

Fractal character of oceanic crustal magnetism determined from drill hole measurements

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A wide range of geophysical processes and rock properties has been described in fractal or scaling terms. For continental crust, well log susceptibilities, surface susceptibilities and aeromagnetic fields all tend to support a model for a 3-D magnetization distribution having a radially-averaged power spectrum proportional to some power of the spatial frequency. This simple model of the scale-invariant behaviour of crustal magnetization and the magnetic fields it produces can be exploited by several applications which require information on such spatial variation. A more realistic power spectrum, and equivalently, covariance model for continental crustal magnetization offers many advantages over the geologically incorrect assumption of a white power spectrum (equivalent to an uncorrelated distribution). Well log susceptibilities and natural remanent magnetization intensities measured for oceanic crust are shown here to exhibit scaling behaviour. Measurements from Ocean Drilling Program holes 504B, 735B, 801C and holes CY1, CY4 in the Troodos ophiolite sequence in Cyprus show overall values for the scaling exponent, α , between -1.36 and -0.68 for susceptibilities and between -1.52 and -0.54 for natural remanent magnetization intensities. Based on this small number of samples, scaling exponents determined for basalt, sheeted dyke and gabbro sequences within these logs show wide variation, indicating no apparent correlation between rock type and scaling behaviour.

[**Key words:** Fractal, magnetic, oceanic crust, drill hole]

Introduction

A wide range of properties of the solid earth have now been discussed in fractal terms¹⁻³ suggesting that ideas from fractal geometry can be exploited in describing geological structure. Seafloor and mountain topography show fractal behaviour^{4,5}. The distribution of fractures from microfractures to large crustal faults has been successfully described in fractal terms based on observations from well logs⁶, outcrop⁷ and the scattering of seismic waves⁸. Sammis & Biegel⁹ demonstrated that fragmented material within fractures also have a fractal distribution.

In order to describe geology, one class of random fractals, the Gaussian scaling noises¹⁰ have proven useful. As the name suggests, a sample of a Gaussian scaling noise has a Gaussian probability distribution function: its histogram will have the normal shape with a characteristic mean (μ) and variance (σ^2). It also has a power spectrum (P) proportional to some power (α) of frequency (f). A Gaussian scaling noise is fully characterized by the three parameters μ , σ^2 , and α . This economy of parameterization is an attractive feature when dealing with limited and noisy data.

Scaling noises have been used to characterize lithological parameters such as reflection sequences derived from acoustic well logs¹¹. Resistivity, porosity, density and natural logs are also found to be scaling^{12,13}. Well logs give a 1-D sample of the 3-D Earth. Brown & Scholz¹⁴ showed that fault patterns in two dimensions are fractal. Tubman & Crane¹⁵ found similar scaling behaviour in both horizontal and vertical porosity logs. It seems likely that the 3-D structure of the Earth has scaling properties, however, there are no 3-D data sets giving direct measurements comparable to well logs. One alternative is to assume scaling behaviour for a parameter and consider the geophysical consequences. This is the approach of Frankel & Clayton¹⁶ who studied the transmission of compressional waves through a scaling crust.

Continental Crustal Magnetization

Gregotski *et al.*¹⁷ found an $\alpha \approx -3$ frequency dependency for power spectra of aeromagnetic surveys over the North American continent at various scales and sampling intervals. They interpreted this result as representing the power-law behaviour of the near-surface susceptibility (or equivalently magnetization, if all magnetization is induced)

distribution using prisms of infinite depth. However, any realistic crustal magnetization model should allow for variations in magnetic properties both horizontally and with depth. Pilkington & Todoeschuck¹⁸ and Maus & Dimri¹⁹ related 3-D magnetization distributions to the field they produced and showed that a scaling magnetization with (3-D) exponent produces a magnetic field (measured at the surface of the distribution) with (2-D) exponent $\alpha-1$. This prediction can be tested directly by examination of susceptibility logs from drill holes and of susceptibility of rock samples taken over a geographic area. One-dimensional samples of crustal magnetization derived from drill hole data, either through well-logs or core samples, support the scaling hypothesis^{18,20-24}. Figure 1 shows an example susceptibility log and its power spectrum. Two-dimensional sampling is also in agreement²⁵. In three dimensions, only inference from 2-D power spectra of measured magnetic fields is available to investigate whether scaling is present or not. Several studies of magnetic data from widely differing geologic terranes and from data sets ranging from tens up to thousands of kilometres have shown scaling magnetic field power spectra^{22,26,27}.

Fedi *et al.*²⁸ pointed out that magnetization distributions other than scaling can also produce a scaling magnetic field power spectrum. They note that uncorrelated distributions of blocks with uniform magnetization, i.e., piecewise correlated, are spectrally equivalent to scaling media characterized by a constant (negative) value of scaling exponent (α). However, the highly variable character of susceptibility measurements from drill holes and rock sample suites which span the range from metres to tens of kilometres suggests their purely “blocky” model is unrealistic. Nevertheless, since areas can be delineated from magnetic field maps that contain similar susceptibilities, and which may represent a single lithology, some blockiness is to be expected. This is due to measured fields not having the resolving power to define the small-scale character within a given lithology or below a certain length scale. Support for true scaling properties has been provided through analyses other than spectral methods. Susceptibility logs were analysed with the rescaled range method and shown to be scaling²¹ and Dolan *et al.*²⁹ used four different methods to consistently determine broad-band fractal scaling for several petrophysical logs. The notion of differing

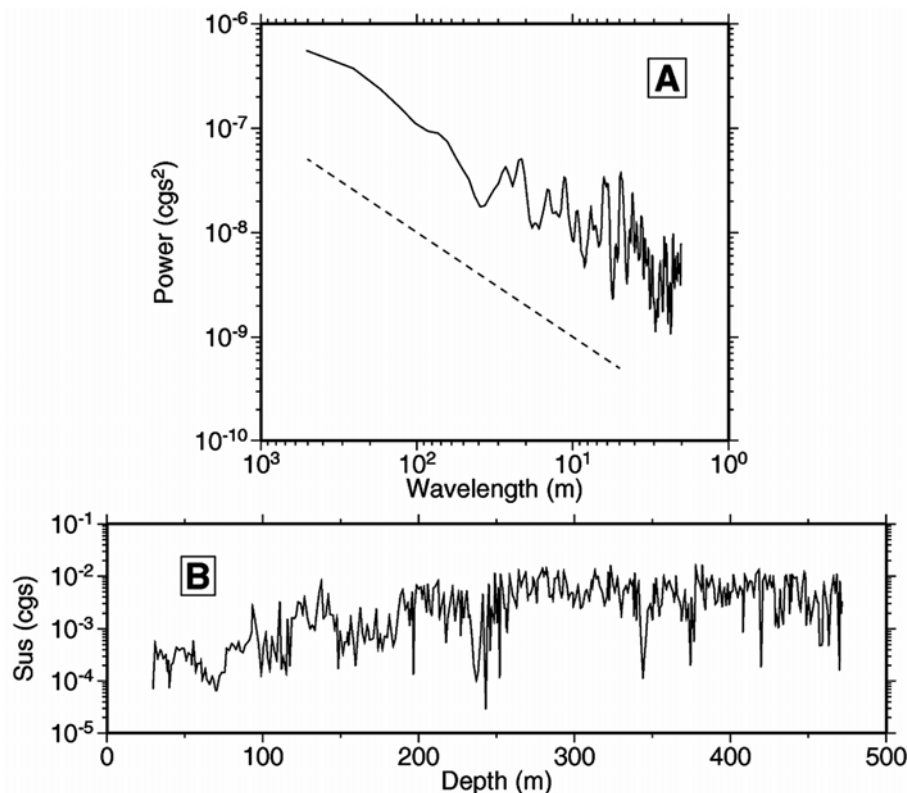


Fig. 1 — A) Power spectrum of Manicouagan susceptibilities (slope = -0.87). Dashed line with slope of -1 is shown for comparison. B) susceptibilities of drill core from the centre of the Manicouagan impact structure, Quebec, Canada.

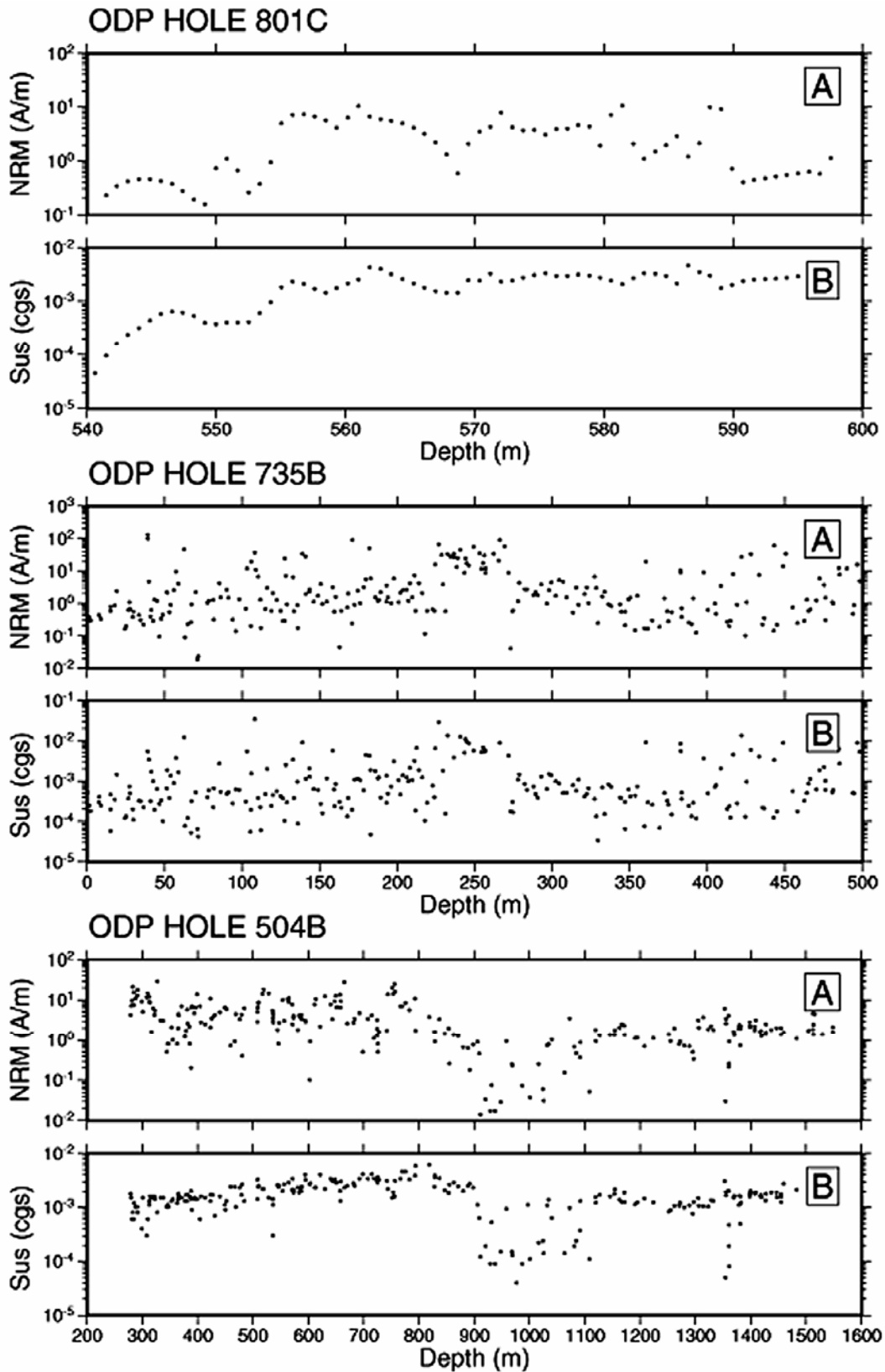


Fig. 2 — Drill-core measurements (A) remanent magnetization; (B) susceptibility) for Ocean Drilling Program holes 504B, 735B and 801C³¹⁻³⁵. Note change in horizontal scales.

descriptions of scaling properties and the possibility of varying scaling exponents has led naturally to multiscaling analyses of magnetic susceptibility and field measurements^{23,24,30}.

All of the power spectra, from fields, logs, and surface collections, are more or less well fitted in log-log coordinates by straight lines with consistent values of α . Of course, individual examples show departures in detail from this behaviour and could be fitted with more complicated functions. However, the limitations of the data may not justify such a step. Using a single straight line to describe the power spectrum of crustal susceptibility is certainly an oversimplification. Nevertheless, much of the common character in the data can be described by a single straight line, whose slope α , can be used to better describe the observed spatial variations in susceptibility.

Oceanic Crustal Magnetization

Most studies of magnetization and scaling, or fractals have addressed the continental case. It is, therefore, appropriate to compare these results with the situation in the oceans. As is well known, the dominant feature of the magnetic field over oceanic crust is a pattern of stripes parallel to a mid-ocean ridge and mirror reversed on each side of it. As newly formed crust cools, it acquires remanent magnetization from the main field, which varies in time in both strength and direction and at intervals reverses. The geology of the ocean floor is much less varied than that on land and because of the general dominance of remanent over induced magnetization it might be expected that susceptibility contrasts have a smaller role to play.

Because the horizontal behaviour of the field is largely governed by the two processes of crustal

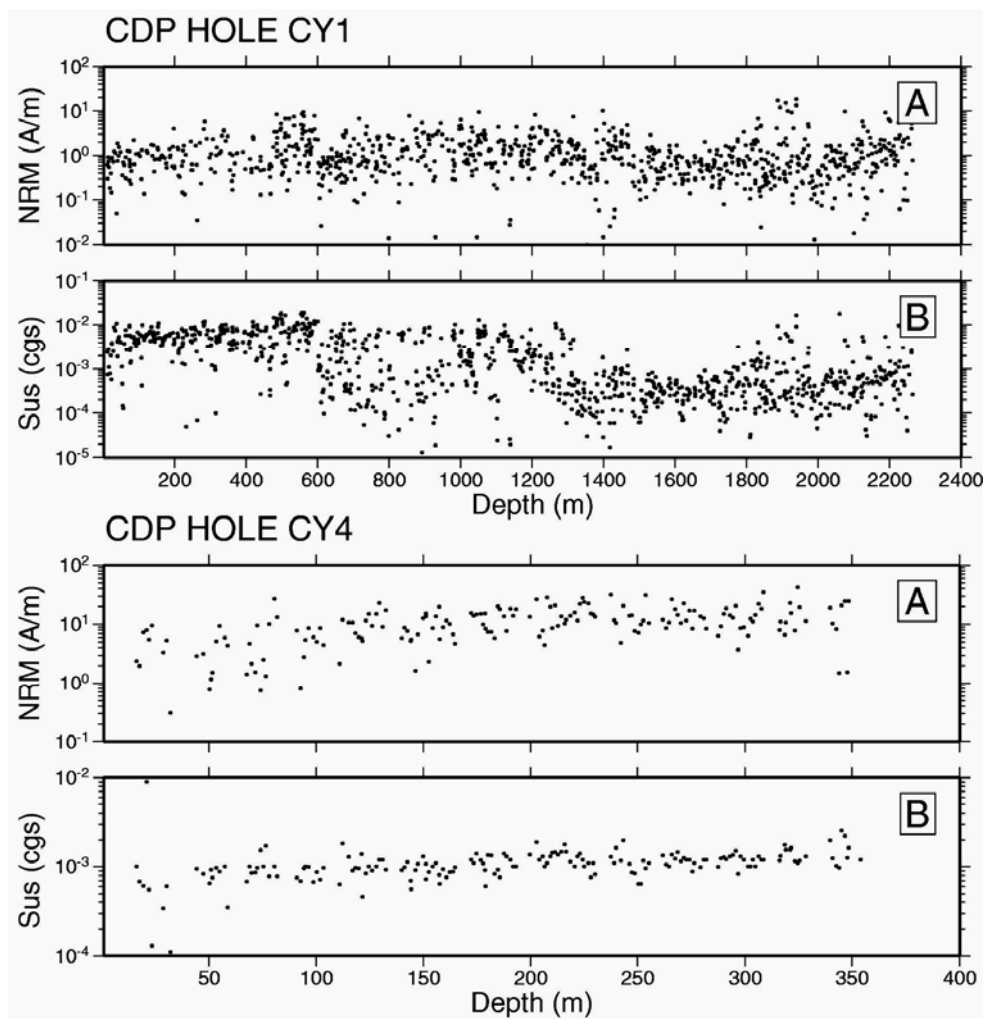


Fig. 3 — Drill-core measurements(A) remanent magnetization; B) susceptibility) for Cyprus Drilling Project holes CY1 and CY4³⁶⁻³⁸. Note change in horizontal scales.

creation and variation in the core field, the connection with the magnetic properties from borehole logs is less direct than may be the case for continental crust. Figure 2 shows the magnetic properties measured on core from three Ocean Drilling Program (ODP) sites that have sampled various parts of the oceanic crust. Hole 504B is located in the Costa Rica Basin of the western equatorial Pacific and at over 1500 m deep, penetrates the complete oceanic crustal section, from a 600-metre thick sequence of extrusive basalts, through a hydrothermally-altered zone and bottoming out in the sheeted dyke complex³¹⁻³³. During ODP Leg 118, Hole 735B was drilled for >500 m into gabbroic rock on top of a shallow platform on the eastern transverse ridge of the Atlantis II Fracture Zone, Southwest Indian Ridge³⁴. Drilling was intended to investigate formation of lower oceanic crust at a slowly spreading ridge. The hole penetrated a sequence of olivine gabbro, troctolite, gabbro, gabbro-norite, and Fe-Ti oxide gabbro with physical properties and seismic velocities appropriate to Layer 3 of oceanic crust. Hole 801C was drilled just west of the Mariana Trench, central Pacific, to sample old (Jurassic), fast-spreading oceanic crust and investigate the magnetic character of the basement in the Jurassic magnetic quiet zone³⁵. The basement units sampled are alkaline in character and are probably best interpreted as basaltic to doleritic sills. In addition to *in situ* oceanic crustal samples, drilling into oceanic crust preserved in the Troodos ophiolite, Cyprus, has provided a complete section through oceanic crust³⁶. Hole CY-1 samples >400 m of extrusives³⁷ while CY-4 extends to >2 km from the lower sheeted dyke complex into the cumulate mafic and ultramafic sequences³⁸. Figure 3 shows magnetic properties from the two Cyprus holes.

Power spectra of natural remanent magnetization (NRM) intensity and susceptibility from all holes are given in Fig. 4. All spectra are characterized by well-defined scaling behaviour in the range from metre to kilometre scale. Spectral slopes range from -0.68 to -1.36 for NRM intensity and -0.54 to -1.68 for susceptibility. Susceptibility values are similar to those observed for continental crust^{18,20,21}, which range from -0.2 to -2.1. No continental examples of NRM scaling exponents are known to this author. Holes 801 and CY1 sample the only the extrusive layer, while 735 consists of just gabbros. Since holes 504, 735 and CY4 sample more than one lithology, the magnetic measurements were divided on the basis of lithology and scaling exponents calculated.

Figures 5 and 6 show the resulting spectra with calculated least-squares slopes listed in Table 1. Unfortunately, the limited number of drill holes and the wide variation in α precludes any firm conclusions on the relation between lithology and scaling behaviour. There is some suggestion, however, that α for remanent magnetization varies much less than α for induced magnetization within a given lithology (particularly for dykes and gabbros). Lithological variations alone can be shown to produce scaling properties; nevertheless, certain physical parameters, e.g., resistivity, density, velocity, are very responsive to the presence of fractures, which

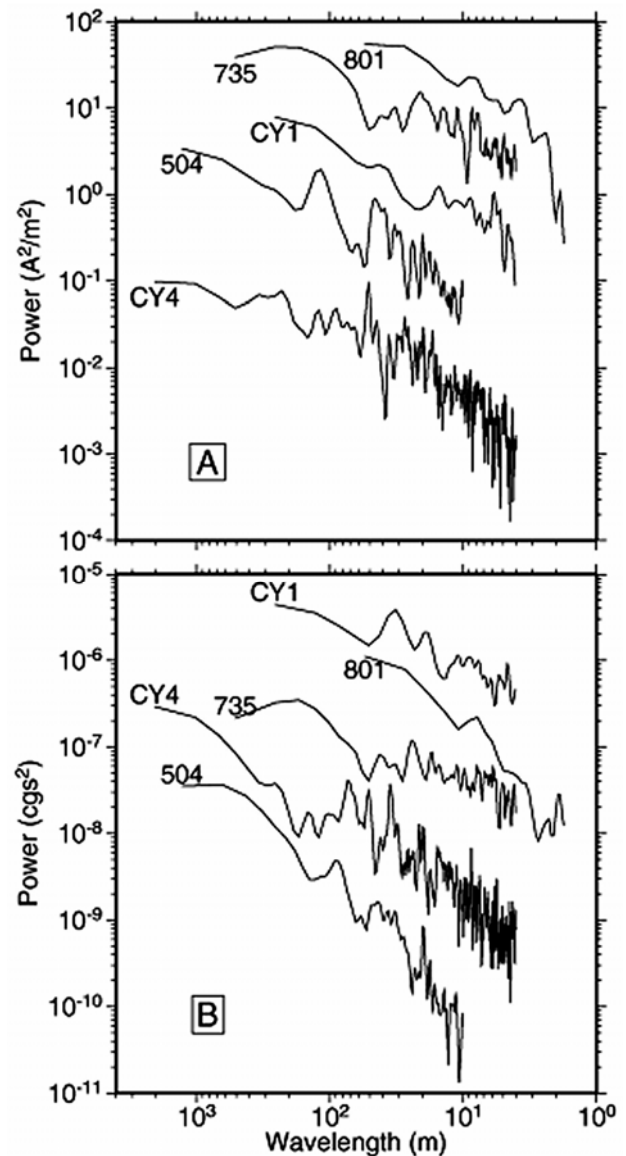


Fig. 4 — Power spectra for all holes: A) remanent magnetization and B) susceptibility. Spectra have been shifted vertically for clarity.

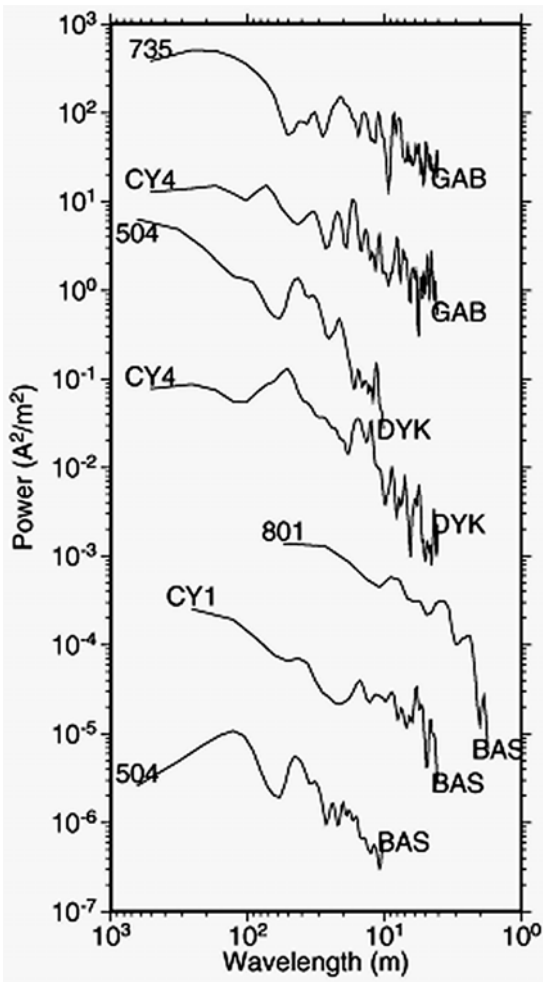


Fig. 5 — Power spectra of remanent magnetization for lithological sections: basalt (BAS: 504B; 801C; CY1), dykes (DYK: 504B; CY4), and gabbros (GAB: 735B; CY4). Spectra have been shifted vertically for clarity.

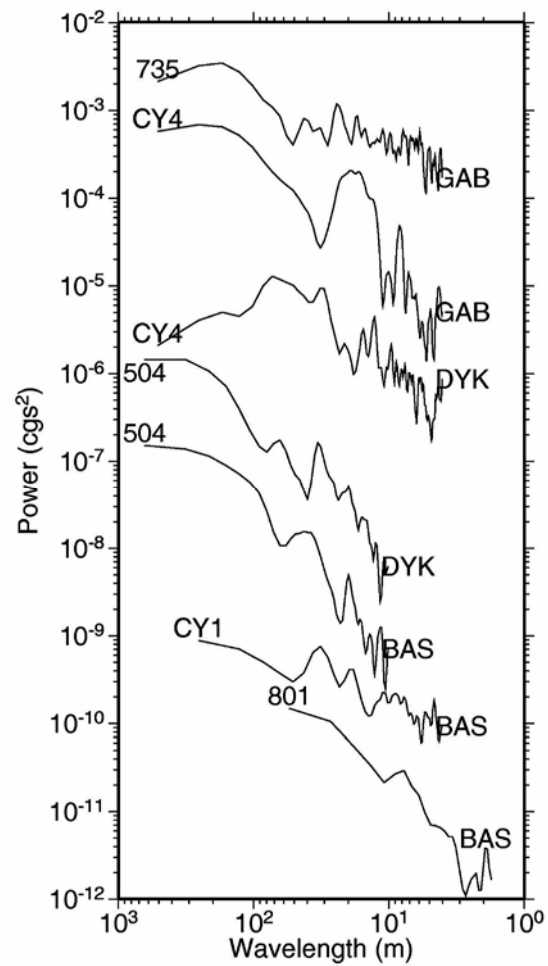


Fig. 6 — Power spectra of susceptibility for lithological sections: basalt (BAS: 504B; 801C; CY1), dykes (DYK: 504B; CY4), and gabbros (GAB: 735B; CY4). Spectra have been shifted vertically for clarity.

themselves show scaling characteristics [e.g., Barton⁷]. For oceanic crust, an additional variable that affects the spatial variability measured magnetizations is alteration. Low-temperature alteration reduces magnetizations within the extrusive layer as a function of age³³ while hydrothermal alteration can produce significant changes in magnetic properties within the sheeted dyke complex^{38,39}. Alteration may vary spatially and cross lithological boundaries, thus adding additional complexity to any relationship between lithology and scaling behaviour.

The practical uses of magnetization scaling exponent values have, so far, been limited to continental crustal applications. Estimating crystalline basement depths from the power spectrum of magnetic data and gridding randomly-spaced magnetic observations using kriging can both incorporate improved statistical descriptions of crustal

Table 1 — Power spectral slopes (scaling exponents) for lithologies in drill holes

Hole	Lithology	Magnetization	Slope
504B	Basalt	Remanent	-0.87
504B	Basalt	Induced	-1.75
801C	Basalt	Remanent	-1.36
801C	Basalt	Induced	-1.52
CY1	Basalt	Remanent	-0.79
CY1	Basalt	Induced	-0.62
504B	Dyke complex	Remanent	-1.3
504B	Dyke complex	Induced	-1.53
CY4	Dyke complex	Remanent	-1.32
CY4	Dyke complex	Induced	-0.89
735B	Gabbro	Remanent	-0.68
735B	Gabbro	Induced	-0.54
CY4	Gabbro	Remanent	-0.77
CY4	Gabbro	Induced	-1.46

magnetization through the use of a specified value of scaling exponent^{41,42}. For the oceanic case, observed α 's could provide additional constraints on crustal emplacement models⁴³ or thermal evolution models used in generating magnetized crusts⁴⁴. In either process, scaling behaviour of the resulting physical property being modelled, will apply to length scales well above and below those used in generating the crust. In addition to forward modelling, use of scaling exponents in the inversion of oceanic magnetic data is straightforward, e.g., Parker⁴⁵ incorporated a scaling magnetization description (however, not based on measured scaling behaviour) in an inversion for magnetization structure along a spreading ridge axis.

Conclusion

A more realistic power spectrum/autocovariance model for crustal magnetization offers advantages over the geologically incorrect assumption of white power spectrum or delta-function autocovariance. This is true whether or not scaling noises are used. However, examination of magnetization data from continental and oceanic crust leads to the conclusion that geological variations can be represented by a scaling noise. It would seem that explaining the origin of this scaling behaviour is a challenge for geophysics. Even though the fractal model provides a succinct description of the observed behaviour, it tells us little about the underlying physical processes. As summarized by Grant⁴⁶, 'The magnetic properties of a rock are determined not only by its original chemistry, but also by nearly everything that has happened to it since it was emplaced'. In view of this comment, it would appear that a simple physical model that explains the spatial variations of crustal magnetization is unattainable. It is, however, interesting to note that several geological processes that influence the pattern of magnetization within a given crustal volume are, themselves, describable in fractal terms.

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