Validation of GPS receiver instrumental bias results for precise navigation

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The positional accuracy of Global Positioning System (GPS) is affected by several errors, the most predominant error being the ionospheric delay. This delay is proportional to the total electron content (TEC). The dual frequency GPS observables can be used to estimate the TEC. The line-of-sight TEC estimated from dual frequency GPS data is corrupted by the instrumental biases of the GPS satellites and the receiver. The instrumental biases exist as the signals at the two GPS frequencies ($f_1=1575.42$ MHz; $f_2=1227.60$ MHz) experience different delays within the GPS satellite and receiver hardware. The estimation of the receiver instrumental bias plays a significant role in achieving required navigation accuracy for civil aviation applications. In this paper, receiver instrumental bias results due to a modified fitted receiver bias method, Kalman filter and singular value decomposition (SVD) algorithms are compared.

Keywords: Global Positioning System (GPS) accuracy, Ionospheric delay, GPS receiver instrumental bias, Fitted receiver bias, Kalman filter, Singular value decomposition (SVD) algorithm

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1 Introduction

Global Navigation Satellite System (GNSS) is a collective term for those navigation systems that provide the user with a three dimensional positioning solution by passive ranging using radio signals transmitted by orbiting satellites. The Global Positioning System (GPS) is the most well known out of all the constellations. The positional accuracy of GPS is affected by several errors such as ionospheric delay, tropospheric delay, satellite and receiver clock offsets, instrumental biases of the satellite and receiver, receiver measurement noise and multipath. The stand alone GPS does not meet the positional accuracy required for Category I Precision Approach and landing phase of an aircraft. To use GPS for all phases of the flight, satellite based augmentation systems (SBAS) have been planned by various countries including USA, Europe, Japan and India. The Indian SBAS is under development stage and is named as GPS Aided Geo Augmented Navigation (GAGAN). The GAGAN network consists of several dual frequency GPS receivers of NovAtel make, located at various airports around the Indian subcontinent.

The ionospheric delay, which is a function of the total electron content (TEC), is one of the main sources of error in GPS precise positioning and navigation. The dual frequency GPS receiver can be used to estimate the TEC, taking advantage of the dispersive nature of ionosphere. However, line-of-sight TEC derived from dual frequency GPS data is corrupted by the instrumental biases of the GPS satellites and the receiver. The instrumental biases occur due to the frequency dependent delays of analog hardware within the GPS satellite and receiver. The estimation of the receiver instrumental biases is particularly important in achieving the Category-I Precision Approach (CAT-I PA) requirements of civil aviation. Several approaches based on the least squares fitting, Kalman filter, Self Calibration of pseudoRange Error (SCORE) algorithm and neural networks are reported in literature for estimation of TEC and instrumental biases. Most of these methods have been applied to data from mid latitude regions. Literature survey suggests that no significant work has been reported on the estimation of TEC and instrumental biases using Kalman filter technique in low latitude regions especially in India.

2 GPS data processing

The dual frequency GPS data provide both code and carrier phase measurements on the two GPS
frequencies and navigation data parameters due to all the visible satellites. In the estimation of TEC and instrumental biases, several data processing steps including extraction of desired data, smoothing, computation of satellite position, receiver position, slant factor, ionospheric pierce point (IPP) coordinates and mean longitude of Sun are to be carried out. These data processing steps are implemented as modules using the MATLAB software. A main program is developed for calling the different modules. The functions performed by various modules are broadly classified into seven categories as:

a. Extraction of satellite ephemeris and time parameters;
b. Extraction of pseudorange and carrier phase data;
c. Computation of smoothed ionospheric delay;
d. Satellite position estimation;
e. Computation of elevation angle, slant factor, IPP coordinates and mean longitude of Sun;
f. Selection of satellites based on lowest position dilution of precision (PDOP) value for receiver position estimation; and
g. Receiver position estimation

The estimation of these parameters is briefly discussed in this paper. The dual frequency GPS data in Receiver INdependent EXchange (RINEX) format is used in the estimation.

2.1 Satellite position estimation

The ephemeris parameters listed in Table 1 are used for satellite position estimation. Important steps of the algorithm for satellite position estimation are:

Table 1 — Ephemeris parameters used in satellite position estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean anomaly at reference time, ( t_{oe} ) ((M_0))</td>
<td>rad</td>
</tr>
<tr>
<td>Mean motion difference from computed value ((\Delta n))</td>
<td>rad s(^{-1})</td>
</tr>
<tr>
<td>Eccentricity ((e))</td>
<td></td>
</tr>
<tr>
<td>Square root of semi-major axis ((\sqrt{a}))</td>
<td>m(^{1/2})</td>
</tr>
<tr>
<td>Longitude of ascending node of orbit plane at weekly epoch ((\Omega_o))</td>
<td>rad</td>
</tr>
<tr>
<td>Inclination angle at reference time ((i_o))</td>
<td>rad</td>
</tr>
<tr>
<td>Argument of perigee at reference time ((\omega))</td>
<td>rad</td>
</tr>
<tr>
<td>Rate of change of longitude of ascending node ((\dot{\Omega}))</td>
<td>rad s(^{-1})</td>
</tr>
<tr>
<td>Rate of change of inclination angle ((d(i)/dt))</td>
<td>rad s(^{-1})</td>
</tr>
<tr>
<td>Amplitude of cosine correction to argument of latitude ((C_{uc}))</td>
<td>rad</td>
</tr>
<tr>
<td>Amplitude of sine correction to argument of latitude ((C_{us}))</td>
<td>rad</td>
</tr>
<tr>
<td>Amplitude of cosine correction to the orbital radius ((C_{rc}))</td>
<td>m</td>
</tr>
<tr>
<td>Amplitude of sine correction to the orbital radius ((C_{rs}))</td>
<td>m</td>
</tr>
<tr>
<td>Amplitude of cosine correction to the inclination angle ((C_{ic}))</td>
<td>rad</td>
</tr>
<tr>
<td>Amplitude of sine correction to the inclination angle ((C_{is}))</td>
<td>rad</td>
</tr>
<tr>
<td>Reference time of ephemeris ((t_{oe}))</td>
<td>seconds of GPS week</td>
</tr>
</tbody>
</table>

 Argument of latitude correction, \( \delta \phi_k = C_{uc} \sin(2\phi_k) + C_{us} \cos(2\phi_k) \) \( \ldots(9) \)

 Radius correction, \( \delta r_k = C_{rc} \sin(2\phi_k) + C_{rs} \cos(2\phi_k) \) \( \ldots(10) \)

 Inclination correction, \( \delta i_k = C_{ic} \sin(2\phi_k) + C_{is} \cos(2\phi_k) \) \( \ldots(11) \)

 Corrected argument of latitude, \( u_k = \phi_k + \delta \phi_k \) \( \ldots(12) \)

 Corrected radius, \( r_k = a(1 - e \cos E_k) + \delta r_k \) \( \ldots(13) \)

 Corrected inclination, \( i_k = i_0 + (d(i)/dt) t + \delta i_k \) \( \ldots(14) \)

 Corrected longitude of node, \( \Omega_k = \Omega_o + (\dot{\Omega} - \Omega_o) t + \dot{\Omega}_o t_o \) \( \ldots(15) \)

 In-plane x-position, \( x_p = r_k \cos u_k \) \( \ldots(16) \)

 In-plane y-position, \( y_p = r_k \sin u_k \) \( \ldots(17) \)

 ECEF x-coordinate, \( x = x_p \cos \Omega_k - y_p \cos i_k \sin \Omega_k \) \( \ldots(18) \)

 ECEF y-coordinate, \( y = x_p \sin \Omega_k + y_p \cos i_k \cos \Omega_k \) \( \ldots(19) \)

 ECEF z-coordinate, \( z = y_p \sin i_k \) \( \ldots(20) \)

Semi-major axis, \( a = \left(\sqrt{a}\right)^2 \) \( \ldots(1) \)

Corrected mean motion, \( n = \sqrt{\frac{\mu}{a^3}} + \Delta n \) \( \ldots(2) \)

Time from ephemeris motion, \( t_k = t - t_{oe} \) \( \ldots(3) \)

Mean anomaly, \( M_k = M_0 + n(t_k) \) \( \ldots(4) \)

Eccentric anomaly (must be solved iteratively for \( E_k \)), \( M_k = E_k - e \sin E_k \) \( \ldots(5) \)

True anomaly, \( \sin v_k = \frac{\sqrt{1 - e^2} \sin E_k}{1 - e \cos E_k} \) \( \ldots(6) \)

\[ \cos v_k = \frac{\cos E_k - e}{1 - e \cos E_k} \] \( \ldots(7) \)

Argument of latitude, \( \phi_k = v_k + \omega \) \( \ldots(8) \)
The following constants are used in the calculations:

Earth’s universal gravitational constant, \( \mu = 3.98600 \times 10^{14} \text{ m}^3 \text{ s}^{-2} \),

WGS-84 value of the earth’s rotation, \( \Omega_e = 7.2921151467 \times 10^{-5} \text{ rad s}^{-1} \)

The ephemeris parameters of all the visible satellites are extracted from the RINEX navigation data and stored in a matrix \( \text{eph} \) for estimation of satellite position. A program \text{satposition.m} computes the position of all the visible satellites at each epoch over the entire observation period and stores for later use. The example estimation for the estimated satellite position is given in Table 2.

### 2.2 Computation of elevation and azimuth angle

For computation of elevation and azimuth angle, receiver position in Earth Centered Earth Fixed (ECEF) coordinates is extracted from the header of RINEX observation data and is converted to geodetic coordinates \((\lambda, \phi, z)\). The satellite position coordinates \((x_s, y_s, z_s)\) from ECEF coordinate frame is transformed to a local coordinate frame defined at the user position. Such a coordinate frame is called a local coordinate or East North Up (ENU) system. These ENU coordinates are used in the calculation of elevation angle and azimuth angle. Elevation \(E\) and azimuth \(A\) angles of the satellite are given by:

\[
E = \arctan \left( \frac{x_U}{\sqrt{x_N^2 + x_E^2}} \right) \quad \text{...(21)}
\]

\[
A = \arctan \left( \frac{x_E}{x_N} \right) \quad \text{...(22)}
\]

### 2.3 Computation of ionospheric pierce point coordinates

The line-of-sight signal traveling from the GPS satellite to the user receiver intersects the ionosphere surface at a point called ionospheric pierce point (IPP). The location of the IPP on the thin shell is shown in Fig. 1 (Ref. 11).

If the geographic latitude and longitude of the GPS receiver \((\lambda_U, \phi_U)\) are known, the geographic latitude and longitude of an IPP can be computed according to the observed azimuth and elevation angle to the tracked satellite. The latitude of a pierce point \(\phi_{IPP}\) is computed as:

\[
\phi_{IPP} = \sin^{-1} \left( \sin \psi_{pp} \cos \phi_U + \cos \phi_U \sin \psi_{pp} \cos A \right) \quad \text{...(23)}
\]

where, \(\psi_{pp}\), is the earth centered angle given by:

\[
\psi_{pp} = \frac{\pi}{2} - E - \sin^{-1} \left( \frac{R_E}{R_E + h_{IPP}} \cos E \right) \quad \text{...(24)}
\]

and \(E\), the satellite elevation angle (radians); \(A\), the azimuth angle of the satellite at the user’s location (radians); \(R_E\), the mean radius of earth (m); and \(h_{IPP}\), the mean height of the ionosphere chosen as 350 km.

The longitude of the pierce point \(\lambda_{IPP}\) is given as:

\[
\lambda_{IPP} = \lambda_U + \sin^{-1} \left( \frac{\sin \psi_{pp} \sin A}{\cos \phi_{IPP}} \right) \quad \text{...(25)}
\]

The geomagnetic coordinate system with its dipole axis intersecting the geographic sphere, or Boreal pole \((\phi_p, \lambda_p)\), at approximately 78.7°N latitude and 290.1°E longitude, is used to compute the geomagnetic latitude, \(\phi_M\), of the IPP.

\[
\sin \phi_M = \sin \phi_{IPP} \sin \phi_p + \cos \phi_{IPP} \cos \phi_p \cos (\lambda_{IPP} - \lambda_p) \quad \text{...(26)}
\]

The geomagnetic longitude \(\lambda_{GP}\) is:

\[
\lambda_{GP} = UT + \lambda_{IPP} - \pi \quad \text{...(27)}
\]

where, \(UT\), is the universal time.
2.4 Computation of mean longitude of Sun

The mean longitude of Sun ($\lambda_m$) is computed using:

$$\lambda_m = M_k + \mu_k + \Omega_k$$

...(28)

where, $M_k, \mu_k,$ and $\Omega_k$, are as defined in Eqs (4), (12), and (15), respectively.

2.5 Computation of slant factor

Slant factor converts the measured slant ionospheric delay at the user location to vertical delay at an IPP on the thin shell and vice versa. The slant factor ($SF$) is given by:

$$SF = \left(1 - \frac{R_E \cos(E)}{R_E + h_{IPP}}\right)^{1/2}$$

...(29)

where, $R_E, E,$ and $h_{IPP},$ are as defined earlier.

A program (slantfactor.m) is developed that calculates elevation angle, azimuth angle, slant factor, IPP latitude and longitude, mean longitude of Sun using the satellite position and receiver position. The sample output estimation due to this module is shown in Table 3.

2.6 Receiver position estimation

A program lowdopfn.m is used to choose four satellites from among all visible satellites, at each epoch, based on minimum position dilution of precision (PDOP) value. This module uses the satellites position information obtained from the satellite position calculation module, and receiver position extracted from the header of the RINEX observation data for calculation of PDOP. The position information of selected four satellites having minimum PDOP value is used for the calculation of receiver position using the Bancroft algorithm in the program receiverpos.m.

3 Instrumental bias estimation

An algorithm based on the Kalman filter is developed for estimation of TEC and instrumental biases using single station data. The slant TEC obtained from the dual frequency code observables is unambiguous, but is affected by the measurement noise and multipath errors. In order to reduce these errors, a code smoothing technique based on the Hatch filter is used. The biased phase smoothed slant TEC obtained from the Hatch filter is still corrupted by the instrumental biases of the GPS receiver and satellites. A five state Kalman filter is used to remove the instrumental biases from the TEC. The measurement and system model of the Kalman filter are formulated. The use of Kalman filter requires a prior estimate of the bias values. The satellite biases are already estimated by the Centre for Orbit Determination (CODE), Europe and are available in public domain. In order to obtain an initial estimate of the receiver bias, a modified fitted receiver bias (FRB) method is used. The computed biased phase smoothed slant TEC obtained using Hatch filter, slant factor, geomagnetic latitude and longitude of IPP, geomagnetic latitude of receiver, and the mean longitude of Sun form the inputs to the Kalman filter. The estimated states include the vertical TEC and the differential instrumental biases.

The International GNSS Service (IGS) network provides GPS data from over 200 agencies worldwide. One such agency is the National Geophysical Research Institute (NGRI) located at Hyderabad. The receiver at NGRI (17.41°N, 78.55°E) is a dual frequency GPS receiver of Ashtech make. The receiver bias value for IGS stations estimated by CODE is available in public domain. Normally, the fitted receiver bias (FRB) method is used to obtain a rough estimate of the receiver bias using 24 hours of GPS data. It is based on the minimization of vertical TEC derived from various satellites. To study the suitability of FRB method to equatorial and low latitude regions such as India, GPS data of select number of days (both quiet and disturbed days) corresponding to the Hyderabad IGS station is analysed for estimation of receiver bias. Three different time periods are considered for estimation of receiver bias using FRB method to identify the period that best fits the CODE derived value. It is proposed to use this best fit FRB estimate as an initial state value in the Kalman filter for further improving the prediction.

<table>
<thead>
<tr>
<th>PRN</th>
<th>Year</th>
<th>Month</th>
<th>Day</th>
<th>Hours</th>
<th>Min</th>
<th>Elevation, deg</th>
<th>Azimuth, deg</th>
<th>Geomagnetic coordinates of IPP</th>
<th>Slant factor</th>
<th>Mean longitude of Sun, deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>30</td>
<td>41.80</td>
<td>49.14</td>
<td>9.59</td>
<td>31.43</td>
<td>1.413</td>
</tr>
</tbody>
</table>
Table 4 — Estimated receiver instrumental bias using FRB method (Hyderabad IGS station)

<table>
<thead>
<tr>
<th>Days of the year 2005</th>
<th>$K_p$ index</th>
<th>Receiver bias provided by CODE, ns</th>
<th>Estimated receiver bias using FRB method in different time durations, ns</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 - 10</td>
<td>0-2</td>
<td>0.44</td>
<td>9.4 1.4 15.9</td>
</tr>
<tr>
<td>17 - 18</td>
<td>5-8</td>
<td>0.378</td>
<td>5.3 1.5 11.9</td>
</tr>
<tr>
<td>26 - 27</td>
<td>0-1</td>
<td>-1.232</td>
<td>12.9 0.4 22.6</td>
</tr>
<tr>
<td>35 - 36</td>
<td>0-1</td>
<td>-1.109</td>
<td>6.6 0.4 15.9</td>
</tr>
<tr>
<td>38 - 39</td>
<td>3-6</td>
<td>-1.937</td>
<td>4.7 -1.6 17.8</td>
</tr>
<tr>
<td>62 - 63</td>
<td>0-2</td>
<td>-0.931</td>
<td>7.4 0.2 11.2</td>
</tr>
<tr>
<td>81 - 82</td>
<td>0-2</td>
<td>0.439</td>
<td>7.6 1.2 14.9</td>
</tr>
</tbody>
</table>

4 Results due to fitted receiver bias method

Table 4 summarises the results of estimated receiver instrumental bias in different time periods for various days obtained using the FRB method for the Hyderabad IGS station. For each data set, two consecutive days of data is taken, and the required GPS observables are extracted in the time period 20:00 – 20:00 hrs LT (24 h) for further processing. The receiver bias is estimated using data of three different time periods, viz. 20:00 – 19:59, 20:00 – 08:00, and 08:00 – 20:00 hrs LT. Comparing these values with the value provided by Centre for Orbit Determination (CODE), Europe, it is found that the fitted receiver bias estimated using nighttime data (20:00 – 08:00 hrs LT) is in close proximity to CODE estimated value. Many studies have been made of the behaviour of TEC. These studies show that the TEC has high and highly variable values in the daytime with larger latitudinal gradients. In the nighttime, TEC values are much lower with smaller variations. In the Klobuchar time delay model, a constant value of 5 ns is chosen for the ionospheric delay during the nighttime (~22:00 – ~06:00 hrs LT)\(^\text{12}\). In view of this, three different time periods are studied for the estimation of receiver bias using FRB method to identify the period which gives the best fit.

5 Validation of receiver instrumental bias

For validation of the estimated receiver instrumental bias, one month GPS data (1-31 July 2004) of Hyderabad GAGAN station (17.45°N, 78.47°E) is considered. The receiver bias is estimated for each day using the FRB methods. Kalman filter approach and the results are compared with that reported for the SVD algorithm\(^8\). Results due to these three methods are compared in Table 5. DIB\(_{\text{SFRB}}\) is the receiver bias estimate obtained from the standard FRB method. One complete day data (00:00 – 24:00 hrs LT) is used for receiver bias estimation in this method. DIB\(_{\text{MFRB}}\) represents the estimate obtained using modified FRB method where nighttime data (20:00 – 08:00 hrs LT) is used for estimation of receiver bias. DIB\(_{\text{SVD}}\) is the receiver bias estimated using SVD algorithm. It is observed from the estimates that the receiver bias obtained using nighttime data (DIB\(_{\text{MFRB}}\)) is relatively much closer to the bias value obtained using SVD algorithm, as compared to the value obtained using complete day data (DIB\(_{\text{SFRB}}\)). The receiver bias computed using Kalman filter is also tabulated, which shows further improvement in the estimated receiver bias.

The mean value and standard deviation of the receiver bias over one month period is tabulated in Table 6. It is found that the day-to-day variation of the estimated receiver biases with respect to the monthly mean is small. The estimated monthly mean receiver bias of Hyderabad station obtained using Kalman filter is -4.15 ns and that obtained using SVD algorithm is -3.94 ns. These values are compared with the hardware calibrated value -4.07 ns (-11.6 TECU). The difference in receiver bias estimated using Kalman filter and the hardware calibrated value is only 0.08 ns (Ref. 20). This study indicates that the Kalman filter technique is giving better results than the SVD algorithm.

6 Conclusions

Precise estimation of TEC is necessary for accurate position fixing of user. For achieving this, a five-state Kalman filter is implemented. A fitted receiver bias method is used for estimating the initial state of the receiver bias. However, only nighttime data is considered in the estimation of receiver bias using FRB method as TEC variations are relatively small and constant during nighttime. The GPS data of Hyderabad IGS and GAGAN stations are considered for estimation of receiver instrumental bias and the bias results are compared with SVD algorithm and
hardware calibrated value reported in literature. It is observed that the mean value of receiver bias of Hyderabad GAGAN station obtained using Kalman filter is -4.15 ns and that using SVD algorithm is -3.94 ns. The hardware calibrated value reported in literature is -4.07 ns. The difference in receiver bias between Kalman filter and SVD approach is found to be 0.21 ns and that between Kalman filter and hardware calibrated value is 0.08 ns. It is observed that the day-to-day variability of receiver bias with respect to the monthly mean is small for the period considered. The proposed technique proved to be very promising and can be applied easily to many other stations.

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