Effect of transverse inhomogeneous electric field and loss-cone distribution on electrostatic ion-cyclotron instability in the ionosphere

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Dispersion relation, resonant energy transferred, growth rate and marginal stability of the electrostatic ion-cyclotron wave with general loss-cone distribution function in low-$\beta$ homogeneous plasma in the presence of a transverse inhomogeneous electric field are discussed by investigating the trajectories of the particles. The wave is assumed to propagate obliquely to the static magnetic field. The whole plasma is considered to consist of resonant and non-resonant particles. It is assumed that resonant particles participate in energy exchange with the wave, whereas non-resonant particles support the oscillatory motion of the wave. Effects of the steepness of the loss-cone distribution and inhomogeneity in electric field on resonant energy transferred and growth rate of the instability are discussed. It is found that the effect of transverse electric field is to stabilize the wave, whereas the inhomogeneity in electric field acts as a source of free energy for the electrostatic ion-cyclotron wave and enhances the growth rate. Effect of steepness of loss-cone is also to enhance the growth rate. The results are interpreted for the space plasma parameters appropriate to the auroral acceleration region.

Keywords: Ion-cyclotron instability; Electrostatic ion-cyclotron instability; Ionospheric ion-cyclotron instability; Loss-cone distribution

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1 Introduction

Plasma wave measurements from S3-3 satellite, sounding rocket, backscatter radar have observed electrostatic ion-cyclotron (EIC) waves at a broad range of altitudes, which includes low altitude ionosphere (300-600 km)\(^1\), topside ionosphere (900 km)\(^2\) and higher altitudes (2600 km)\(^3\). Recently a polar satellite\(^4\) has observed EIC waves in the perpendicular electric field\(^5\). The same satellite has also observed local EIC waves in a region containing perpendicularly heated ion distribution. Recent observations by the FAST satellite\(^6\) also indicate the existence of EIC waves in the upward current auroral region\(^7\).

Investigations by Basu et al.\(^8\), Ganguli\(^9\) and Gavrishchaka et al.\(^10\) point to the importance of localized transverse electric fields in space for the generation of the waves that are responsible for the ion energisation of the ionospheric ions. Recently electric fields of the order of hundreds of milli-volts per meter have been depicted by the Freja satellite in high latitude ionosphere, the auroral zone, magnetotail and the plasmasheet\(^11\). The field varies rapidly and unevenly, indicating that the flow and its shear are inhomogeneous.

EIC waves in a low magnetized plasma have been analysed by Alba et al.\(^12\). Simulations of ion-cyclotron mode in magnetoplasma with transverse inhomogeneous electric field for Maxwellian plasma have been carried out by Ganguli and others\(^9,11,13\). Gavrishchaka et al.\(^14\) have studied the EIC mode in a two-ion component plasma with transverse velocity shear. Effect of neutral collisions on the excitation threshold of the EIC waves have been studied recently by Koeke et al.\(^15\) Sharma et al.\(^16\) have studied the excitation of current driven EIC waves in the presence of transverse direct current electric fields in a magnetized plasma. Agrimson et al.\(^17\) have experimentally studied the effect of parallel velocity shear on the excitation of EIC waves in a single and double ended Q-machine.

In most of the theoretical work reported so far, the velocity distribution functions have been assumed to be either ideal Maxwellian or bi-Maxwellian\(^9,11,17\), ignoring the steep loss-cone feature. Plasma in mirror-like devices and in the auroral region with curved and converging field lines considerably depart from Maxwellian distribution, and have steep loss-cone distribution\(^18,21\). In the present work, for the first time, the loss-cone distribution function is used to study the EIC waves in the presence of inhomogeneous transverse electric field. The present analysis is based on Dawson’s theory\(^22\) of Landau damping, which has been further extended by Terashima\(^23\), Misra and
Tiwari, Varma and Tiwari and Baronia and Tiwari.

2 Basic assumptions

The basic assumptions are the same as in earlier work by Terashima and others. The plasma is considered to be homogeneous and collisionless, consisting of resonant and non-resonant particles. The ions are supposed to have unit charge. The wave is considered to be propagating obliquely to the static uniform magnetic field \( B_0 \) that is along the z-direction. The non-resonant particles support the oscillatory motion of the ElC wave, while the resonant particles participate in energy exchange with the wave. An EIC wave is assumed to start at \( t = 0 \) when the resonant particles are not disturbed. The trajectories of particles are then evaluated within the framework of linear theory. Using the particle trajectory in the presence of ElC wave, the dispersion relation and the growth rate are derived for different distribution indices.

The wave is assumed to have the form

\[
\mathbf{k} \cdot \mathbf{E}_1 \mathbf{k} = (k_x, 0, k_z), \quad \mathbf{E}_1 = (E_x(t), 0, E_z(t))
\]

with

\[
E_x(r, t) = E_1 \cos (k_x x + k_z z - \omega t), \quad E_z(r, t) = \kappa E_1 \cos (k_x x + k_z z - \omega t)
\]

\[
\kappa = \left( \frac{k_x}{k_z} \right) < 1
\]

The amplitude \( E_1 \) is a slowly varying function of \( t \), i.e.

\[
\frac{1}{E_1} \left( \frac{dE_1}{dt} \right) \ll \omega.
\]

In the present analysis, the EIC instability in the system of hot electrons and hot ions is considered under the condition where \( a \) is taken to be comparable to the mean ion gyroradius but much larger than the Debye length. When \( y^2/a^2 < 1 \), \( E(y) \) becomes a constant uniform field. In the case \( y > a \), the electric field changes sign and is oppositely directed.

The solution for the particle orbit in terms of the field \( E(y) \) and \( B_0 \) is obtained by the Bogoliubov-Mitropolskii method as

\[
x(t) = x_0 + \Delta \frac{V_1}{\Omega_{ki}} \left[ 1 + \frac{3}{4} \left( \frac{E(y)}{\Omega_{ki}} \right)^2 \right] \sin(\theta - \Omega_{ki} t) - \sin \theta
\]

\[
y(t) = y_0 + \frac{V_1}{\Omega_{ki}} \left[ 1 + \frac{1}{4} \left( \frac{E(y)}{\Omega_{ki}} \right)^2 \right] \cos(\theta - \Omega_{ki} t) - \cos \theta
\]

\[
z(t) = z_0 + V_1 t
\]

where \( \theta \) is the phase of \( V_1 \) at \( t = 0 \) and

\[
\mathcal{E}(y) = \frac{q}{m} E(y)
\]

\[
\Delta = \frac{\mathcal{E}(y)}{\Omega_{ki}} \left[ 1 + \left( \frac{E'(y)}{E(y)} \right) \frac{1}{4} \left( \frac{V_1}{\Omega_{ki}} \right)^2 + \ldots \right]
\]

It should be noted that \( d\Delta/dt \) represents the drift velocity and the second term in the square brackets on the right hand side of Eq. (6) represents the finite gyro radius correction.

3 Particle trajectories and velocities

In the present mathematical analysis, the procedure adopted by Terashima and Bajaj and Tiwari is followed. The equation of motion of a particle is given by

The gyro-frequency of the ion, here, \( \ell = 1, 2, \ldots \) represents the harmonics of the wave, \( \rho_{ix} \), the mean gyro-radii of the ions and electrons respectively, \( \omega \) the wave frequency, \( k_x \) and \( k_z \) the components of the wave vector along and across the magnetic field, respectively.
\[ \frac{d\mathbf{v}}{dt} = q \left[ \mathbf{E} + \left( \frac{1}{c} \right) \mathbf{v} \times \mathbf{B}_0 \right] \]  
\[ \ldots (7) \]

where \( \mathbf{E} \) consists of the wave electric field and the impressed inhomogeneous electric field and other symbols have their usual meaning.

If \( \mathbf{E} \) is considered to be a small perturbation, velocity \( \mathbf{v} \) can be expressed in terms of unperturbed velocity \( \mathbf{V} \) and perturbed velocity \( \mathbf{u} \). The perturbed velocity \( \mathbf{u} \) is determined by

\[ \frac{du}{dt} = \frac{qkE}{m} \sum J_n(\mu) \cos(\Lambda_n t + \Psi_n^0) \]

\[ \ldots (8) \]

where \( u_{1,2} = u_n + iu_{\phi} \) represents the perturbed velocity in transverse direction and \( u_{\phi} \) represents the perturbed velocity in parallel direction. The basic trajectories are the same as derived by Terashima \(^2\) and Bajaj and Tiwari \(^3\). The resonance criterion is given by

\[ \Lambda_n V_l = k_i V_l - \omega + \ell \Omega_\parallel + k_{\parallel} \Delta' = 0; \]

\[ \ell = \pm 1, \pm 2 \]  
\[ \ldots (9) \]

where \( V_l \) is the resonance velocity of the particles and the particles with parallel unperturbed velocity \( V_l \) near to

\[ V_l = \frac{\omega - \ell \Omega_\parallel - k_{\parallel} \Delta'}{k_i} \]

which in this case are the ions.

This resonance condition means that for ions the waves appear to be independent of \( t \) in the particles frame. \( J_n(\mu) \) and \( J'_n(\mu) \) are Bessel's function, which arise from the different periodical variations of charged particle trajectories, where

\[ \mu = \frac{k_i V_l}{\Omega_i} \left[ 1 + \frac{3}{4} \left( \frac{E'(y)}{\Omega_i^2} \right) \right] \]

and

\[ \Delta' = \frac{E'(y)}{\Omega_i} \left[ 1 + \left( \frac{E''(y)}{E'(y)} \right) \left( \frac{1}{4} \frac{V_l}{\Omega_i} \right)^2 + \ldots \right] \]

The term represented by the Bessel’s function indicates the reduction in the field intensity due to finite gyro-radius effect. The oscillatory solution of \( u(t) \) is given by

\[ u_n(r,t) = \frac{qE_i}{m} \sum J_n(\mu) \sum J'_n(\mu) \left[ \frac{\Lambda_n}{\Lambda_n^2 - \Omega_i^2} \sin \chi_{\text{el}} + \frac{\delta}{2 \Lambda_n} \sin(\chi_{\text{el}} - \Lambda_n t) \right] \]

\[ \ldots (10) \]

\[ \chi_{\text{el}} = \frac{\omega - \ell \Omega - k_{\parallel} \Delta'}{\delta} \]

\[ \delta = 0 \] for non-resonant particles and \( \delta = 1 \) for resonant particles.

### 4 Density perturbation

To find out density perturbation associated with the velocity perturbation \( \mathbf{u}(r,t) \), we consider the equation

\[ \frac{dn}{dt} = -N(V)(\nabla \cdot \mathbf{u}) \]  
\[ \ldots (12) \]

Expressing the right hand side of Eq. (12) as the function of \( t \) and the initial parameters and integrating we get \( n_i(r,t) \) the perturbed density, for the non-resonant and resonant particle as

\[ n_i(r,t) = \frac{qE_i N}{m} \sum J_n(\mu) \sum J'_n(\mu) \left[ \frac{\Lambda_n}{\Lambda_n^2 - \Omega_i^2} \sin \chi_{\text{el}} - \frac{\delta}{2 \Lambda_n} \cos(\chi_{\text{el}} - \Lambda_n t) \right] \]

\[ \ldots (13) \]

\[ \chi_{\text{el}} = \frac{\omega - \ell \Omega - k_{\parallel} \Delta'}{\delta} \]

\[ \delta = 0 \] for non-resonant particles and \( \delta = 1 \) for resonant particles.
provided that $\omega \sim \ell \Omega$,

and $\kappa^2 = \left( \frac{k_\perp}{k_\parallel} \right)^2 \left( \frac{\Lambda_n^2}{\Lambda_0^2 - \Lambda_n^2} \right) = \frac{\Lambda_n^2}{\Omega^2} \quad \ldots (15) \]

5 General distribution function

To calculate the dispersion relation and growth rate the general loss-cone distribution function of the following form \cite{20,21,22} is used

\[
N(y, V) = N_0 \left( 1 - \varepsilon \left( \frac{y + \frac{V^2}{\Omega_0}}{\Lambda_n} \right) \right) \times \frac{V_{\parallel}^{2J}}{\pi^{3/2} V_{\perp}^J V_\perp} \exp \left( - \frac{V_{\parallel}^2}{V_{\perp}^2} + \frac{V_{\perp}^2}{V_{\perp}^J} \right) \quad \ldots (16) \]

\[
f_J(V_\perp) = \frac{V_{\parallel}^{2J}}{\pi V_{\perp}^{2J+1}} \exp \left( - \frac{V_{\parallel}^2}{V_{\perp}^2} \right) \]

\[
f_\parallel(V_\perp) = \frac{1}{\sqrt{\pi V_{\perp}^J}} \exp \left( - \frac{V_{\parallel}^2}{V_{\perp}^2} \right)
\]

where $\varepsilon$ is a small parameter of the order of inverse of ‘density gradient scale length’ and is zero for homogeneous plasmas, $J = 0, 1, 2, \ldots$ the distribution index, also known as the steepness of the loss-cone. For $J = 0$ this distribution represents a bi-Maxwellian distribution and for $J \to \infty$ this reduces to Dirac-Delta function \cite{23}. $V_{\parallel}^2 = 2T_\parallel/m$ and $V_{\perp}^J = (J+1)^J (2T_\perp/m)$ are the squares of parallel and transverse thermal velocities with respect to the external magnetic field. Index $J$ characterizes the width of the loss-cone. Moreover, $f_J(V_\perp)$ is peaked about $J^{1/2}V_\perp$ and has a half-width of $\Delta V_\perp \sim J^{-1/2}V_\perp$.

6 Dispersion relation

Applying the charge neutrality condition, $n_\parallel = n_\parallel^\ast$, where $n_\parallel^\ast$ are the integrated perturbed densities for the non-resonant particles, and using Eqs (13) and (16), we obtain

\[
n_\parallel = \left( \frac{1}{k_\perp d_{ie}^2} \right) E_\parallel \sin (k_\parallel r - \omega t) \quad \ldots (17) \]

\[
n_\parallel = -\frac{k_\perp^2 \omega^2}{(\omega - \omega_\parallel) - \ell \Omega} \frac{V_{\parallel}^2}{V_{\perp}^2} \left( J_{\perp}^2 \right) E_\parallel \sin (k_\parallel r - \omega t) \quad \ldots (18) \]

where $\omega_{pe}^2 = \frac{4\pi n_0 e^2}{m_e}$

$\omega_{pe} = \omega_{pe} (1 - \varepsilon)$ is the Doppler shift in wave frequency, $\varepsilon = \frac{\beta^2}{2a^2}$ the degree of inhomogeneity; $\omega_{pe} = \frac{k_\parallel E_0}{B_0}$

where $E_0$ is the transverse dc electric field.

Also $< J_{\perp}^2 > = \frac{2\pi}{6} \frac{\pi V_{\parallel}^J d_{ie}}{(J+1)^{3/2}} (\omega_{pe} \parallel) \quad \ldots (19)$

where $I_1 \left( \frac{k_\perp^2 \rho^2}{2} \right)$ is modified Bessel’s function.

The Debye length $d_{ie}$ corresponding to mean parallel energy is given by

\[
d_{ie} = \frac{T_{ie}}{\mu_\parallel} \quad \ldots (20) \]

Using the Poisson’s equation

\[
\nabla E = -k_\parallel (1 + \kappa^2) E_\parallel \sin (k_\parallel r - \omega t) = 4\pi e (n_\parallel - n_\parallel^\ast) \quad \ldots (21) \]

and perturbed ion and electron density $n_\parallel$ and $n_e$ the dispersion relation is obtained as

\[
1 + \left( \frac{1}{1 + \kappa^2} \right) \left( \frac{1}{k_\perp^2 d_{ie}^2} \right) - \left( \frac{\kappa^2}{1 + \kappa^2} \right) \left[ \frac{\omega_{pe}^2}{\omega_{pe}^2 - \ell \Omega^2} \right] \times \left( J_{\perp}^2 + J_{\parallel}^2 \right) = 0 \quad \ldots (22) \]

For $\ell = 1$, $< J_{\perp}^2 + J_{\parallel}^2 > = 1 - (J+1)\beta_1$.

$< J_{\perp}^2 > = 1 - \frac{1}{2} (J+1)\beta_1 \beta_1 = \frac{k_\perp^2 \rho^2}{2} \quad \ldots (23)$

For $J = 0$ and $E_{\parallel}(y) = 0$, this dispersion relation reduces to that given by Terashima \cite{24}.

7 Energy balance and growth rate

The wave energy density $W_w$ per unit wavelength is the sum of pure field energy and the changes in the energy of the non-resonant particles, i.e., $W_w = \frac{\lambda E_\parallel^2}{8\pi} + W_e + W_i$, which comes out to be
\[ W_i = \frac{\lambda E_i^2}{8\pi} \left[ \frac{\omega_i^*}{\omega_i^*} \right]^{1/2} \left( J_{\perp} + J_{\parallel} \right)^2 \]

Here, the ions contribution is dominant unless \( k_i^2 d_i^2 < 1 \).

The transverse energy and parallel energy of the resonant ions are calculated to be

\[ W_{\perp} = \frac{\lambda E_{\perp}^2}{8\pi} \left[ \frac{\omega_{\perp}}{\omega_{\perp}} \right]^{1/2} \left( J_{\perp} + J_{\parallel} \right)^2 \]

\[ W_{\parallel} = \frac{\lambda E_{\parallel}^2}{8\pi} \left[ \frac{\omega_{\parallel}}{\omega_{\parallel}} \right]^{1/2} \left( J_{\perp} + J_{\parallel} \right)^2 \]

where \( \omega_i^* = \omega - \Omega_i \) is the Doppler-shifted wave frequency

and \( R = \left( \frac{J_{\perp} + J_{\parallel}}{J_{\perp} + J_{\parallel}} \right)^2 \)

Using the law of conservation of energy

\[ \frac{d}{dt} (W_{\perp} + W_{\parallel}) = 0 \]

The growth rate is derived as

\[ \gamma = \frac{1}{\Omega_i} \left( \frac{dE_i}{dt} \right) = -\frac{dW_r}{dt} \left/ \frac{2W_r} \right. \]

Hence the growth rate defined in Eq. (29) is given by

\[ \gamma \approx \sqrt{2} \left( \frac{\omega_i^*}{\omega_i^*} \right) \left( 1 - \frac{\ell \Omega_i}{\omega_i^*} \right)^2 \]

where \( \ell = 1,2,3, \ldots \) is to be substituted and \( R \) is given by Eq. (27). Here it is noted that the distribution index \( J \) and the transverse inhomogeneous electric field \( E(y) \) affects the growth rate. For \( J = 0 \) and \( E(y) = 0 \), the result is the same as derived by Terashima.

8 Marginal instability

For the marginal instable condition \( \gamma = 0 \), we then arrive at the result

\[ \omega_{\perp} = \omega - \ell \Omega_i \left[ 1 - \left( \frac{T_{\parallel}}{T_{\perp}} \right) \right] \]

which shows that transverse inhomogeneous electric field may be a source of EIC wave generation besides the temperature anisotropy and the steep loss-cone. When both \( T_{\parallel}/T_{\perp} \) and \( R \) are greater than unity, wave generation by transverse inhomogeneous electric field is possible. Marginal instability condition also signifies that transverse inhomogeneous electric field lowers the threshold condition for the EIC wave generation, depending on the direction of \( E_0 \).

9 Results and discussion

In the present analysis, the expressions for the dispersion relation, resonant energies and growth rate are derived in the presence of an inhomogeneous electric field and the steepness of the loss-cone. The
following parameters relevant to the auroral acceleration region\textsuperscript{1, 25, 26} are used to evaluate the dispersion relation, resonant energies and growth rate:

$$B_0 = 4300 \text{ nT} \at 1.4 R_E, \quad \Omega_i = 412 \text{ s}^{-1}, \quad \ell = 1, \quad \lambda = 300 \text{ m},$$

$$E_i = 50 \text{ mV/m}, \quad \omega_{pi}^2 \Omega_i^2 = 2, \quad b = 0.1, \quad k_j = 0.002 \text{ m}^{-1}, \quad k_i = 0.0025 \text{ m}^{-1}, \quad T_i/m_i = 25 \text{ and } \Omega_i = 10$$

The effect of $E_0$ and $J$, keeping $\partial$ constant, on the growth rate and the resonant energies transferred is depicted in Figs 1-3. Figure 1 depicts the variation of growth rate ($\gamma/\omega$) with the wave frequency ($\omega/\Omega_i$) for different values of transverse electric field ($E_0$) and distribution index ($J$), keeping the inhomogeneity as constant for the first harmonic of the ion-cyclotron wave. It is observed that the effect of increasing transverse electric field is to reduce the growth rate, which may be due to the shifting of resonance condition. The value of $\gamma/\omega$ increases with $E_0$ and the frequency is shifted towards lower values and hence the growth rate decreases with $E_0$. The effect of higher $J$ is to enhance the growth rate. Thus, the mirror-like structure of the magnetosphere with a steep value of $J$ may be unstable for the EIC wave emission. It is also observed that the growth rate decreases with the increasing values of $\omega/\Omega_i$, which may be due to the shifting of the resonance condition. Hence the wave energy is being transferred to the particles.

Figure 2 shows the variation of transverse resonant energy ($W_{\perp}$), in joules, with $\omega/\Omega_i$ for different values of $E_0$ and $J$. It is observed that the effect of increasing $E_0$ is to increase the $W_{\perp}$, which may be due to the fact that the perpendicular electric field shifts the resonant criteria due to its effect involved in the modification of resonance condition. The effect of increasing $J$ is to decrease the value of $W_{\perp}$. Thus, the steep loss-cone distribution of the magnetosphere stabilizes the $W_{\perp}$. It is also observed that $W_{\perp}$ increases with the increasing values of $\omega/\Omega_i$. The increase in heating of the particles by the $E_0$ is supported by the decrease in growth rate, as the wave energy is being transferred to the particles by the resonance interaction process.

Figure 3 depicts the variation of parallel resonant energy ($W_{\parallel}$), in joules, with $\omega/\Omega_i$ for different values of $E_0$ and $J$. Here it is observed that $W_{\parallel}$ decreases when $\omega < \Omega_i$, becomes minimum for $\Omega_i - \omega$, and then for $\omega > \Omega_i$, $W_{\parallel}$ increases. Thus, the wave energy is being transferred to the parallel resonating particles only for $\omega > \Omega_i$. For $\omega > \Omega_i$, effect of $E_0$ is to increase the $W_{\parallel}$. Thus, the heating of resonant ions parallel to magnetic field may be enhanced by the $E_0$. The effect of increasing values of $J$ is to decrease the $W_{\parallel}$ but $J$ is not very effective for $\omega \sim \Omega_i$. The steep loss-cone distribution of the magnetosphere stabilizes the $W_{\parallel}$ of the EIC wave.

The effect of electric field inhomogeneity ($\partial$) and $J$ on the growth rate and the energy transferred for the
first harmonic of the wave is depicted in Figs 4-6. Figure 4 shows the variation of growth rate with wave frequency for different values of \( \vartheta \) and \( J \). It is observed that the growth rate increases with the increase in the inhomogeneity, which may also be due to the shifting of resonance condition. The effect of inhomogeneous electric field appears due to the difference in the \( \mathbf{E} \times \mathbf{B} \) motion of the ions and the electrons. The average electric field experienced by the ions differs from that experienced by the electrons because of their different Larmor radii. Thus, due to the \( \mathbf{E} \times \mathbf{B} \) drift of the particles the wave frequency gets a Doppler-shift by \( \omega_E \). It is also observed and is evident from the equation \( \omega_E = \omega_H (1-\vartheta) \) that \( E_0 \)
Fig. 4—Variation of $\gamma/\omega$ with $\omega/\Omega_1$ for different values of $\vartheta$ and $J$ for $E_0 = 20$ mV/m [Series-1: $J = 0$, $\vartheta = 0.9$; Series-2: $J = 0$, $\vartheta = 0.5$; Series-3: $J = 0$, $\vartheta = 0.1$; Series-4: $J = 2$, $\vartheta = 0.9$; Series-5: $J = 2$, $\vartheta = 0.5$; Series-6: $J = 2$, $\vartheta = 0.1$; Series-7: $J = 4$, $\vartheta = 0.9$; Series-8: $J = 4$, $\vartheta = 0.5$; Series-9: $J = 4$, $\vartheta = 0.1$]

reverses its direction for $\vartheta \geq 1$. The effect of $J$ is to increase the growth rate as is discussed earlier. Fig. 5 shows the variation of $W_{rL}$ (in joules) with wave frequency for different degrees of inhomogeneity ($\vartheta$) and $J$. The effect of $\vartheta$ and $J$ is to reduce $W_{rL}$. From Fig. 6 it is observed that $\vartheta$ and $J$ show reducing effect on $W_{rL}$ (in joules) as well. The reduction in transverse and parallel energy by the inhomogeneous electric field is supported by the increase in the growth rate, as the particle energy is being transferred to the wave by the resonance interaction processes. Thus, the wave may be generated by extracting energy from the resonant particles in the presence of the inhomogeneous electric field.
Thus, the transverse inhomogeneous electric field acts as a source of free energy for the EIC waves. It modifies the wave-particle resonance condition and leads to weakening of Landau damping effects and enhances the growth rate$^{9,13,16,29}$. The Doppler shift in frequency by $\omega_{0k}$ results in different longitudinal phase velocities, which improves the ability of the plasma to meet the Landau resonance condition through dissipative (resonance) effects$^{13}$. This leads to broadening of the spectrum and reduction in the threshold current of EIC instability. As the magnitude of the transverse inhomogeneous electric field increases, the effect becomes more drastic, as it can now create a localized region of negative energy density. Convection of energy away from the region of the negative energy density can sustain a wave growth by means of reactive (non-resonant) processes.

The effect of distribution index is also to increase the growth rate of the wave. The destabilizing effects due to the steep loss-cone on different instabilities have also been reported by various workers$^{20,25}$. The steep loss-cone structures are analogous to mirror-like devices with higher mirror ratio that may accelerate the charged particles moving perpendicular to the magnetic field. Thus, more energetic particles may be available to provide energy to the wave by wave-particle interaction.

The EIC waves are often detected in the inverted-V structures of the auroral acceleration region$^{9,31}$. The coherent ion-cyclotron waves in association with the perpendicular electric field up to over 50 mV/m have been observed recently$^5$ by a Polar satellite$^9$. A new structure consisting of spiky electric and magnetic fields with individual spikes of durations of about 100 $\mu$s to 2 ms and repetition rates of the order of 100 Hz, the local hydrogen ion frequency, at least for the slower structures, has also been observed by the Polar satellite. The naturally occurring electric fields as large as 1500 mV/m have also been observed that produce ion-cyclotron waves of 0.5 s duration. The perpendicular component of the electric field exhibits coherent, non-linear oscillations at a frequency that is only a fraction of the local ion-cyclotron frequency$^5$ of 105 Hz.

The results obtained here have a direct bearing on the interpretation of space observations in the auroral acceleration region. Observations from S3-3 satellite$^{9,31}$ have shown that large transverse electric fields exist on auroral field lines at altitudes from 1000 km to satellite apogee of 8000 km and are inferred to as electrostatic shocks. Recently, Freja satellite$^{32}$ has also identified large-amplitude electrostatic shock-like electric field structures associated with black aura$^{13}$.

The EIC waves in the presence of the inhomogeneous electric field have lower threshold current and are less sensitive to increasing ion temperature$^{13}$. Hence, these can be excited and heat the ions effectively for smaller field-aligned drifts.
which are typical for lower altitudes. In the case of large transverse inhomogeneous electric fields, the reactively driven (non-resonant) ion-cyclotron instability can be triggered and lead to ion heating. Since the instability is attributed to an inhomogeneity in energy density, it is referred to as inhomogeneous energy density-driven instability (IEDD)\textsuperscript{13,23}.

Recent observations by the Freja satellite\textsuperscript{35} indicate that the localized transverse electric fields give rise to broad band extremely low frequency (BB-ELF) wave instabilities in the magnetosphere. BB-ELF emissions are extremely low frequency electric and magnetic field fluctuations observed\textsuperscript{36} in the range 1 Hz-3 kHz. These emissions have been detected within regions of auroral inverted-V electron precipitation at a few 1000 km altitude as well as in the magnetospheric tail at several earth-radii, in the magnetospheric day side cusp/cleft and in the topside auroral ionosphere from altitudes\textsuperscript{36} of a few 100 km to a couple of 1000 km. Gyro resonant heating by these waves around the gyro-frequency also gives rise to intense events of transverse acceleration of ions\textsuperscript{37}. At least at altitudes from 1000 km up to several 1000 km, most of the ion energisation is associated with BB-ELF waves\textsuperscript{35}.

The effects of the EIC waves on the plasma transport properties in the auroral acceleration region are known to be quite significant\textsuperscript{38}. These effects may be more extensive in the presence of transverse inhomogeneous electric field, since in this case ion-cyclotron waves are not so quickly quenched by ion heating\textsuperscript{39}.

Recent observation of low frequency, oblique ion acoustic-like waves in the auroral ionosphere and in association with interplanetary shocks and bow shocks by the FAST and Freja satellite can also be explained by transverse inhomogeneous electric field\textsuperscript{39}. These low frequency waves in fact are the ion-cyclotron wave modified by the transverse inhomogeneous electric field, which can be sustained in the realistic ionosphere conditions, where the temperature ratio is of the order of unity and the magnitude of the field-aligned drift is low\textsuperscript{40}. The transverse electric field needed is relatively small and is frequently observed in the ionosphere.

The EIC turbulence plays an important role in the loss-cone current-potential relationship. It leads to spatial variations in the double layer potential and thereby produces thin auroral arcs embedded in inverted-V precipitation\textsuperscript{30,31}. It has also been suggested that the loss-cone effect can enhance the anomalous resistivity for a given turbulence level. Since the steep loss-cone distribution in the presence of EIC wave and the transverse inhomogeneous electric field enhances the growth rate, the anomalous resistivity and transport resulting from this instability is likely to play a crucial role in the auroral acceleration region. The equilibrium dipolar magnetic field of the earth is curved in the meridional plane and may introduce loss-cone effects in the particle distribution function\textsuperscript{21}. Thus, the behaviour studied for the EIC wave may be of importance in the electrostatic emission in the auroral acceleration region.

**10 Conclusions**

The EIC instability is investigated in the presence of transverse inhomogeneous electric field. The loss-cone distribution, using particle aspect analysis, is considered suitable to the auroral acceleration region. The findings of the investigations are:

The transverse electric field ($E_0$) has the stabilizing effect on the growth rate, but it increases the transverse energy of the ions. Thus, perpendicular heating of the ions is the consequence of $E_0$ only. The acceleration of ions along the magnetic field depends on the frequency of the EIC wave. The effect of electric field is to enhance the parallel acceleration of ions in the region $\omega > \Omega_c$.

The effect of electric field inhomogeneity ($\vec{\phi}$) is to enhance the growth rate, reducing the parallel and perpendicular heating of ions. Thus, inhomogeneous electric field may excite the EIC wave in the auroral acceleration region.

The distribution index ($J$) enhances the growth rate. Thus, the wave may be generated due to the converging magnetic field lines of the auroral acceleration region. The effect of $J$ is to decrease the transverse and parallel energy of the ions.

The observations of EIC wave activities in the regions of transverse inhomogeneous electric fields are explained as the inhomogeneity acts as the source of free energy to generate the EIC wave. The EIC emissions and heating of ions in loss-cone distribution are examined suitable to auroral acceleration observations.

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