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Optimal common cycle time for a multi-item production system with discontinuous delivery policy and failure in rework

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This study is concerned with the optimal common cycle time for a multi-item production system with discontinuous delivery and failure in rework. In real life manufacturing environments, managements often plan to produce multiple products in turn on a single machine in order to maximize machine utilization. Also, dealing with random defective items during production seems to be an inevitable task, and the multi-delivery policy is commonly adopted to distribute the finished products to buyers. The objective of this study is to determine the optimal common production cycle that minimizes the total production-inventory-delivery costs per unit time for a multi-item production system with failure in rework and multi-delivery policy. Mathematical modeling along with an optimization procedure is used to derive the optimal common cycle time for the aforementioned production problem.

Keywords: multi-item production, optimization, common cycle time, failure in rework, scrap, multi-shipment

Introduction

A mathematical technique was first introduced[1] to solve the economic production quantity (EPQ) problem for single product under the assumption of perfect production and the policy of continuous inventory issuing. However, in real life production environments, managements often plan to produce multiple products in turn on a single machine in order to maximize machine utilization. Gordon and Surkis[2] presented a simple and practical approach to determine the control policies for a multi-item inventory environment where the items are ordered from a single supplier and the demand for items are subject to severe fluctuations. The time between the orders can either be fixed or based on a system of accumulating a fixed order quantity for all products. Their model balanced the stock carrying and stock-out costs. An operational system structure was developed and a simulation procedure adopted to determine the appropriate value of their inventory factor in the model. Zahorik *et al*.[3] investigated a multi-item, multi-level production scheduling problem with linear costs and production and inventory constraints at a key facility. Two multi-item problems were considered, one where the constraint was on shipping capability and the other where there was a final stage bottleneck machine. A multi-item Facilities-in-series problem was formulated as a linear program, and a three-period result was used as the basis for a rolling heuristic for T-period problems. They discussed the conditions under which this heuristic fails to find optimal solutions, and provide computational comparisons to standard linear programming. Rosenblatt[4] compared two policies for the joint replenishment problem with a general ordering cost function. The fixed-cycle policy used a dynamic programming approach, resulting in partitioning the items into groups. The basic-cycle policy used a heuristic approach to partition the items into only two groups. A simulation model was developed to compare the effectiveness of the two policies and the economic order quantity approach. Leachman and Gascon[5] proposed a heuristic scheduling policy for multi-item, single-machine production systems facing stochastic, time-varying demands. Their dynamic cycle length heuristic, integrated feedback control based on the monitoring of inventory levels with the maintenance of economic production cycles. The policy could be applied time period by time period to decide on which items to produce and in what quantities during the next time period. Extensive studies related to various aspects

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of multi-item production planning and optimization issues have since been conducted[6-11].

Unlike the economic production quantity model that adopts a continuous inventory issuing policy, in the real world supply chain environments, a multi-delivery policy is often used for distributing the finished products to customers. Goyal[12] studied an integrated single supplier-single customer problem. With examples to demonstrate his proposed model, he presented a method typically applicable to the inventory problems where a product is procured by a single customer from a single supplier using examples to demonstrate his proposed model. Kim and Hwang[13] developed a formulation of a quantity discount pricing schedule for a supplier. They assumed a single incremental discount system and proposed an algorithm to derive an optimal discount schedule. They investigated cases in which both the discount rate and the break point are unknown but either one is prescribed, and used a numerical example to illustrate their algorithm. Banerjee and Banerjee[14] developed an analytical model for a coordinated, orderless inventory system for the single product, single vendor-multiple purchaser case. Such a system was used in electronic data interchange (EDI) at the time for the exchange of information between trading partners. On the basis of the potential benefits of this technique, they proposed a common cycle replenishment approach, where the supplier alone makes all replenishment decisions, without any ordering on the part of the customers. Their model and concepts were demonstrated through a simple numerical example, and they concluded that EDI-based inventory control can be attractive from economic as well as other perspectives. Sarker and Khan[15] addressed the problem of a manufacturing system in which raw materials are procured from suppliers in a lot and processed into finished products. They proposed an ordering policy for procuring raw materials to meet the requirements of a production facility. In turn, the facility must deliver its finished products demanded by outside buyers at fixed interval time points. A general cost model was first developed by considering both raw materials and finished products. This model was then used to develop a simple procedure to determine an optimal ordering policy for procuring raw materials as well as the manufacturing batch size so as to minimize the total cost for meeting the customer demand on time. Additional studies have since been made to address various aspects of the periodic or multiple delivery issues[16-22].

In real life production environments, dealing with random nonconforming items seems to be another challenging and inevitable task. Porteus[23] believed that an in-control production process may shift to out of control during each production cycle. Therefore, a two-state Markov chain is modeled and the optimal lot size accordingly derived. Cheung and Hausman[24] developed an analytical preventive maintenance model and safety stock strategies in a production environment subject to random machine breakdowns. They illustrated the trade-off between investing in the two options, preventive maintenance and safety stock, and provided optimality conditions under either one or both of the strategies that minimize the associated costs. They also analyzed both the deterministic and exponential repair time distributions in detail in their study. Studies have also been carried out on the different aspects of imperfect production systems and quality assurance issues during the past decades[25-33].

This paper is concerned with determining the optimal common production cycle time for a multi-item production system with discontinuous delivery and failure in rework. Since little attention has been paid on this area, this paper is intended to bridge the gap.

Problem description and mathematical analysis

A multi-item production system incorporating discontinuous delivery and failure in rework is examined in this study. For the purpose of maximizing machine utilization, L products are produced in turn on a single machine. The items produced are screened, and the inspection cost for each item is included in the unit production cost C_i. During the production of each product i (where $i = 1, 2, …, L$), a portion x_i of the nonconforming items is randomly produced at a rate d_i. Under normal operations, the constant production rate P_{1i} for product i satisfies $(P_{1i}-d_{1i}-\lambda_i)>0$, where λ_i is the annual demand rate for product i and d_{1i} can be expressed as $d_{1i}= x_iP_{1i}$. All of the nonconforming items are reworked at the rate of P_{2i} right after the end of regular production in each cycle with an additional cost C_{Ri}. During the reworking process, a portion φ_i of the reworked items fails and is scrapped at a disposal cost C_{Si} per item, and the excess demand due to scraps during a cycle is considered as lost sales with a shortage cost of C_{Li} per item. All the finished goods for each product i are delivered to customers only if the whole production lot is quality assured at the end of the rework process. A discontinuous inventory issuing policy is used in which fixed quantity n installments of the finished batch are delivered at fixed intervals of time during delivery time t_{3i} (see Fig. 1).

In such a multi-item production system with rework, we need to assume that $\sum_{i=1}^{L}\left(\left(\lambda_i / P_{1i}\right)+\left(x_i\lambda_i / P_{2i}\right)\right) < 1$.

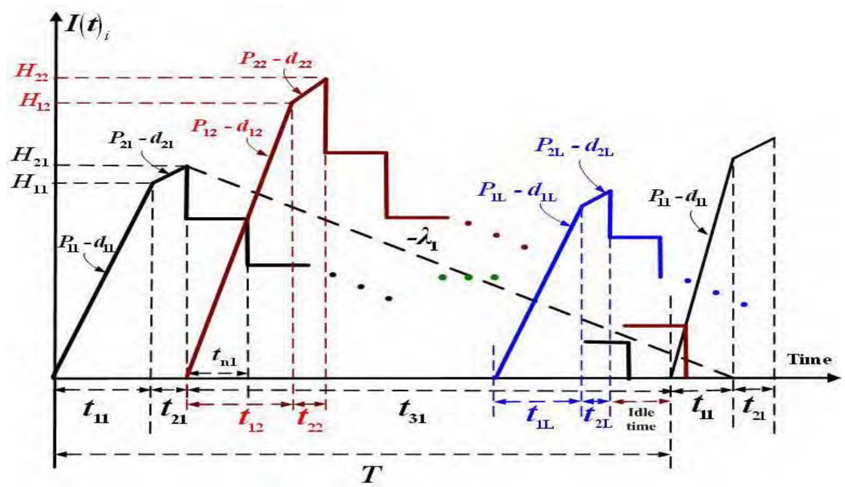

Fig. 1ð The on-hand inventory of the perfect quality product *i* in the proposed multi-item production system under a common cycle policy

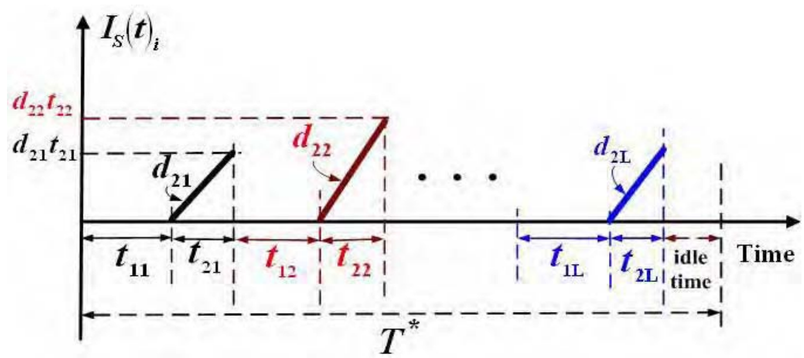

Fig. 2ð The on-hand inventory of the scrapped product *i* in the proposed multi-item production system under a common cycle policy

This assumption is necessary to ensure that the facility has a sufficient capacity for the regular production and rework so as to satisfy the demand for all L products.

The on-hand inventory of the scrapped product i in the proposed multi-item production system is illustrated in Fig. 2.

For each product i, the cost-related parameters used in this study include the production setup cost K_i, unit holding cost h_i, unit holding cost h_{1i} for each reworked item, fixed delivery cost K_{1i} per shipment, and unit shipping cost C_{Ti}. The additional notation includes the following:

T = common production cycle length, a decision variable,

t_{1i} = production uptime for product i in the proposed EPQ model,

t_{2i} = the rework time for product i in the proposed EPQ model,

Q_i = production lot size per cycle for product i,

d_{2i} = production rate of scrap items during the reworking time for product i,

H_{1i} = maximum on-hand inventory level for product i when regular production ends,

H_{2i} = maximum on-hand inventory level in units for product i when rework process ends,

n = number of fixed-quantity installments of the finished batch to be delivered to customers in each cycle, which is assumed to be a constant for all products,

t_{ni} = a fixed interval of time between each installment of finished products delivered during t_{2i}, for product i,

$I(t)_i$ = on-hand inventory level of perfect quality product i at time t,

$I_S(t)_i$= on-hand inventory level of scrapped product i at time t,

$TC(Q_i)$ = total production-inventory-delivery costs per cycle for product i,

$E[TCU(Q)]$ = total expected production-inventory-delivery costs per unit time for L products in the proposed system, E[$TCU(T)$] = total expected production-inventory-delivery costs per unit time for L products in the proposed system, using common production cycle time T as the decision variable.

Based on the aforementioned model description and Figs. 1 and 2, the following formulas can be obtained directly[28]:

$$H_{1i} = \left(P_{1i} - d_{1i}\right) t_{1i} \quad \text{é} \quad (1)$$

$$H_{2i} = H_{1i} + \left(P_{2i} - d_{2i}\right) t_{2i} \quad \text{é} \quad (2)$$

$TC(Q_i)$, the total production-inventory-delivery cost per cycle for L products consists of the setup cost, variable production cost, reworking cost, disposal cost, cost for lost sales, fixed and variable delivery costs, holding costs during the production uptime t_{1i} and rework time t_{2i}, and holding cost for the finished goods kept during the delivery time t_{3i}[16].

$$\sum_{i=1}^{L} TC\left(Q_i\right) = \sum_{i=1}^{L} \left\{ \begin{array}{l} K_i + C_i Q_i + C_{Ri}\left(x_i Q_i\right) + C_{Si}\left[x_i Q_i \varphi_i\right] + C_{Li}\left[x_i Q_i \varphi_i\right] + nK_{1i} \\ + C_{Ti}\left[Q_i\left(1 - \varphi_i x_i\right)\right] + h_{1i}\left[\dfrac{P_{1i} t_{2i}}{2}\left(t_{2i}\right)\right] \\ + h_i\left[\dfrac{H_{1i} + d_{1i} t_{1i}}{2}\left(t_{1i}\right) + \dfrac{H_{1i} + H_{2i}}{2}\left(t_{2i}\right) + \dfrac{n-1}{2n}\left(H_{2i} t_{3i}\right)\right] \end{array} \right\}$$

é (3)

Since the defective rate x is assumed to be a random variable with a known probability density function, in order to take the randomness of x into account, this study uses the expected values of x in cost analysis. Substituting all the variables[16] and $T=Q_i/\lambda_i$ in Eq. (3), and with further derivations we obtain the following expected E[$TCU(T)$]:

$$E\left[TCU(T)\right] = \sum_{i=1}^{L} \left\{ \begin{array}{l} \dfrac{K_i}{T} + C_i\lambda_i + C_{Ri}\lambda_i E(x_i) + C_{Si}\lambda_i E(x_i)\varphi_i + C_{Li}\lambda_i E(x_i)\varphi_i + \dfrac{nK_{1i}}{T} + C_{Ti}\lambda_i E_{0i} \\ + \dfrac{h_{1i}T\lambda_i^2}{2}\left(\dfrac{E(x_i)^2}{P_{2i}}\right) + \dfrac{h_i T\lambda_i^2}{2}\left[\dfrac{\left[E_{0i}\right]^2}{\lambda_i} - \dfrac{\left[E_{0i}\right]^2}{\lambda_i n} + \dfrac{E_{0i}}{P_{1i} n} + \dfrac{\varphi_i E\left(x_i\right)}{P_{1i}}\right] \\ + \dfrac{h_i T\lambda_i^2}{2}\left[\dfrac{E_{1i}}{P_{2i} n} + \dfrac{E\left(x_i\right)\left[1 - E\left(x_i\right)\right]}{P_{2i}}\right] \end{array} \right\}$$

é (4)

where E_{0i} and E_{1i} denote $\left[1-\varphi_i E\left(x_i\right)\right]$ and $E(x_i)\left[1-\varphi_i E\left(x_i\right)\right]$, respectively.

Optimal common production cycle time

If the expected cost function E[$TCU(T)$] is convex, one can locate its minimum point and hence find the optimal common production cycle time. By differentiating Eq. (4) with respect to T, one obtains the following second derivative:

$$\frac{\partial^2 E\left[TCU(T)\right]}{\partial T^2} = \sum_{i=1}^{L}\left\{\frac{2\left(K_i + nK_{1i}\right)}{T^3}\right\} \quad \text{é} \quad (5)$$

Note that in Eq. (5) the results for K_i, n, K_{1i}, and T are all positive. Hence, *E[TCU(T)]* is a convex function for all T different from zero. The optimal common production cycle time T^* can be obtained by setting the first derivative of *E[TCU(T)]* equal to zero. Further derivations, one obtains

$$T^* = \sqrt{\frac{2\sum_{i=1}^{L}\left(K_i + nK_{1i}\right)}{\sum_{i=1}^{L}\left\{h_i\lambda_i^2\left[\dfrac{\left[E_{0i}\right]^2}{\lambda_i} - \dfrac{\left[E_{0i}\right]^2}{\lambda_i n} + \dfrac{E_{0i}}{P_{1i} n} + \dfrac{\varphi_i E\left(x_i\right)}{P_{1i}} + \dfrac{E_{1i}}{P_{2i} n} + \dfrac{E\left(x_i\right)\left[1 - E\left(x_i\right)\right]}{P_{2i}}\right] + \dfrac{h_{1i}\lambda_i^2 E(x_i)^2}{P_{2i}}\right\}}}$$

... (6)

Discussion on production setup time

In general, the production setup time is relatively small compared to the production uptime. However, if the setup time is a factor, one must check whether there is enough time in each cycle to account for the setup, production, and reworking of L products[34]. Assuming S_i denotes the setup time for product i, to ensure that each cycle has sufficient time for the setup, production, and reworking of L products, the following must hold:

$$\sum_{i=1}^{L}\left(S_i + \left(Q_i / P_{1i}\right) + \left(x_i Q_i / P_{2i}\right)\right) < T \quad \text{é} \quad (7)$$

Using $T=Q_i / \lambda_i$, we can rearrange Eq. (8) as

$$T > \frac{\sum_{i=1}^{L} S_i}{1 - \sum_{i=1}^{L}\left(\left(\lambda_i / P_{1i}\right) + \left(x_i\lambda_i / P_{2i}\right)\right)} = T_{\min.} \quad \text{é} \quad (8)$$

Finally, if the setup time is a factor, one should choose the optimal common production cycle time from max $(T^*, T_{\min})$[34].

Numerical example

Assume that a multi-item production system has scheduled five products to be produced in turn on a single machine. For the five different products, the annual demands λ_i are 3000, 3200, 3400, 3600, and 3800 and annual production rates P_{1i} 58000, 59000, 60000, 61000, and 62000, respectively. The random defective rates x_i

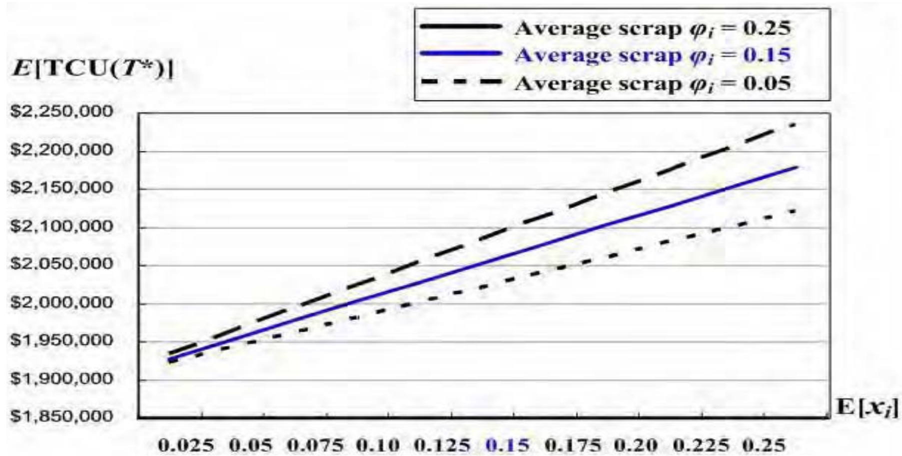

Fig. 3ð Variation of average defective rate and average scrap rate effects on the optimal E[*TCU*(*T**)] of the proposed multi-item production system

associated with the five products follow a uniform distribution over the intervals of [0, 0.05], [0, 010], [0, 0.15], [0, 020], and [0, 0.25], respectively. All of the nonconforming items for the five products are reworked respectively at the rates P_{2i} of 1800, 2000, 2200, 2400, and 2600 with additional reworking costs C_{Ri} of \$50, \$55, \$60, \$65, and \$70 per reworked item. During the reworking, a portion φ_i of 0.5, 0.1, 0.15, 0.2, and 0.25 of the five reworked products, respectively, fails and becomes scrap, with additional disposal costs C_{Si} of \$20, \$25, \$30, \$35, and \$40 per scrapped item. The values of the other parameters used for the five products are as follows:

K_i = production set up costs \$3800, \$3900, \$4000, \$4100, and \$4200, respectively.

C_i = unit manufacturing costs \$80, \$90, \$100, \$110, and \$120, respectively.

C_{Li} = unit cost for lost sales \$160, \$180, \$200, \$220, and \$240, respectively.

h_i = unit holding costs \$10, \$15, \$20, \$25, and \$30, respectively.

h_{1i} = unit holding costs per rework \$30, \$35, \$40, \$45, and \$50, respectively.

n = the number of shipments per cycle; in this study, this is assumed to be a constant, 4.

K_{1i} = the fixed delivery costs per shipment \$1800, \$1900, \$2000, \$2100, and \$2200, respectively.

C_{Ti} = unit transportation costs \$0.1, \$0.2, \$0.3, \$0.4, and \$0.5, respectively.

Applying Eq. (6) one obtains the optimal common production cycle time T*=0.6115 (years), and from Eq. (4), one can obtain the expected production-inventory-delivery costs per unit time for L products, E[*TCU*(*T**=0.6115)]=\$2,070,314.

The variation of the effects of the average defective rate and average scrap rate on the optimal E[*TCU*(*T**)] of the proposed multi-item production system are illustrated in Fig. 3. Note that as the average scrap rate E[φ_i] increases, the expected system cost E[*TCU*(*T**)] increases slightly, and as the average random defective rate E[x_i] increases, the expected system cost E[*TCU*(*T**)] increases significantly.

Concluding remarks

In real life manufacturing environments, for the purpose of maximizing machine utilization, production planners often schedule multiple products to be produced in turn on a single machine. Also, dealing with random defective items during the production run seems to be an inevitable task, and multi-delivery is commonly adopted for distributing the finished items to customers. Therefore, it is important for managements to look into the effects of the multi-delivery and failure in rework on the common production cycle decisions of multi-item production systems. In this study, we used mathematical modeling to determine the optimal common cycle that minimizes the long-run average cost for such a specific multi-item production system. These research results are intended to assist managements in the fields to better plan and control such a realistic multi-item production system. For future study, one interesting topic will be to examine the effect of the variable demand rates on the common cycle time for the same model.

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