GPS signal Rician fading model for precise navigation in urban environment

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Global Positioning System (GPS) usage is not limited to the aircraft en-route navigation and missile guidance where the user receives the satellite signals from the open sky. Currently, GPS has become an essential utility in the car navigation, mobile phones, surveying and aircraft landing applications. The received signal strength of a GPS satellite at a given location on or near the earth surface can be predicted by analyzing the propagation characteristics of the channel with an appropriate propagation model. The signal propagation characteristics particularly the short term variations severely affect the quality, availability and continuity of the system. The short term propagation characteristics of GPS signal is modeled and analyzed in this paper. Short term variations are mainly due to multipath reflections and Doppler shift which degrades the quality of received signal particularly in urban environments. The variation of signal quality with respect to user velocity is observed using Rician fading model.

Keywords: GPS signal, Precise navigation, Multipath effect, Rician fading model

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1 Introduction

Global Positioning System (GPS) is an all-weather navigation and timing system. It is a space based navigation system that provides three dimension position, velocity and time by measuring the distance from the user location to the precise locations of the GPS satellites\(^1\). It is primarily designed as a land, marine, and aviation sector navigation system. Each GPS satellite transmits two spread spectrum pseudo random noise (PRN) ranging codes along with 50 bps navigation data message at two frequencies L1 (1575.42 MHz) and L2 (1227.60 MHz), which are derived from highly stable on-board atomic clocks. The GPS receiver calculates its position by precisely timing the signals sent by satellites above the earth’s surface.

The accuracy of the computed position depends on the received signal strength, which may be degraded due to several reasons, such as travelling long distances through vacuum, dense clouds, dust particles; different layers of the earth’s atmosphere such as troposphere, ionosphere; protonosphere natural elements such as mountains; and even man-made flying objects like airplanes, etc. In addition, the random fluctuations in the received signals due to different fading phenomena also affect the signal quality, system availability and are ultimately a major cause of system outages. In general, GPS signal variations can be classified into two types: large scale variations and short term variations. The large scale variations in a signal are mainly due to path loss and shadowing. The average value of the signal strength at any point depends on its distance, carrier frequency, type of antennas used, atmospheric conditions and so on. It may also vary because of shadowing caused by terrain and clutter such as hills, buildings, and other obstacles. This type of signal variation, which is observable over relatively long distances, has a log normal distribution. The second type of variation is due to multipath reflections. In urban or dense urban areas, there may not be any direct line-of-sight path between a satellite and a receiver antenna\(^2\). Instead, the signal may arrive at a GPS receiver over a number of different paths after being reflected from tall buildings, towers, and so on. Because the signal received over each path has a random amplitude and phase, the instantaneous value of the composite signal is found to vary randomly about a local mean\(^3\).
In this paper, the short term fluctuations in the GPS signal amplitude due to phase change is analyzed using Rician fading model. Multipath error is one of the predominant error sources in all GPS applications. Particularly, the multipath error has to be precisely estimated in the Global Navigation Satellite Systems (GNSSs) as it is the major error source (2-4 m) that limits the GPS receiver’s performance. Whenever, a signal is transmitted from a GPS satellite it follows a multiple number of propagation paths on its way to receiving antenna. These multiple signal paths are due to the fact that the signal gets reflected back to the antenna off surrounding objects, including the earth’s surface.

The GPS receiver tracks both the direct and reflected signal components. The radio wave transmitted from a satellite radiates in all directions. These radio waves include reflected waves that are reflected off due to various obstacles, diffracted waves, scattering waves, and the direct wave from the satellite to GPS receiver (Fig. 1).

In this case, since the path lengths of the direct, reflected, diffracted, and scattering waves are different, the time each takes to reach the GPS receiver will be different. In addition, the phase of the incoming wave varies because of reflections. As a result, the receiver receives a superposition consisting of several waves having different phases and times of arrival. The generic name of a radio wave in which the time of arrival is retarded in comparison with this direct wave is called a delayed wave. Then, the reception environment characterized by a superposition of delayed waves is called a multipath propagation environment. In a multipath propagation environment, the received signal is sometimes intensified. This phenomenon is called multipath fading and the signal level of the received wave changes from moment to moment.

3 Fading
Fading is a common phenomenon in GPS caused by the superposition of two or more versions of the transmitted signals, which arrive at the receiver at slightly different times. The resultant received signal can vary widely in amplitude and phase, depending on various factors such as the relative propagation time of the waves and bandwidth of the transmitted signal. At the receiver, these multipath waves with randomly distributed amplitudes and phases combine to give a resultant signal that fluctuates in time and space. Therefore, a receiver at one location may have a signal that is much different from the signal at another location, only a short distance away because of the change in the phase relationship among the incoming radio waves. This causes significant fluctuations in the signal amplitude. This phenomenon of random fluctuations in the received signal level is termed as fading. The short term fluctuation in the signal amplitude caused by the local multipath is called Rayleigh fading, and is observed over distances of about half a wavelength. The most commonly known statistical representations of fading are: Rayleigh, Rician. The received signal envelope in a typical multipath environment is shown in Fig. 2.

4 Rician fading model
It is a stochastic model for radio propagation anomaly caused by partial cancellation of radio signal by itself. If the environment is such that in addition to scattering, there is a strong dominant signal seen at the receiver caused by line-of-sight, the mean of the
random process is no longer zero and varies around the power level of the dominant path. Such a situation may be better modelled as Rician fading. Rayleigh fading is a specialized model for Rician fading when there is no line-of-sight (LOS) signal. In a microcellular environment, the transmitter antennas are often placed below the skyline of buildings and are surrounded by local scatterers, such that the plane waves arrive at the base station with a larger angle of arrival (AOA) spread. Furthermore, LOS path sometimes exist between the satellite and receiver, while at other times LOS path does not exist. Even in the absence of LOS propagation conditions, there often exist a dominant reflected or diffracted path between the satellite and receiver.

LOS or dominant reflected or diffracted path produces the specular component and the multitude of weaker secondary paths contributes to the scatter component of the received envelope. In this type of propagation environment, the received signal envelope still experiences fading. However, the presence of the specular component changes the received envelope distribution, and very often a Rician distributed envelope is assumed. In this case, the received envelope is said to exhibit Rician fading. Such GPS signals are modelled by Rician fading model. Rician distribution is used to model the signal when a direct line-of-sight component exists between the obstacle and receiver in addition to the multipath components. It can be expressed as a phasor sum of a constant and a number of scattering point sources:

\[ R = re^{j\theta} = C + \sum_{j=1}^{n} A_j e^{j\phi_j} \quad \cdots(1) \]

where, \( C \), is constant coherent signal with clear LOS; \( A_j \), amplitude of \( j^{th} \) incoming wave; and \( \phi_j \), initial phase shift of \( j^{th} \) incoming wave. Rician probability density function is given by:

\[ p(r) = \frac{2r}{\beta} e^{-\frac{r^2+\beta^2}{2\beta}} I_0\left(\frac{2r\beta}{\beta}\right) \quad r \geq 0 \quad \cdots(2) \]

\[ p(r) = 0 \quad r < 0 \quad \cdots(3) \]

where, \( \beta \) is the mean square value of the Rayleigh distributed component of \( r \); and \( I_0 \), the modified Bessel function of order zero. The phase distribution is no longer uniform like Rayleigh distribution. The phase distribution of Rician distribution is derived by Beckmann as:

\[ p(\theta) = \frac{1}{2\pi} e^{-\frac{C^2}{\beta}} \left[ 1 + G\sqrt{\pi} e^{G^2} \left( 1 + \text{erf}(G) \right) \right] \quad \cdots(4) \]

where,

\[ G = \frac{C \cos \theta}{\sqrt{\alpha}}, \quad 0 \leq \theta \leq 2\pi \]

and

\[ \text{erf}(G) = \frac{2}{\sqrt{\pi}} \int_0^G e^{-y^2} dy \]

Clarke’s simulation model for simulating Rician fading is given by the signal transfer function which is given by:

\[ Y(t) = Y_c(t) + jY_s(t) \]

\[ Y_c(t) = \frac{1}{\sqrt{N_s}} \sum_{n=1}^{N_s} \cos \left( \omega_d t \cos \alpha_n + \phi_n \right) \quad \cdots(5) \]

\[ Y_s(t) = \frac{1}{\sqrt{N_s}} \sum_{n=1}^{N_s} \sin \left( \omega_d t \cos \alpha_n + \phi_n \right) \quad \cdots(6) \]

where, \( N_s \), is the number of propagation paths; \( \omega_d \), the maximum Doppler frequency in radians; \( \alpha_n \) and \( \phi_n \), the angle of arrival and initial phase of the \( n^{th} \) propagation path, respectively. Both \( \alpha_n \) and \( \phi_n \) are uniformly distributed over \((-\pi, \pi)\) for all \( n \) and they are mutually independent. The signal transfer function after including the direct LOS component is given by:

\[ Z(t) = Z_c(t) + jZ_s(t) \]

\[ Z_c(t) = \left( Y_c(t) + \sqrt{K} \cos \left( \omega_d t \cos \theta_0 + \phi_0 \right) \right) / \sqrt{1+K} \quad \cdots(7) \]

\[ Z_s(t) = \left( Y_s(t) + \sqrt{K} \sin \left( \omega_d t \cos \theta_0 + \phi_0 \right) \right) / \sqrt{1+K} \quad \cdots(8) \]

where, \( K \), is the ratio of multipath to direct component; \( \theta_0 \) and \( \phi_0 \), angle of incidence and initial phase of direct component, respectively.
5 Results and Discussion

The response of Rician fading of GPS transmitted signal for different velocities of receiver is obtained, in the present paper, using sum of sinusoids method. The response is obtained by considering the carrier frequency, $f_c = 1575.42$ MHz; number of scatterers, $N_s = 8$, $K = 3$, 6 and by changing the value of Doppler frequency ($\omega_d$). The value of $N_s$ is determined from the environment. The response of Rician fading of GPS transmitted signal for a receiver moving velocity of 50 km h$^{-1}$ and $K = 3$ and $K = 6$ are shown in Figs 3 and 4. From these figures, it is observed that the normalized amplitude of GPS signal changes with respect to time when a receiver is moving with a certain velocity. From Figs 3 and 4, it is also observed that as the value of $K$ is increased, the normalized amplitude of response is reduced. The response of Rician fading of GPS transmitted signal for a receiver moving velocity of 100 km h$^{-1}$ and $K = 3$ and $K = 6$ are shown in the Figs 5 and 6. From Figs 5 and 6, it is observed that the normalized amplitude of GPS signal is changing with respect to time when a receiver is moving with velocity of 100 km h$^{-1}$. It is also observed that as the value of $K$ is increased, the normalized amplitude of response is reduced.

From Figs (3 to 6), it is concluded that for a constant value of $K$, the mean value of the response decreases with the increase in velocity of the user; and for a constant velocity, as the value of $K$ is increased, the mean value is reduced.
For constant $K$, the level crossing rate of the receiver moving with a velocity of 100 km h$^{-1}$ is more as compared with the level crossing rate of the receiver moving with a velocity of 50 km h$^{-1}$ at a fade level of 1. The average fade duration of a receiver with a velocity 100 km h$^{-1}$ is more as compared with the average fade duration of a receiver moving with a velocity of 50 km h$^{-1}$ at a fade level of 1. The values of minimum amplitude, maximum amplitude, mean and standard deviations of GPS transmitted signal with receiver velocities of 50 and 100 km h$^{-1}$ are also calculated.

6 Conclusions

In this paper, modeling of short term variations is presented for the GPS transmitted signal. Short term variations of the GPS signal using Rician fading model is analysed for different configurations, e.g. receiver with velocities 50 and 100 km h$^{-1}$ for $K = 3$ and 6. From the analysis, it is concluded that the level crossing rate and the average fade duration of the receiver’s response due to Rician fading is found higher at a fade level of 1 for a receiver moving with a velocity of 100 km h$^{-1}$ in comparison to the receiver moving with a velocity of 50 km h$^{-1}$.

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References