Tropospheric error correction in assisted GPS signals

Mohammed Mushthaq Ahmed¹,², Quddusa Sultana¹,₃, A Supraja Reddy² & M A Malik³

¹Department of Electronics and Communication Engineering, Deccan College of Engineering and Technology, Hyderabad 500 001, AP, India  
²Research and Training Unit for Navigational Electronics (NERTU), Osmania University, Hyderabad 500 007, AP, India  
³Deccan Group of Institutions, Hyderabad 500 001, AP, India  
E-mail: ¹mushthaq.md@gmail.com; ³quddusas@gmail.com

Received 7 November 2012; revised 19 March 2013; accepted 23 March 2013

Though standalone cell phone technology is sufficient to navigate, presently, in the developed and the under developed countries of the world, almost every cell phone has a Global Positioning System (GPS) receiver embedded in it. This combined system is called as assisted GPS. There are various issues to be dealt critically while using such systems. For instance, cellular networks and GPS signals encounter various types of errors in which tropospheric error is one. This paper focuses on the estimation of tropospheric error and its correction in cell phone signals as well as GPS signals. In order to estimate the tropospheric delay, four different tropospheric error correction models are implemented. Two models, namely Hopfield model and European Geo-stationary Navigation Overlay Service (EGNOS) model are implemented for GPS signals and other two models, namely Radio Technical Commission for Aeronautics (RTCA) and Locata Tropospheric (LTC) model are implemented for cellular signals. The results indicate that in case of GPS signals, as the altitude increases, tropospheric delay (TD) gradually decreases. The maximum TD is observed at the surface of earth for Hopfield model (13 m) as well as for EGNOS model (13.2 m). In case of cellular signals, TD increases as the altitude increases since the base stations are on the ground. At an altitude of 14 km, TD is observed for RTCA model as 9.08 m and for LTC model, it is 11.41 m.

Keywords: Tropospheric error correction, Tropospheric delay, Assisted GPS signal

PACS No: 92.60.hf; 91.10.Fc

1 Introduction

There is an increasing interest towards location based mobile services. Currently, global system mobile (GSM) handsets with integrated GPS navigator are available on mass level in the world market. Mobile phones can estimate their position using time of arrival (TOA), angle of arrival (AOA), time difference of arrival (TDOA) techniques. However, these are not accurate. Hence, cell phone can be integrated with GPS receiver to improve accuracy. The GPS system was established by the United States Department of Defense (DOD) to provide real time navigation for the US military. Civil use was the secondary objective of GPS and civil users were limited in 1990 but presently, the GPS users grew at an amazing rate due to its vast applications.

There are various types of errors that affect the cell phone signals as well as the GPS signals. Some of the major errors in cellular communications are signal fading, co-channel interference, multipath and tropospheric error. The prominent errors in GPS are ionospheric error, tropospheric error, multipath and clock bias. In this paper, tropospheric error is focused. Investigations are performed on the estimation and correction of tropospheric time delay error in case of cell phone signals as well as GPS signals. As the tropospheric time delay error depends on the meteorological data, this data is collected from various cities and is incorporated in tropospheric error models.

2 Tropospheric error

The layer of the atmosphere which is closest to the earth is troposphere. Generally, troposphere extends up to 14 kms from the surface of the earth. This layer is most dense than other layers and mainly consists of dry gases and water vapour. Most of the water vapour is below 4 kms. Sometimes, the atmospheric layer till 50 kms altitude is known as troposphere. The troposphere thickness is not same everywhere, it stretches to 16 kms at equator and reduces to 9 kms
above the pole. Due to the refractive index \((N_1 > 1)\) of the earth's neutral atmosphere, GPS signals suffer from tropospheric propagation delays. The total tropospheric delay in the direction of a particular satellite, i.e. slant path delay (SPD) can be divided into a hydrostatic (dry) and a wet component. About 90% of the tropospheric refraction is due to dry component and about 10% is due to wet component. Though the tropospheric delay is much smaller than the ionospheric delay at L-band frequencies, it greatly affects the signal propagation.

3 Tropospheric error correction models applicable to GPS signals

Many models are proposed in the available literature but few prominent and efficient models for estimating the tropospheric delay in GPS signals are considered in this paper. Two models, namely Hopfield model\(^6\) and European Geo-stationary Navigation Overlay Service (EGNOS) model\(^7\) are implemented for the GPS signals. Two models Radio Technical Commission for Aeronautics (RTCA)\(^8\) and Locata Tropospheric (LTC) models\(^9\) are implemented for cellular signals.

3.1 Hopfield model

The Hopfield model was basically proposed by a scientist, Helen Hopfield\(^6\). This model is based on the relationship between the dry refractivity at height ‘h’ to the surface of earth. The thickness of the dry layer, \(h_{dry}\), is given as:

\[
 h_{dry} = 40136 + 148.72 \times (T_o - 273.16) \tag{1}
\]

where, \(T_o\) is the temperature \((^\circ K)\) at the surface of earth; and \(h_{dry}\), the wet layer thickness, i.e. where humidity exists. It is calculated for Hyderabad city on 28 Feb 2012 as 4 km.

The total tropospheric delay \((\Delta_{Trop})\) in zenith direction is expressed as:

\[
 \Delta_{Trop} = \Delta_{dry} + \Delta_{wet} \tag{2}
\]

\[
 = 10^{-6} \times \sum \left[ N_{dry} \times h_{dry} + N_{wet} \times h_{wet} \right] \tag{3}
\]

where, \(N_{dry} = K1 \times p_o / T_o\) \tag{4}

\[
 N_{wet} = K2 \times e_p / T_o + K3 \times e_p / T_o^2 \tag{5}
\]

The constants are given as \(K1 = 77.64\) Kmb, \(K2 = -12.96\) Kmb and \(K3 = 3.718 \times 10^5\); \(p_o\) is the total pressure (mb); and \(e_p\) is the partial pressure of water vapour both in milli bar at surface.

If \(\theta\) is the elevation angle (in deg) of the satellite with respect to the user then mapping functions can be included in Eq. (3) to get the respective delay as:

\[
 \Delta_{dry} = \frac{\beta H}{g} \left[ 1 - \frac{\delta T}{T_o} \right] \tag{6}
\]

where, \(m_d(\theta)\) and \(m_w(\theta)\), are the mapping functions for dry and wet component delays for Hopfield model. These are given as:

\[
 m_d(\theta) = \frac{1}{\sin \left( \sqrt{\theta^2 + 6.25} \right)} \tag{7}
\]

\[
 m_w(\theta) = \frac{1}{\sin \left( \sqrt{\theta^2 + 22.5} \right)} \tag{8}
\]

3.2 European Geo-stationary Navigation Overlay Service (EGNOS) model

The EGNOS model is the European contribution to the Global Navigation Satellite System (GNSS). The EGNOS tropospheric correction model is considered as a possible tool for estimating and correcting the tropospheric delay in order to obtain error free GPS signals. The total tropospheric delay due to dry and wet components, using mapping function \((\theta_z)\) is modelled as is in Eq. (2). The zenith dry and wet delays are computed as: \(^{10}\)

\[
 \Delta_{dry} = Z_{dry} \times \left[ 1 - \frac{\beta H}{g} \left[ 1 - \frac{\delta T}{T_o} \right] \right] \tag{9}
\]

\[
 \Delta_{wet} = Z_{wet} \times \left[ 1 - \frac{\beta H}{g} \left[ 1 - \frac{\delta T}{T_o} \right] \right] \tag{10}
\]

where, \(g\), is acceleration due to gravity \((9.80665\) m s\(^{-2}\)); \(H\), height of the receiver above mean sea level (m); \(\lambda^\prime\) = -2.6 mb km\(^{-1}\), the temperature lapse rate \((2.6\) mb km\(^{-1}\)); and \(R_o = 287.054\) J kg\(^{-1}\) K\(^{-1}\).

The two expressions for the zenith dry delay and the zenith wet delay at mean sea level are as follows:

\[
 Z_{dry} = (10^{-6} \times y_1 \times R_o \times p_o) / g_{sm} \tag{11}
\]

\[
 Z_{wet} = [(10^{-6} \times y_2 \times R_o) / g_{sm} (\lambda^\prime + 1) - \beta R_o \times p_o / T_o] / g_{sm} \tag{12}
\]

where, \(y_1\), \(y_2\) and \(g_{sm}\), are constants: \(y_1 = 77.604\) k mb\(^{-1}\);

\(y_2 = 382000\) k^2 mb\(^{-1}\); \(g_{sm} = 9.78\) m s\(^{-2}\).

The mapping function (MF) at elevation angle \((\theta_z)\) is expressed as:

\[
 MF(\theta_z) = \frac{1.001}{\sqrt{0.002001 + \sin(\theta_z)^2}} \tag{13}
\]
4 Tropospheric error correction models applicable to cellular signals

The cellular network is a ground based system, hence, the signals from the base station propagate through the tropospheric layer and reach the receiver on ground. Therefore, the models which could estimate the tropospheric delay in such ground based systems are RTCA model and LTC model. RTCA and LTC models are basically developed for the area of navigation, however, in this paper their feasibility is investigated for cellular signals.

4.1 RTCA model

RTCA has defined a tropospheric delay model for use with local area augmentation systems (LAAS). The tropospheric correction consists of a dry (hydrostatic) and a wet component expressed as in Eq. 2. The equation for dry and wet delays are given as:

\[ \Delta_{dry} = 10^{-6} \times N_{dry} \times D \times [1 - (h_{MS} - h_{BS}) / h_{dry,o}] \]  
\[ \Delta_{wet} = 10^{-6} \times N_{wet} \times D \times [1 - (h_{MS} - h_{BS}) / h_{wet,o}] \]

where, \( h_{MS} \) stands for user height (m); \( h_{BS} \), stands for the base station tower height (m); and \( D \), is the slope distance (m) between the user and the tower. Refractivity \( N \) for dry is calculated as in Eq. (5) and wet components is given as:

\[ N_{wet} = 2.277 \times 10^4 \times f_o / T_o^{1.2} \times 10^{7.4475 \times T_o - 273} / (T_o - 38.3) \]

where, \( f_o \) is humidity (%) which is given as:

\[ f_o = (e_o / 0.01) \times \exp(-37.2465 + 0.213166 \times T_o - 0.000256908 \times T_o^2) \]

4.2 Length based tropospheric (LBT) model

A modified length ratio based tropospheric (LBT) model is referred as the Locata Tropospheric Model (LTC). The LTC model uses the principle of LBT model. It is introduced for use in locata positioning, and its performance is investigated for use during the Known Point Ambiguity Resolution (KPAR) procedure. This model uses new constants for calculating refractivity index (N). Here, the tropospheric correction is given as:

\[ \Delta_{Trop} = 10^{-6} \times N \times D \]

where,

\[ N_{wet} = 71.2962 \times e_o / T_o + 375463 \times e_o / T_o^2 \]
\[ N_{dry} = 77.689 \times (P_o - e_o) / T_o \]

4.3 Locata Tropospheric (LTC) model

In LTC, the tropospheric correction is applied to the difference between the distance of transmitter and user and the distance of transmitter and the known point (Fig. 1). By taking the difference between the two distances, LTC takes care of the distances that are not included in the ambiguity resolution and also removes the radio interference and biases. Mathematically, the tropospheric error can be calculated as in Eq. (15).

\[ D = D_1 - D_2 \]

where, \( D_1 \) is the slope distance between the base station and user receiver; \( D_2 \), the slope distance between the base station and a known point.

5 Interpolation of meteorological data

Tropospheric models depend upon MET data. Meteorological (MET) data consists of total pressure, temperature, partial pressure due to water vapour and relative humidity. If the airborne data is available, it can be directly used by different tropospheric models to find the tropospheric delay at different heights. Otherwise, the surface meteorological data should be interpolated at various heights for troposphere, tropopause and stratosphere (Table 1).

6 Results and Discussion

Available surface MET data for various cities of India is interpolated at different altitudes of the atmosphere. Balloon data, which is collected from a southern region of India is compared with the MET data obtained using interpolation techniques. If the real meteorological data is not available, then the Minimum Operational Performance Standards (MOPS) tropospheric algorithm can also be applied. This approach uses standard meteorological data dependent on latitude and takes seasonal variations into account. The advantage of this model is that the
meteorological measurements are not necessary and meteorological parameters are modelled with the help of default data sets. However, the accuracy of this model is not sufficient.

6.1 Interpolation of meteorological data and comparison with balloon data

The surface meteorological data is collected from various cities of India on 28 February 2012 (Table 2). Temperature of three different cities is interpolated and compared at different altitudes of troposphere, tropopause and stratosphere (Fig. 2). The data provided by the MOPS data sheet and balloon data is also used to compare interpolated temperature at different altitudes.

It can be observed that the temperature is maximum at the surface of earth (approximately 299°K) for all three cities and starts decreasing in the troposphere, i.e. up to a height of 14 km and remains constant in tropopause, i.e. up to 18 km. Thereafter, the temperature gradually increases to 278°K approximately in stratosphere up to 50 km. There is a slight variation in the surface temperature of the three different cities. From the balloon data, the temperature is found to be 310°K at the surface and starts decreasing till 16 km and it slowly increases up to 212°K in stratosphere (balloon data could be collected till 23 km altitude only due to various constraints).

The interpolation of pressure at different altitudes is shown in Fig. 3. The pressure is inversely proportional to altitude, i.e. as the altitude increases the pressure decreases. The pressure is maximum at the surface of earth. It can be observed that the pressure decreases from 1000 mb at surface to 0 mb at
an altitude of 45 km and then remains constant (0 mb) up to 50 km. There is an insignificant difference in pressure of three cities and MOPS. From the given balloon data, the pressure is maximum at the surface (1000 mb) and decreases gradually and becomes 20 mb at 23 km altitude. The interpolation of partial pressure (PP) due to water vapour with respect to altitude is shown in Fig. 4.

PP decreases with increase in altitude. It is approximately 12 mb at the surface of earth for all three cities. It is seen that the PP is significant within 5 km and above it becomes negligible. From balloon data, PP fluctuates randomly and is observed as 17 mb at surface and remains constant till 1.5 km altitude. It suddenly reduces to 7.5 mb at approximate 2 km, then it gradually increases till 3.8 km altitude and again reduces to approximately 0 mb at approximate 4 km. After few more variations it becomes 0 mb at 5 km altitude. According to MOPS provided data sheet, the PP is 26 mb at surface and becomes 0 mb at 10 km altitude.

6.2 Estimation of tropospheric delay in GPS signals using interpolated meteorological data

For investigating the tropospheric delay (TD) corrections in the GPS signals, the GPS receiver integrated with mobile phone may apply the tropospheric error correction models. In this paper two prominent models, namely Hopfield model and EGNOS model are used to estimate the tropospheric delay in signals transmitted by the GPS satellite to the mobile receiver. For the analysis, initially, Hyderabad surface and interpolated MET data and eventually balloon data is utilized.

Estimation of TD with respect to elevation angle is shown in Fig. 5. As the elevation angle increases, TD exponentially decreases. When elevation is 5°, the tropospheric delay is found to be maximum for Hopfield model (22.5 m) as well as for EGNOS model (15.5 m). The TD reduces to approximately 1.5 m for Hopfield model and 3 m for EGNOS model, when elevation increases to 90°.

The estimation of TD with respect to altitude is shown in Fig. 6. The TD decreases with increase in altitude for both the models. The TD is observed high at the surface of the earth, for Hopfield model (13 m) as well as for EGNOS model (13.2 m). The TD gradually decreases and becomes 0 m for both the models near 40 km altitude.

6.3 Estimation of tropospheric delay in GPS signals using balloon data

The balloon data is collected from a southern region of India from surface of earth to 23 km altitude. It is incorporated by GPS models to estimate the tropospheric time delay efficiently. The TD is estimated with respect to elevation angle and altitude. Estimation of TD with respect to elevation angle is shown in Fig. 7. As the elevation angle increases, TD exponentially decreases. When elevation is 5°, the tropospheric delay is found to be maximum for Hopfield model (24 m) as well as for EGNOS model (13.5 m). The TD reduces to approximately 2.5 m for Hopfield model and 1.5 m for EGNOS model, when
elevation increases to 90°. The maximum difference (10 m) in TD is observed between points A and B, i.e. at 5° elevation angle.

The estimation of TD with respect to altitude is shown in Fig. 8. The TD is high at the surface of the earth, for Hopfield model (12.88 m) as well as for EGNOS model (12.96 m). This is because the tropospheric layer is the thickest for GPS signals at the surface of earth. As the altitude increases, there is a sharp decrease in TD till 16 kms altitude for both the models. Then, the TD gradually decreases further and becomes approximately 0 m at 23 km altitude.

Maximum difference (i.e. 6 m) between both the models is observed at points A (10 km for EGNOS) and B (12 km for Hopfield). The curves intersect at point C, when they reach 17 km altitude, and the TD is observed as 2 m. Table 3 shows the comparison of GPS tropospheric error correction models. At the surface of earth, the TD is observed maximum for Hopfield model (12.88 m) as well as for EGNOS model (12.96 m) at 0 km altitude. The TD becomes approximately 0 m for both the models at 23 km altitude.

### 6.4 Effect of tropospheric error on GPS user position

The position of a GPS receiver can be determined by calculating pseudo ranges from a minimum of four GPS satellites. The ranges measured do not represent the true ranges because the signal coming from a GPS satellite will be affected by various errors. After making necessary corrections to the observed pseudo ranges, the receiver position and the receiver clock bias with GPS time can be determined by using the popular algorithm known as Bancroft Algorithm. For the analysis, GPS data is collected by DL4 plus dual channel GPS receiver kept at Osmania University for a particular day (19 January 2008). Figure 9 shows the variations in user position error, with and without incorporating tropospheric corrections, with respect to local time. Comparatively, error in the user position is found to be more when tropospheric error correction is not incorporated. Table 4 shows the minimum, maximum and mean values of the user position with and without incorporating tropospheric error correction. The mean value of user position before incorporating tropospheric error correction was 9.37 m and after introducing the tropospheric error correction it reduced to 8.47 m approximately.

<table>
<thead>
<tr>
<th>Model</th>
<th>Min TD, m</th>
<th>Max TD, m</th>
<th>Mean TD, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hopfield</td>
<td>0.015</td>
<td>12.88</td>
<td>2.88</td>
</tr>
<tr>
<td>EGNOS</td>
<td>0.132</td>
<td>12.96</td>
<td>2.45</td>
</tr>
</tbody>
</table>

### Error in user position with and without incorporating tropospheric correction

<table>
<thead>
<tr>
<th>Error in user position</th>
<th>Without tropospheric correction, m</th>
<th>With tropospheric correction, m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>7.0349</td>
<td>6.3657</td>
</tr>
<tr>
<td>Maximum</td>
<td>16.5106</td>
<td>14.979</td>
</tr>
<tr>
<td>Mean</td>
<td>9.3710</td>
<td>8.4718</td>
</tr>
</tbody>
</table>
6.5 Estimation of tropospheric delay in cellular signals using interpolated meteorological data

To estimate the tropospheric time delay in cellular signals, RTCA and LTC models are implemented and analyzed. The LTC model is based on the well-known Length Based Tropospheric (LBT) model. Hence, LBT model is also implemented. Hyderabad MET data (Table 2), which is interpolated for various altitudes is used by these models to estimate the TD.

The estimation of TD with respect to elevation angle is shown in Fig. 10. It is observed that the TD is minimum when the elevation angle is maximum and the delay is maximum when the elevation angle is minimum. At 5° elevation angle, the maximum delay is observed for RTCA model (7.5 m) and for LTC model (1.4 m). The maximum TD difference (6 m) is observed between point A and B at 5° elevation angle for both the models.

In cellular system, with respect to earth surface, the TD gradually increases with increase in altitude upto the end of troposphere, i.e. 14 km. The TD becomes maximum above the tropospheric layer because the signals received by the mobile user are transmitted from base station, which is located within the troposphere at fixed height.

Estimation of TD with respect to altitude is shown in Fig. 11. The maximum TD is observed for the LTC model (11.41 m) as well as for RTCA model (9.08 m) at an altitude of 14 km, i.e. end of troposphere. The delay is minimum (0 m) at the earth surface, i.e. at 0 km altitude. At 7 km altitude, the maximum TD difference (2.3 m) is observed between point A and B. The two curves intersect at point C at 11 km altitude where the TD is noted as 9 m.

Table 5 shows the comparison of tropospheric error correction models for cellular signals. The maximum TD is found for RTCA (9.08 m) and LTC (11.4 m) models at 14 km altitude. The TD is negligible (0 m) at earth surface for both the models. At 7 km altitude the maximum TD difference is observed for RTCA (4.3 m) and LTC (7 m) models.

6.6 Tropospheric effect on cellular signals in real time scenario

A standalone GSM module (representing cell phone) is kept at the Research and Training Unit for Navigational Electronics (NERTU), Osmania University, Hyderabad. It receives signals from nearest base stations. GSM data sheet provides the position of the base stations in geodetic coordinate system along with the code and name of the base station. GPS receiver (Novatel DL4 plus) is also mounted at NERTU along with GSM module to determine user position. From the GSM and GPS data, the slope distances and the elevation angles between the user and the base stations are calculated (Table 6). As the user moves from one cell to another cell, the elevation angle also changes, which results in the tropospheric delay (TD) change. TD with respect to elevation angle is estimated. From Fig. 12, it can be observed that the TD reduces as the elevation angle increases. When elevation is 1.5°, TD is approximately

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>0</th>
<th>7</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD, m</td>
<td>0</td>
<td>4.3</td>
<td>9.08</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 5 — Comparison of tropospheric error correction models used for cellular signals

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>0</th>
<th>7</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD, m</td>
<td>0</td>
<td>4.3</td>
<td>9.08</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 6 — Details of base stations and respective tropospheric delay

<table>
<thead>
<tr>
<th>Base stations</th>
<th>Location</th>
<th>El. deg</th>
<th>Slant distance, km</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS1</td>
<td>Amber Nagar</td>
<td>4.9178</td>
<td>0.6355</td>
</tr>
<tr>
<td>BS2</td>
<td>OU</td>
<td>5.6563</td>
<td>0.7307</td>
</tr>
<tr>
<td>BS3</td>
<td>Vidya Nagar</td>
<td>3.5256</td>
<td>0.6476</td>
</tr>
<tr>
<td>BS4</td>
<td>Zamistanpur</td>
<td>2.1872</td>
<td>1.0186</td>
</tr>
<tr>
<td>BS5</td>
<td>Padmarao Nagar</td>
<td>1.7959</td>
<td>1.2443</td>
</tr>
<tr>
<td>BS6</td>
<td>Ramnagar</td>
<td>4.6622</td>
<td>1.2425</td>
</tr>
<tr>
<td>BS7</td>
<td>Boiguda</td>
<td>1.5992</td>
<td>1.9033</td>
</tr>
<tr>
<td>BS8</td>
<td>Warsiguda</td>
<td>2.8897</td>
<td>1.9988</td>
</tr>
<tr>
<td>BS9</td>
<td>Parsigutta</td>
<td>2.9778</td>
<td>2.2445</td>
</tr>
</tbody>
</table>
maximum TD is found at 14 km altitude for RTCA model (9.08 m) and LTC model (11.41 m) and TD is negligible at earth surface. In real time scenario, TD is estimated with respect to elevation angle as well as slant distance between the user and base station. The maximum TD is observed as 0.7 m for the RTCA model and 0.4 m for LTC model when the elevation angle was 1.5°. The behaviour of these two models is found to be similar.

Acknowledgment

The present work has been carried out under the project entitled, “Investigation of Atmospheric Effects on Future Ground Based Augmentation for GPS System” sponsored by Department of Science and Technology, New Delhi, India, vide sanction letter No: SR/S4/AS:53/2010, dated:12th July 2010.

References

5. Ashraf Farha & Terry Moore, High spatial variation tropospheric model (University of Nottingham, ION GPS/GNSS, UK), September 2003.