Investigation of scattering characteristics for a conducting sphere moving at high velocity

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The scattered field of a conducting sphere moving at high velocity is investigated, when the incident wave is of arbitrary frequency. First the transformation of coordinates, transformation of vectors and the electromagnetic transformation are derived in detail, when one system of coordinates moves to another one along their Z-axis. The transformation of trigonometric function and that of the components of vector in spherical coordinates are presented for the first time. These transformations establish a theoretic basis for studying the scattered field by a target moving in high velocity. Utilizing the reference, the scattering characteristics of a conducting sphere moving in high velocity are investigated in detail. The expression of scattering section is obtained and simulations of that are presented. Results show that the forward scattered field is larger than that of the backward, as the sphere moves at a high velocity. The forward and backward scattered fields are not much influenced by the operating frequency and the Doppler effect plays the main role. The second radiation on the surface of the target is mainly electric dipole radiation.

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1 Introduction
In recent years, development of space aircraft is moving in the direction of supersonic speed, high flexibility, high temperature tolerance, high secretiveness, high obdurability, and high accuracy in control and guidance systems. These require break through both in relevant basic theories and source of the experiment. Chinese National Natural Science Foundation have determined structure characteristic of scattering field and control method of space targets moving at high speed, as the great basic questions in its program both in 2003 and 2004. Obviously, research on scattering structure for a macro object moving at a high velocity might assume great importance theoretically, as also for project application.

It is well known that both the electromagnetic radiation and scattering characteristic of the micro particles of high-speed has already been studied in detail and applied to real problems. Literature on study of electromagnetic scattering characteristics about a conducting sphere at high-speed is actually rare. On the other hand, an isotropic conductor target moving at high speed turns into an anisotropic ellipsoid. Thus to obtain an analytic expression for a moving target a theoretical foundation may be required for studying the property of scattering field from an anisotropic ellipsoid.

This paper first presents a study of the relation of trigonometric functions between two coordinate systems moving at high velocity with respect to each other. The scattered field of a moving conducting sphere is then investigated in detail. Finally a study of the simulations and a particular analysis are presented.

2 Transformation of physical quantities in the spherical coordinate system
As shown in Fig. 1, we suppose that a system $\Sigma'$ is moving with speed $v$ with respect to the system $\Sigma$ along its z-axis. For the sake of simplicity, we use the symbol $'$ to denote various parameters in $\Sigma'$ system and that no mark denotes corresponding parameters in $\Sigma$ system. The relations between the rectangular system and the spherical system of coordinates are respectively written as

$$
\begin{align}
    x &= r \sin \theta \cos \phi \\
    y &= r \sin \theta \sin \phi \\
    z &= r \cos \phi
\end{align}
$$

and

$$
\begin{align}
    x' &= r' \sin \theta' \cos \phi' \\
    y' &= r' \sin \theta' \sin \phi' \\
    z' &= r' \cos \phi'
\end{align}
$$

... (1)
It is easy to prove that the relations between the two systems of rectangular coordinates are the following:

\[
\begin{align*}
x' &= x, \\
y' &= y \\
z' &= \gamma(z - vt) \\
t' &= \gamma \left( t - \frac{vc}{c^2} \right)
\end{align*}
\]  

The four-dimensional form of Eq. (2a) may be written as:

\[
\begin{bmatrix}
x' \\
y' \\
z' \\
t'
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \gamma & \beta \gamma \\
0 & 0 & -\beta \gamma & \gamma
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z \\
t
\end{bmatrix}
\]  

Combining Eqs (1) and (2a) and measuring the distance \( z \) at the same time \( t \), namely \( z' = \gamma z \), one can easily obtain

\[
\begin{align*}
\cos \theta' &= \frac{\gamma \cos \theta}{\left(1 + \beta^2 \gamma^2 \cos^2 \theta \right)^{1/2}} \\
\sin \theta' &= \frac{\sin \theta}{\left(1 + \beta^2 \gamma^2 \cos^2 \theta \right)^{1/2}} \\
\sin \phi' &= \sin \phi, \cos \phi' = \cos \phi
\end{align*}
\]

Equation (3) presents the relations for the angle function in the two spherical coordinate systems and \( r \) is the distance from the origin of coordinate system to the observing point. Obviously, the trigonometric functions in two spherical coordinate systems are the same, as the velocity approaches zero. The trigonometric function of angle \( \phi \) is not dependent on speed. In the plane vertical to the moving direction, there is no scale flex due to the fact that system moves along \( z \)-axis. The relation for the four-dimensional wave vector is obtained from Eq. (2b).

\[
k'_x = k_x, k'_y = k_y, k'_z = \gamma \left( k_z - \frac{v}{c} \omega \right) \\
\omega' = \gamma (\omega - vk_z)
\]

The electromagnetic field results in a two-step tensor in the four-dimensional space. The relation for the tensor can be written as:

\[
\begin{bmatrix}
0 & B_z & -B_y & -\frac{i}{c}E_z \\
-B_z & 0 & B_y & \frac{i}{c}E_z \\
B_y & -B_x & 0 & \frac{i}{c}E_z \\
-\frac{i}{c}E_z & -\frac{i}{c}E_y & \frac{i}{c}E_x & 0
\end{bmatrix}
\]

Therefore we can get the relations for electromagnetic fields as:

\[
\begin{align*}
E'_x &= \gamma (E_x - vB_z) \\
E'_y &= \gamma (E_y + vB_z), E'_z = E_z \\
B'_x &= \gamma \left( B_x + \frac{v}{c^2}E_z \right) \\
B'_y &= \gamma \left( B_y - \frac{v}{c^2}E_z \right), B'_z = B_z
\end{align*}
\]

Putting the inverse transformation of Eq. (5) to a matrix form, we get:

\[
\begin{bmatrix}
E_x \\
E_y \\
E_z
\end{bmatrix} =
\begin{bmatrix}
\gamma & 0 & 0 \\
0 & \gamma & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
E'_x \\
E'_y \\
E'_z
\end{bmatrix}
+ \frac{1}{\gamma^2}
\begin{bmatrix}
0 & \gamma & 0 \\
\gamma & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
B'_x \\
B'_y \\
B'_z
\end{bmatrix}
\]
We write the above in the vector form for simplicity, as

\[ \mathbf{E} = \mathbf{T} \mathbf{E}' + \mathbf{T} \mathbf{B} \]  

It is supposed that the relation between vector quantities representing the spherical coordinate system and the rectangle coordinate system is the following:

\[ \mathbf{E} = \mathbf{P} \mathbf{E}', \mathbf{B} = \mathbf{P} \mathbf{B}' \]

After placing the above equations into Eq. (6), we obtain

\[ \mathbf{E}_s = (\mathbf{P}^* \mathbf{T} \mathbf{P}') \mathbf{E}' + (\mathbf{P}^* \mathbf{T} \mathbf{P}') \mathbf{B}' = \mathbf{M} \mathbf{E}' + \mathbf{N} \mathbf{B}', \]  

where

\[ \mathbf{M} = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}, \mathbf{N} = \begin{pmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \end{pmatrix} \]

The elements for matrix \( \mathbf{M} \) are

\[ m_{11} = \gamma \sin \theta \sin \theta' + \cos \theta \cos \theta' \]
\[ m_{12} = \gamma \sin \theta \cos \theta' - \cos \theta \sin \theta' \]
\[ m_{13} = \gamma \cos \theta \sin \theta' + \sin \theta \cos \theta' \]
\[ m_{21} = 0, m_{22} = 0, m_{23} = \gamma \]
\[ m_{31} = 0, m_{32} = 0, m_{33} = \gamma \]

For matrix \( \mathbf{N} \), the elements are

\[ n_{11} = 0, n_{12} = 0 \]
\[ n_{13} = \gamma \nu (\cos \theta \cos \phi - \sin \theta \sin \phi) \]
\[ n_{21} = 0, n_{22} = 0, n_{23} = \gamma \nu \cos \theta \]
\[ n_{31} = -\gamma \sin \theta', n_{32} = -\gamma \cos \theta', n_{33} = 0 \]

Obviously, when the speed is zero, \( \mathbf{M} \) retrogresses into the unit matrix. This conclusion has stated its exactness. Equations (2)-(5) and (7) are the basis for research of the scattered fields of the high speed sphere moving along \( z \)-direction.

3 Scattering field of moving conducting sphere

3.1 Introduction of conducting immobile sphere scatter theory

Several researches\(^4\) have studied the scattered field of a conductor sphere in the situation that the incident wave propagates along \( z \)-axis, polarizes in \( x \)-direction and the sphere lies at the origin. The scattering field can be given by the relations

\[ E_{\theta s} = \frac{j E_0 e^{-j k r'}}{k r'} \cos \phi' s_2 (\theta') \]
\[ E_{\phi s} = \frac{-j E_0 e^{-j k r'}}{k r'} \sin \phi' s_1 (\theta') \]

\[ s_2 (\theta') = \sum_{n=1}^{\infty} a_n \left[ c_n \tau_n + d_n \tau_n' \right] \]
\[ s_1 (\theta') = \sum_{n=1}^{\infty} a_n \left[ \tau_n \tau_n' + c_n \tau_n \right] \]  

\[ \tau_n = \frac{dP_n (\cos \theta')}{d\theta'}, \pi_n = \frac{P_n (\cos \theta')}{\sin \theta'} \]

The other formulae in Eq. (8) can be obtained in the literature. The upper mark (') denotes that we are in system \( \Sigma' \).

3.2 Scattering field of moving conducting sphere

Supposing a medium sphere lies in the origin of system \( \Sigma' \), with \( a \) as its radii, the incident wave polarizes along \( x \)-direction and propagates along \( z \)-direction, namely \( E_x = e^{j k z} \). We then can obtain from Eqs (4) and (5) for the wave in system \( \Sigma' \)

\[ E'_s = \gamma (1 - \beta) e^{-j k z'} \]
\[ E'_s = E'_0 e^{j k z'} \]  

Equation (9) shows that measurement made in \( \Sigma' \) has not changed any more due to polarization. Just its amplitude, wave number and frequency have changed. Thus the scattering field in system \( \Sigma' \) can be obtained as

\[ E_{\theta s}' = \frac{j E_0 e^{-j (k r' - \phi')}}{k r'} \cos \phi' s_2 (\theta') \]
\[ E_{\phi s}' = \frac{-j E_0 e^{-j (k r' - \phi')}}{k r'} \sin \phi' s_1 (\theta') \]

\[ B'_0 = \frac{E'_{\theta s}}{c}, B'_0 = \frac{E'_{\phi s}}{c} \]

\[ r' = \sqrt{r^2 + \gamma^2 \left( \frac{r \cdot r}{c^2} \right)} \]

From Eq. (7), we are able to obtain the scattering field in system \( \Sigma \) as
After the necessary mathematical operation it is concluded that

\[
E_{o}\ = \ m_1 E_{y} + m_2 E'_{x} + n_1 B_{y} + n_2 B'_{x},
\]

\[
E_{o} = m_2 E_{y} + m_1 E'_{x} + n_1 B_{y} + n_2 B'_{x},
\]

It is reasonable to suppose that the corresponding components are equal as the velocity approach near to zero [from Eq. (11)]. This result shows the exactness of the former derivations. Because the weights of the electric field are the complicated functions of the angles, obtaining the scattering section of the target will be difficult. The differential scattered section (DSS) is defined as

\[
\sigma_{\phi} = \lim_{r \to \infty} \frac{r^2 \sigma}{s}
\]

Putting Eq. (11) and the corresponding magnetic field in the above equation, one can get

\[
\sigma_{\phi} = (1-\beta)^2 \left[ \frac{\cos^2 S_{f} (\theta) (\gamma \cos \theta \cos \phi + \beta \cos \theta + \sin \theta \sin \phi)^2}{2\eta (1+\gamma \beta \cos^2 \theta) \left( \frac{k - \beta}{c} \right)^2} \right] + \left[ \frac{\sin^2 S_{f} (\theta) \gamma^2 (1+\beta \cos \theta)^2}{2\eta (1+\gamma \beta \cos^2 \theta) \left( \frac{k - \beta}{c} \right)^2} \right] \]

Equation (12) is the DSS as the sphere moves at high speed. The scattered section can be obtained by the integration of Eq. (12) using a computer.

### 3.3 Numerical results

In the following figures, the radius of the conductor ball is 3 m, which has the same magnitude as the wavelength in order to have more obvious scattering effects. The DSS variation versus the angle \( \phi \) is shown in Fig. 2 as the target's velocity is at a constant angle \( \phi = \pi/25 \). It shows that the forward scattering is greater than backward scattering. The scattering reaches its minimum when \( \theta = \pi/2 \). We can infer that the scattering field is mainly induced by electric dipole on the surface of the moving sphere. The frequency has not got much influence in the forward scattering, as shown in Fig. 3. However, the velocity has a greater effect on it. This is because the frequency received increases with the increase of velocity. It is inferred that the backward scattering is weak since the frequency received is lower, which is shown in Fig. 4. The data plotted along the coordinate axis of Fig. 5 is smaller than that in other figures, and this is in agreement with that in Fig. 2. The scattering in Fig. 5 reaches its maximum when \( \phi = \pi/2 \). This is comprehensible, since the incident wave is polarized in \( \chi \)-direction and the electric dipole radiates strongly in the direction vertical to itself.
moving at high speed is investigated in detail. The analytic expression of the formula for DSS has been obtained and its discussion presented. Simulation shows that the forward scattering is greater than the backward scattering. The operating frequency is not much affected in the scattering field compared with other observing points, and the Doppler effect plays the main role. The second radiation on the surface of the target is mainly of electric dipole radiation.

The work presented in this paper establishes a base for studying the scattering characteristics and resonance property, etc. of a space target moving at high velocity. It makes an attempt at a theoretical study for obtaining the analytic expression of the formula of scattering field for an anisotropic ellipsoid. This work can also be used to investigate the radiating effect of a target moving at high speed. But the work has not considered the scattered characteristic of an object, when the incident wave is at an arbitrary angle. This problem can be solved by circumvolving the coordinates axes, and it will be a complex research problem, which can be taken up as a next step.

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