L^2 (c_m - c_m)}{v^2} \\
P_r &= \frac{\nu}{\alpha}, \quad S_r = \frac{\nu}{D}, \quad \epsilon = \frac{\beta}{\beta L}, \quad N = \frac{\beta}{\beta L (T_m - T_a)}
\end{align*}
\] ... (6)

in relations (1) - (5), which then reduce to:

\[
\begin{align*}
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} &= 0 \\
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} &= \frac{\partial^2 U}{\partial Y^2} + \theta + NC \\
\frac{\partial \theta}{\partial t} + U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} &= \frac{1}{P_r} \frac{\partial^2 \theta}{\partial Y^2} + \epsilon \left( \frac{\partial U}{\partial Y} \right)^2 \\
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} &= \frac{1}{S_r} \frac{\partial^2 C}{\partial Y^2}
\end{align*}
\] ... (7)

and the initial and boundary conditions are:

\[
\begin{align*}
t &\leq 0, \quad U = V = \theta = C = 0 \quad \text{for all} \quad Y \\
t > 0, \quad U = V = \theta = C = 0 \quad \text{for} \quad X = 0 \\
U &= V = 0, \quad \theta = C = 1 \quad \text{at} \quad Y = 0 \\
U = \theta = C = 0 \quad \text{as} \quad Y \to \infty
\end{align*}
\] ... (8)

All the parameters are defined in the nomenclature.

We now have to solve the coupled non-linear partial differential equations. So, we employ an implicit finite-difference method of the Crank-Nicholson. So, we consider a rectangular region with \( X \) varying from 0 to 1 and \( Y \) varying from 0 to \( Y_{max} = 22 \), where \( X = 1 \) corresponds to the height of the plate and \( Y_{max} \) is regarded as \( \infty \) by assuming that \( Y_{max} \) lies well outside the momentum and thermal boundary layers. We now divide \( X \) and \( Y \)-directions into \( M \) and \( N \) grid-spacing respectively. For economy of computer time, we select variable mesh size in the \( X \) and \( Y \)-directions as:

\[
\begin{align*}
\Delta X &= 0.02 \quad (0 \leq X \leq 0.10) \\
&= 0.06 \quad (0.10 < X \leq 0.40) \\
&= 0.1 \quad (0.40 < X < 1.0) \\
\Delta Y &= 0.10 \quad (0 \leq Y \leq 2.0) \\
&= 0.50 \quad (2.0 < Y \leq Y_{max}) \\
\Delta t &= 0.05
\end{align*}
\]

The finite difference equations of Crank-Nicholson type corresponding to (9) is expressed as:
\[
\begin{align*}
&\frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n}{\Delta t} + \frac{U_{i,j}^n}{2} \left[ \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n - \theta_{i+1,j}^n}{\Delta x_j} \right] \\
&+ \frac{V_{i,j}^n}{2} \left[ \frac{\theta_{i,j}^{n+1} - \theta_{i,j}^n - \theta_{i,j+1}^n}{\Delta y_i} \right] \\
&= \frac{\Delta Y_j^+ \theta_{i,j-1}^n - \Delta Y_j \theta_{i,j}^n + \Delta Y_j \theta_{i,j+1}^n}{\Delta Y_j^+ \Delta Y_j \Delta Y_j^-} \\
&+ \left[ \frac{U_{i,j+1}^n - U_{i,j}^n}{\Delta Y_j^-} \right]^2
\end{align*}
\]

which simplifies to:

\[
A_1 \theta_{i,j+1}^{n+1} + B_1 \theta_{i+1,j}^{n+1} + D_1 \theta_{i,j}^{n+1} = E_1
\]  

(12)

where

\[
A_1 = \frac{-V_{i,j}^n}{2\Delta Y_j} - \frac{1}{P \Delta Y_j \Delta Y_j^-} \\
B_1 = \frac{1}{\Delta t} + \frac{1}{2\Delta X_j} - \frac{1}{P \Delta Y_j \Delta Y_j^-} \\
D_1 = \frac{-V_{i,j}^n}{2\Delta Y_j^-} - \frac{1}{P \Delta Y_j \Delta Y_j^-}
\]

\[
E_1 = \frac{1}{\Delta t} \theta_{i,j+1}^{n+1} + \frac{U_{i,j+1}^n}{2} \left[ \frac{\theta_{i,j+1}^{n+1} - \theta_{i,j}^n + \theta_{i,j-1}^n}{\Delta x_j} \right] \\
- \frac{V_{i,j}^n}{2} \left( \frac{\theta_{i,j+1}^n - \theta_{i,j-1}^n}{\Delta y_i} \right) \\
+ \frac{1}{\Delta t} \left[ \frac{\Delta Y_j^+ \theta_{i,j-1}^n - \Delta Y_j \theta_{i,j}^n + \Delta Y_j \theta_{i,j+1}^n}{\Delta Y_j^+ \Delta Y_j \Delta Y_j^-} \right]
\]

\[
+ \varepsilon \left[ \frac{U_{i,j+1}^n - U_{i,j}^n}{\Delta Y_j^-} \right]^2
\]

Eq. (10) reduces to the form:

\[
A_2 C_{i,j+1}^{n+1} + B_2 C_{i+1,j}^{n+1} + D_2 C_{i,j}^{n+1} = E_2
\]  

(13)

where,

\[
A_2 = \frac{-V_{i,j}^n}{2\Delta Y_j} - \frac{1}{S \Delta Y_j \Delta Y_j^-} \\
B_2 = \frac{1}{\Delta t} + \frac{1}{2\Delta X_j} - \frac{1}{S \Delta Y_j \Delta Y_j^-} \\
D_2 = \frac{-V_{i,j}^n}{2\Delta Y_j^-} - \frac{1}{S \Delta Y_j \Delta Y_j^-}
\]

\[
E_2 = \frac{1}{\Delta t} C_{i,j+1}^{n+1} + \frac{U_{i,j+1}^n}{2} \left[ \frac{C_{i,j+1}^{n+1} - C_{i,j}^n + C_{i,j}^n}{\Delta x_j} \right] \\
- \frac{V_{i,j}^n}{2} \left( \frac{C_{i,j+1}^n - C_{i,j-1}^n}{\Delta y_i} \right) \\
+ \frac{1}{\Delta t} \left[ \frac{\Delta Y_j^+ C_{i,j-1}^{n+1} - \Delta Y_j C_{i,j}^n + \Delta Y_j C_{i,j+1}^{n+1}}{\Delta Y_j^+ \Delta Y_j \Delta Y_j^-} \right]
\]

\[
+ \frac{1}{S} \left[ \frac{\Delta Y_j C_{i,j-1}^{n+1} - \Delta Y_j C_{i,j}^n + \Delta Y_j C_{i,j+1}^{n+1}}{\Delta Y_j^+ \Delta Y_j \Delta Y_j^-} \right]
\]

The finite-difference equation of Crank-Nicholson type corresponding to relation (8) is expressed as:

\[
\frac{C_{i,j}^{n+1} - C_{i,j}^n}{\Delta t} + \frac{C_{i,j+1}^{n+1} - C_{i,j}^{n+1} + C_{i,j}^n - C_{i,j-1}^n}{2 \Delta X_j} \\
+ \frac{C_{i,j+1}^n - C_{i,j-1}^n}{2 \Delta Y_j} \\
= \frac{1}{S} \left[ \frac{\Delta Y_j^+ C_{i,j-1}^{n+1} - \Delta Y_j C_{i,j}^n + \Delta Y_j C_{i,j+1}^{n+1}}{\Delta Y_j^+ \Delta Y_j \Delta Y_j^-} \right]
\]  

(14)
\[ \frac{U_{i,j}^{n+1} - U_{i,j}^n}{\Delta t} + \frac{1}{2} \left[ \frac{U_{i,j}^{n+1} - U_{i+1,j}^n + U_{i,j-1}^n - U_{i-1,j}^n}{\Delta Y_j} \right] \\
+ \frac{V_{i,j}^n}{2} \left[ \frac{U_{i,j}^{n+1} - U_{i,j-1}^n + U_{i+1,j}^n - U_{i-1,j}^n}{\Delta Y_j} \right] \\
+ \frac{1}{2} \left[ \frac{\Delta Y_j^+ U_{i,j}^{n+1} - \Delta Y_j U_{i,j}^n + \Delta Y_j U_{i,j+1}^n + \Delta Y_j U_{i,j-1}^n}{\Delta Y_j^+ \Delta Y_j} \right] \\
+ \frac{1}{2} \left[ \frac{\Delta Y_j^+ U_{i,j}^n - \Delta Y_j U_{i,j}^{n+1} + \Delta Y_j U_{i,j+1}^n - \Delta Y_j U_{i,j-1}^n}{\Delta Y_j^+ \Delta Y_j} \right] \\
+ \frac{1}{2} \left( \theta_{i,j}^{nn} + \theta_{i,j}^n \right) + \frac{N}{2} \left( C_{i,j}^{nn} + C_{i,j}^n \right) \]

which simplifies to:

\[ V_{i,j}^{n+1} = E_4 \]

where,

\[ E_4 = \frac{V_{i,j+1} - V_{i,j+1}}{\Delta Y_j} \]

with the following initial and boundary conditions:

\[ U_{i,j}^0 = 0, U_{i,N}^0 = 0, U_{i,j}^n = 0, U_{i,j}^n = 0 \]

\[ V_{i,j}^0 = 0, V_{i,j}^n = 0, V_{i,j}^n = 0 \]

\[ C_{i,j}^0 = 0, C_{i,j}^n = 1, C_{i,j}^n = 0, C_{i,j}^n = 0 \]

\[ \theta_{i,j}^0 = 0, \theta_{i,j}^n = 1, \theta_{i,j}^n = 0, \theta_{i,j}^n = 0 \]

Here, the subscript \( i \) designates the grid-point with \( X \)-coordinate \( \sum_{k=1} \Delta X_k \cdot j \) designates the grid-point with \( Y \)-coordinate \( \sum_{k=1} \Delta Y_k \), and the subscript \( n \) designates a value of time \( t = n \Delta t \).

Also,

\[ \Delta X_i = X_i - X_{i-1}, \quad i = 1 \to M \]

\[ \Delta Y_j = Y_j - Y_{j-1}, \quad j = 1 \to N - 1 \]

\[ \Delta Y_{i,j} = Y_{i,j} - Y_{i,j-1}, \quad j = 0 \to N - 1 \]

\[ \Delta Y_{i,j} = Y_{i,j} - Y_{i,j+1}, \quad j = 1 \to N - 1 \]
Also, during the computations, the coefficients \(U^{n}_{i,j}, V^{n}_{i,j}\) appearing in Eqs (13) and (15) are treated as constants during a given time-step. The values of \(U, V, \theta\) and \(C\) are known at all grid-points at \(\tau = 0\) from the initial conditions. We then calculate the values of \(U, V, \theta\) and \(C\) at \(\tau = \Delta \tau\) as:

On the line \(i = 1\), Eq. (13) constitutes a tridiagonal system of equations for \(j=1\) to \(N-1\) in \(N-1\) unknowns, which are solved by using the Thomas algorithm described by Carnahan et al.\(^{6}\). We have thus computed the values of \(0, C\) and then \(U\) on the line \(i = 1\). This helps us to compute the values of \(V\) from equation (19). This procedure is repeated for finding the values of \(0, C, U\) and \(V\) on all \(i\)-lines.

In this way, the procedure is repeated for \(n = 2, 3, 4\) until the steady-state is reached. The steady-state is determined when the difference between two consecutive values of \(U\) or \(\theta\) or \(C\) or \(V\) is less than \(10^{-6}\).

We have computed the time required to reach the steady-state for different values of the dissipation parameter \(\varepsilon\) for fluid with Prandtl number \(P_{r} = 0.733\) (air). On Fig. 1, the transient concentration profiles are shown for \(\varepsilon \) = 0.0, 3.0 and \(N = 2.0\) and \(S_{c} = 0.16, 0.78\). We observe from this figure that in the absence of viscous dissipative heat \(\varepsilon = 0\), and for small values of \(S_{c} = 0.16\), the time required to reach steady-state is 4.0 whereas when \(\varepsilon = 3.0\), under similar conditions, the time required to reach steady-state is 4.4 from which we conclude that in the presence of viscous dissipative heat, it needs more time to reach steady-state when \(S_{c}\) is small. However, when \(S_{c} = 0.78\), for high value of the Schmidt number, the steady-state is reached at \(t = 2.1\) when \(\varepsilon = 0.0\) and for \(\varepsilon = 3.0\), the steady-state time is \(t = 2.2\). Hence, we conclude that at high values of \(S_{c}\), the time required to reach steady-state is less as compared to that at low values of \(S_{c}\).

An increase in \(S_{c}\) leads to a fall in the concentration -value. From Fig. 2, we study the effect of the buoyancy force parameter \(N\) on time to reach the steady-state-condition. For \(N = 0.5\), the time required to reach steady-state when \(\varepsilon = 3.0\) is 3.4 whereas for large \(N = 2.0\) it is 2.2 which leads to conclude that as the buoyancy force parameter \(N\) increases, time required to reach steady-state is reduced.

The transient temperature profiles at \(X = 1.0, P_{r} = 0.733\) and \(N = 2\) are shown on Fig. 3 for \(\varepsilon = 0.0, 3.0\), and \(S_{c} = 0.16\) and 0.78. We observe from this figure that time required to reach steady-state temperature is not affected by either \(\varepsilon\) or \(S_{c}\). However, due to viscous

![Fig. 2—Transient concentration profiles \(x=1.0, P_{r}=0.733, S_{c}=0.78\)](image)

![Fig. 3—Transient temperature profiles \(x=1.0, P_{r}=0.733, N=2.0\)](image)
dissipative heat $\varepsilon = 3.0$ the transient temperature increases more at small values of $S_4$ ($= 0.16$) as compared to large values of $S_4$ ($= 0.78$).

From Fig. 4, we conclude that the transient temperature is more when the buoyancy force parameter $N$ is large.

The transient velocity profiles are shown on Fig. 5 at $X = 1.0$, $\varepsilon = 0.0, 0.3, 0.7$ and $S_4 = 0.16, 0.78$. We conclude that the time required to reach steady-state velocity is not significantly affected by an increase in Schmidt number when $\varepsilon = 3.0$. It is $t = 2.2$ when $S_4 = 0.16$ and $t = 2.4$ when $S_4 = 0.78$. However, the transient velocity increases due to more viscous dissipative heat, but decreases due to an increase in the Schmidt number $S_4$.

On Fig. 6, the effect of the buoyancy force parameter $N$ on transient velocity is shown. We conclude from this figure that an increase in $N$ leads to an increase in the transient velocity.

We now study the effect of viscous dissipative heat and mass transfer on the steady-state local skin-friction, which is given by:

$$
\tau_L = \frac{1}{G^2} \left( \frac{\partial U}{\partial Y} \right)_{Y=0}
$$

This is evaluated by using five-point approximate formula for the derivative in (21) and these are shown in Fig. 7. We observe from this figure that in the absence of viscous dissipative heat, local skin-friction decreases with increasing the Schmidt number. Same
is the effect in the presence of viscous dissipative heat. However, due to viscous dissipative heat, the local skin-friction increases with increasing $\varepsilon$. However, an increase in the buoyancy ratio parameter $N$, the local skin-friction increases.

Average skin-friction is given by:

$$\bar{f} = \frac{1}{Gr^{1/4}} \int_0^1 \left( \frac{\partial U}{\partial Y} \right)_{y=0} dX \quad \ldots \quad (22)$$

The integral is approximated by a five-point formula and then the integral is evaluated by using Newton-Cotes closed integration formula and these are shown on Fig. 8. It is observed from this figure that the average skin-friction increases with increasing time $t$ at small values of $t (<1.5)$ and then for $t > 1.5$, it is not significantly affected by time. It increases due to more viscous dissipative heat or owing to an increase in the buoyancy ratio parameter $N$.

From the knowledge of temperature and species concentration field, we study the local and average Nusselt number $\left( \text{Nul} \right)$ and Sherwood number $\left( \text{Shl} \right)$ which is given by:

$$\text{Nul} = -Gr^{1/4} \int_0^1 \left( \frac{\partial \theta}{\partial Y} \right)_{y=0} dX \quad \ldots \quad (23)$$

$$\text{Shl} = -Gr^{1/4} \int_0^1 \left( \frac{\partial C}{\partial Y} \right)_{y=0} dX$$
As in case of $\tau$, we have computed the numerical values of local and average $Nu$ and $Sh$ and these are shown on Figs. 9-12.

From Fig. 9, we observe that in the absence of viscous dissipative heat, the local Nusselt number increases with increasing distance $X$ along the vertical plate and it is not significantly affected when the Schmidt number is increased. Transient velocity increases due to more viscous dissipative heat but decreases with increasing the Schmidt number. Transient velocity increases with increasing the buoyancy ratio parameter $N$.

The effective average Nusselt number is shown in Fig. 10 and we observe that the nature of the behaviour of $Nu$ is same in the absence or presence of the viscous dissipative heat. But $Nu$ decreases due to the presence of viscous dissipative heat and it also increases with an increase in $S_v$.

From Fig. 11, we observe that the local Sherwood number $Sh_L$ increases with increasing the distance $X$ along the vertical plate and it increases more due to more viscous dissipative heat. An increase in the buoyancy ratio parameters $N$ or $S_v$ leads to an increase in $Sh_L$.

On Fig. 12, average Sherwood number $\overline{Sh}$ is shown. It is observed that at small values of time $t<1$, an increase in $t$ leads to a fall in $\overline{Sh}$ but at large values of time $t$, it is not significantly affected by time $t$. Due to more viscous dissipative heat, there is a small increase of $\overline{Sh}$. Also $\overline{Sh}$ increases with increasing the Schmidt number, but decreases with increasing $N$.

**Conclusions**

In the presence of viscous dissipative heat, it takes more time to reach steady-state concentration level. Time to reach steady-state concentration is more as Schmidt numbers decreases. Time required to reach steady-state concentration is less as the buoyancy ratio parameter $N$ increases. Time required to reach steady-state temperature is not affected by viscous dissipative heat or Schmidt number.

In the presence of viscous dissipative heat, the transient temperature increases more at small values of the Schmidt number as compared to large $S_v$. The transient temperature increases due to an increase in buoyancy ratio parameter. In the presence of viscous dissipative heat, time required to reach steady-state velocity is not significantly affected when the Schmidt number is increased. Transient velocity decreases due to more viscous dissipative heat but decreases with increasing the Schmidt number. Transient velocity increases with increasing the buoyancy ratio parameter $N$.

Local skin-friction decreases with increasing the Schmidt number for both $\varepsilon = 0$ or $\varepsilon \neq 0$. Local skin-friction increases with increasing $\varepsilon$ or $N$. Average skin-friction increases with increasing time $t$ when $t<1.5$, but for $t>1.5$, it is not affected by time. Average skin-friction increases with increasing $\varepsilon$ or $N$.

Local Nusselt number $Nu_L$ increases with increasing the distance $X$ when $\varepsilon = 0$, but when $\varepsilon \neq 0$, $Nu_L$ decreases with decreasing $N$. An increase in $S_v$ leads to an increase in $Nu$ but due to the presence of viscous dissipative heat, $Nu$ also decreases.

The local Sherwood number $Sh_L$ increases with increasing $X$ and increases with increasing $\varepsilon$ or $N$ or $S_v$. The average Sherwood number $\overline{Sh}$ decreases with increasing time $t$ when $t<1.0$ and then its remains constant for all $t>1.0$. Viscous dissipative heat does not affect the average Sherwood number significantly. $\overline{Sh}$ increases with increasing $S_v$ but decreases with increasing $N$.

**Nomenclature**

- $c$ - species concentration in fluid
- $c_w$ - species concentration in fluid far away from the plate
- $c_p$ - species concentration at the plate
- $C$ - non-dimensional species concentration
- $\varepsilon$ - specific heat at constant pressure
- $D$ - chemical molecular diffusivity
- $Gr_t$ - Grashof number
- $Gr_c$ - modified Grashof number
- $g$ - acceleration due to gravity
- $\overline{Sh}$ - average Sherwood number
- $\overline{Sh}$ - Sherwood number
- $\overline{Nu}$ - effective average Nusselt number
- $Nu$ - Nusselt number
- $Sh$ - Sherwood number
- $\tau$ - modified Grashof number
- $\alpha$ - acceleration due to gravity
\[ \begin{align*}
K & \quad \text{thermal conductivity} \\
L & \quad \text{length of the plate} \\
Nu_{st} & \quad \text{local Nusselt number} \\
Nu_X & \quad \text{average Nusselt number} \\
N & \quad \text{buoyancy ratio parameter} \\
P_r & \quad \text{Prandtl number} \\
Sh_{st} & \quad \text{local Sherwood number} \\
Sh_X & \quad \text{average Sherwood number} \\
S_f & \quad \text{Schmidt number} \\
T & \quad \text{temperature of the fluid} \\
T_m & \quad \text{temperature of the fluid far away from the plate} \\
T_w & \quad \text{plate temperature} \\
t & \quad \text{time} \\
t^* & \quad \text{non-dimensional time} \\
x, y & \quad \text{velocity components in the } x \text{ and } y \text{-directions} \\
U_x, V_y & \quad \text{non-dimensional velocity components} \\
X, Y & \quad \text{coordinates axes} \\
\beta & \quad \text{coefficient of volume expansion} \\
\beta' & \quad \text{volumetric coefficient of expansion with concentration} \\
\mu & \quad \text{dynamic viscosity} \\
v & \quad \text{Kinematic viscosity} \\
\rho & \quad \text{density} \\
\alpha & \quad \text{thermal diffusivity } K/\rho c_p \\
\tau & \quad \text{local skin-friction} \\
\overline{\tau} & \quad \text{average skin-friction} \\
\tau_s & \quad \text{dissipation parameter} \\
\theta & \quad \text{non-dimensional temperature}
\end{align*} \]

References