Evaluation of Uncertainty of Measurement in Calibration of Electrostatic Voltmeter — A Case Study

Shiv Kumar Jaiswal*, S R Gupta, L Sridhar and Daleep Singh

Electrical and Electronic Standards Division, National Physical Laboratory, Dr K S Krishnan Marg, New Delhi 110 012

Received: 26 July 2003; accepted: 03 October 2003

In the present paper the calibration of AC high voltage using the precision grade standard voltage transformer (used as a voltage divider) and standard digital voltmeter taking a case study of Electrostatic Voltmeter are discussed. Various sources of the uncertainty in measurement and their estimation based on Type A and Type B method as per ISO/IEC 17025 guidelines are also discussed. The results are reported at \( k = 2 \) for approx. 95 per cent confidence level. The standards used for calibration are traceable to the ‘National Standards’.

Keywords: Calibration, Uncertainty of measurement, Voltage divider, Electrostatic voltmeter, Digital voltmeter, Voltage transformer

1 Introduction

The objective of a measurement is to determine the value of the measurand, i.e. the value of the particular quantity to be measured. In calibration experiments the value of measurand is estimated against the appropriate reference standard. The result of a measurement is the value attributed to a measurand, obtained by the measurement.

In general the result of measurement is only an approximation or estimate of the value of the measurand due to lack of complete knowledge of the value of the measurand. Complete knowledge requires an infinite number of information. Phenomena that contribute to the doubt about the validity of the result of the measurement and are called sources of uncertainty.

The uncertainty of measurement means doubt about the validity of the result of a measurement. The statement of the result is complete only if it contains both the value attributed to the measurand and the uncertainty of measurement associated with the value. It is well known that the result of the measurement is the best estimate of the value of measurand and that all components of uncertainty contribute to the dispersion.

The AC High Current & High Voltage Standards at National Physical Laboratory maintains the primary standards of AC current and voltage ratios at 50 Hz. This Standards laboratory is involved in the calibration of current transformers, voltage transformers, and allied equipments like, kA meters, tong testers, clamp meters, kV meters, electrostatic voltmeters, and instrument transformer test sets.

The evaluation and expression of uncertainty in measurement is very important in the calibration of these instruments. In the present paper the uncertainty evaluation in the calibration of electrostatic voltmeter (ESVM) has been presented using Universal Method, i.e., Type A and Type B method as per ISO/IEC 17025 guidelines.

2 Methodology of Calibration

ESVM is basically a variable capacitor on which deflecting torque is produced by the action of electric field or charged conductors. They are used in the laboratory for the measurement of high voltages. These types of instruments can be directly used up to the voltage of 20 kV and for using them at higher voltages; these are used in conjunction with the range...
extension devices like, capacitive voltage divider or resistive voltage divider. 

ESVM is calibrated by standard digital voltmeter in conjunction with standard voltage transformer. The standard voltage transformer has been used here as a voltage divider. Figure 1 shows the block diagram for the calibration of electrostatic voltmeter by comparison method.

For the calibration of this device the same high voltage is applied to the Standard Arm and Under Calibration Arm in the set-up. The value of input voltage, applied to the electrostatic voltmeter (i.e. Device under calibration) is indicated by the voltmeter of the device itself. The standard voltage transformer, used as a voltage divider, step downs this voltage to the range of standard digital voltmeter. The standard digital voltmeter, connected across the secondary of the standard voltage transformer, measures this divided output. The measured value on standard against indicated value on electrostatic voltmeter is equal to digital voltmeter reading multiplied by nominal ratio of the voltage transformer (i.e. ratio of the rated primary voltage to the rated secondary voltage).

\[ V_{\text{std}} = K_n \times V_{\text{DVM}} \]  

(1)

For each value of Indicated Value on ESVM, corresponding value is measured by standard digital voltmeter in conjunction with standard voltage transformer and results are presented in Table 1.

<table>
<thead>
<tr>
<th>SL No.</th>
<th>Indicated value by electrostatic voltmeter (kV)</th>
<th>Standard digital voltmeter reading (V)</th>
<th>Nominal ratio of standard voltage transformer</th>
<th>Measured value on the standard (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4000</td>
<td>81.152</td>
<td>5 kV/100V = 50</td>
<td>4057.6</td>
</tr>
<tr>
<td>2</td>
<td>81.078</td>
<td>81.524</td>
<td></td>
<td>4053.9</td>
</tr>
<tr>
<td>3</td>
<td>81.173</td>
<td>81.392</td>
<td></td>
<td>4069.6</td>
</tr>
<tr>
<td>4</td>
<td>81.257</td>
<td></td>
<td></td>
<td>4062.9</td>
</tr>
</tbody>
</table>

\[ V_{\text{DVM}} = 81.263 \text{ V} \]

\[ V_{\text{std}} = 4063.2 \text{ V} \]

3 Mathematical Model Used for the Evaluation of Uncertainty

In calibration of electrostatic voltmeter, comparison method is used, where the voltage indicated by the electrostatic voltmeter and measured value on standard are compared. The following mathematical model is used to estimate the uncertainty in this measurement.

\[ V_{\text{est}} = K_n \times V_{\text{DVM}} + \Delta V_{\text{res}} \]  

(2)

Since \( V_{\text{std}} = K_n \times V_{\text{DVM}} \) therefore Eq. (2) reduces to the following form

\[ V_{\text{est}} = V_{\text{std}} + \Delta V_{\text{res}} \]  

(3)

where \( V_{\text{est}} \) is the voltage indicated by the electrostatic voltmeter, \( V_{\text{std}} \) is the measured value on
the standard against indicated value on electrostatic voltmeter, $K_n$ is the nominal ratio of standard voltage transformer, $V_{	ext{DVM}}$ is the standard digital voltmeter reading, and $\Delta V_{\text{res}}$ is the correction due to resolution of device under calibration, i.e., electrostatic voltmeter.

4 Uncertainty Equation

For uncorrelated input quantities, the combined standard uncertainty $u_c$ is given, as shown in Eq. (4).

$$u_c^2(y) = \sum_{i=1}^{N} \left[ \frac{\partial f}{\partial x_i} u_i^2(x_i) \right], \quad \ldots (4)$$

where $u_c(y) = \text{Combined standard uncertainty}$, $u_i(x_i) = \text{Components of uncertainty}$, $\partial f/\partial x_i = \text{Sensitivity coefficients}$ $(c_i)$, and $N = \text{Number of uncertainty components}$.

$$u_c^2(V_{\text{Std}}) = (c_1)^2 u_1^2(V_{\text{Std}}) + (c_2)^2 u_2^2(K_n) + (c_3)^2 u_3^2(V_{\text{DVM}}) + (c_4)^2 u_4^2(\Delta V_{\text{res}}), \quad \ldots (5)$$

where $u_1(V_{\text{Std}}) = \text{Standard uncertainty in measured value of voltage on standard due to repeatability in measurement}$, $u_2(K_n) = \text{Standard uncertainty in nominal ratio of standard voltage transformer}$, $u_3(V_{\text{DVM}}) = \text{Standard uncertainty of standard digital voltmeter}$, and $u_4(\Delta V_{\text{res}}) = \text{Standard uncertainty due to resolution of device under calibration}$, i.e., electrostatic voltmeter.

The corresponding sensitivity coefficients [calculated from Eq. (3 and 4)] are

$c_1 = \partial V_{\text{Std}} / \partial V_{\text{Std}} = 1$ (for Type A evaluation),

$c_2 = \partial V_{\text{Std}} / \partial K_n = V_{\text{DVM}}$, $c_3 = \partial V_{\text{Std}} / \partial V_{\text{DVM}} = K_n$, and

$c_4 = \partial V_{\text{Std}} / \partial \Delta V_{\text{res}} = 1$.

5 Evaluation of Uncertainty

Six readings were taken for an electrostatic voltmeter of accuracy class 0.5, for 0-5 kV range. For the indicated value of 4 kV on ESVM the readings shown by standard digital voltmeter in conjunction with the standard voltage transformer of ratio 50 V/100 V are shown in the Table 1.

The standards used are Standard Voltage Transformer of uncertainty ±0.005 per cent (in nominal ratio) and Digital Voltmeter of uncertainty ±0.1 per cent.

5.1 Type A Evaluation of Standard Uncertainty

Average value $\bar{V}_{\text{Std}} = (\Sigma V_{\text{Std}})/n = 4063.2$ V.

Experimental variance $s^2(V_{\text{Std}}) = (\Sigma(V_{\text{Std}} - \bar{V}_{\text{Std}})^2)/(n-1) = 348.135/5 = 69.627$ V$^2$.

Experimental standard deviation $= +\sqrt{s^2(V_{\text{Std}})} = 8.344$ V.

Variance of the arithmetic mean $s^2(\bar{V}_{\text{Std}}) = s^2(V_{\text{Std}})/n = 11.605$ V$^2$.

Standard deviation of mean $u(\bar{V}_{\text{Std}}) = \sqrt{11.605} = 3.407$ V.

and degree of freedom $v_1 = n-1 = 6-1 = 5$.

5.2 Type B Evaluation of Standard Uncertainty

(a) The uncertainty in nominal ratio 50 (i.e., 5 kV/100 V = 50) of standard voltage transformer from its calibration certificate is $a_1 = \pm 0.005$ per cent (i.e., ± 50 ppm) at 95.45 per cent confidence level and $k = 2$. For normal distribution the standard uncertainty $u_2(K_n)$ is:

$$u_2(K_n) = a_1(K_n)/2 = [(0.005/100) \times 50]/2 = 12.5 \times 10^{-4}$$

and degree of freedom $v_2 = \infty$.

(b) The output of the standard voltage transformer is 100 V for 5 kV input. Therefore, input to the digital voltmeter is in 100 V range. The uncertainty of Standard Digital Voltmeter from its calibration certificate for 100 V range is $a_2 = \pm 0.1$ per cent (i.e., ± 1000 ppm) at 95.45 per cent confidence level and $k = 2$. The average value of the digital voltmeter reading is 81.263 V. For normal distribution the standard uncertainty $u_3(V_{\text{DVM}})$ is:

$$u_3(V_{\text{DVM}}) = a_3(V_{\text{DVM}})/2 = [(0.1/100) \times 81.263 V]/2 = 0.0406 V$$

and degree of freedom $v_3 = \infty$.

(c) The resolution of ESVM is 0.01 kV (i.e., 0.01 × 1000 = 10 V). When we consider the limits to be half of the resolution the limits of this uncertainty component become $a_3 = \pm 5$ V. Since upper and lower limit of this uncertainty component is given, therefore
assuming rectangular distribution. For rectangular distribution the standard uncertainty \(u_t(\Delta V_{res})\) due to resolution of device under calibration (i.e. electrostatic voltmeter) is:

\[ u_t(\Delta V_{res}) = \frac{a(V)}{\sqrt{3}} = \frac{5}{\sqrt{3}} = 2.8868 \text{ V} \]

and degree of freedom \(v_t = \infty\).

### 5.3 Combined Standard Uncertainty (\(u_c\))

\[ u_c^2(V_{std}) = (c_1)^2 u_t^2(V_{std}) + (c_2)^2 u_t^2(K_0) + (c_3)^2 \]

\[ u_c^2(V_{DVM}) + (c_4)^2 u_t^2(\Delta V_{res}) \]

\[ u_c(\Delta V_{res}) = 4.907 \text{ V} \]

### 5.4 Effective Degree of Freedom (\(v_{eff}\))

Effective degree of freedom of standard uncertainty \(u_t(y)\) associated with the output estimate \(u_t(y)\) is given by Welch-Satterthwaite formula:

\[ v_{eff} = \left[ \sum_{j=1}^{N} [u_t(y)_j]^2 / v_j \right]^{1/2} \]

\[ (4.907)^4 \]

\[ v_{eff} = \frac{\{(1 \times 3.407)^4 + (12.5 \times 10^{-5} \times 81.263)^4 + \{(0.0406 \times 50)^4 + (1 \times 2.8867)^4 \}}{\}}{\} \]

\[ v = 21.52 = 21 \text{ (truncating to the lower side)} \]

### 5.5 Expanded Uncertainty (\(U\))

It is given by \(U = k \times u_c\), where \(k\) is a coverage factor.

From student’s \(t\)-distribution, for 95.45 per cent confidence level and for \(v_{eff} = 21\), the value of coverage factor \(k\) is 2.13.

\[ U = 2.13 \times 4.907 = 10.45 \text{ V} = 10.5 \text{ V} \]

The uncertainty budget is summarized in Table 2.

### 5.6 Results

The measured value on the standard with 95.45 per cent confidence level and coverage factor \(k = 2.13\) is 4063.2 V ± 10.5 V.

The reported expanded uncertainty of measurement is stated as the combined standard uncertainty multiplied by the coverage factor \(k=2\) which for a normal distribution corresponds to coverage probability of approx. 95 per cent.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Estimates</th>
<th>Limits</th>
<th>Probability distribution</th>
<th>Standard uncertainty</th>
<th>Sensitivity coefficient</th>
<th>Uncertainty contribution</th>
<th>Degree of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x_i)</td>
<td>(x_i)</td>
<td>(\pm \Delta x_i)</td>
<td>Type A or B</td>
<td>(u(x_i))</td>
<td>(c_i)</td>
<td>(u(y) = c_i u(x_i))</td>
<td>(v_i)</td>
</tr>
<tr>
<td>Standard VT</td>
<td>50×10^{-4}</td>
<td>25×10^{-4}</td>
<td>Normal</td>
<td>12.5×10^{-4}</td>
<td>81.263 V</td>
<td>0.1016 V</td>
<td>\infty</td>
</tr>
<tr>
<td>Resolution of ESVM</td>
<td>10 V</td>
<td>5 V</td>
<td>Rectangular</td>
<td>2.8868 V</td>
<td>1.0</td>
<td>2.8868 V</td>
<td>\infty</td>
</tr>
<tr>
<td>Repeatability</td>
<td>4063.2 V</td>
<td>—</td>
<td>Normal</td>
<td>3.407 V</td>
<td>1.0</td>
<td>3.407 V</td>
<td>5</td>
</tr>
<tr>
<td>(V_{wre})</td>
<td>4063.2 V</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Expanded uncertainty</td>
<td>—</td>
<td>—</td>
<td>coverage</td>
<td>10.5 V</td>
<td>21</td>
<td>4.907 V</td>
<td>21</td>
</tr>
</tbody>
</table>

\(U\)
6 Conclusions

In this paper, we have dealt with the calculation of uncertainty for the case of ESVM calibration. The calibration was done using the comparison method. The detailed analysis of the various components contributing to the uncertainty in the measurement has been done and expanded uncertainty in measurement calculated as per ISO guidelines. The uncertainty budget has been also prepared which shows the various sources of uncertainty, its estimation and the result of the measurement. For indicated value of 4 kV on electrostatic voltmeter the measured value on the standard is 4063.2 V with expanded uncertainty of ±10.5 V (i.e. ±0.26 per cent) at \( k = 2.13 \) and approx. 95 per cent confidence level.

Acknowledgement

The authors are grateful to Dr A K Gupta, Head, Electrical and Electronic Standards Division, National Physical Laboratory for constant encouragement. They are also thankful to Dr Rina Sharma, Sr Scientist, Length and Dimensional Standards, NPL for useful discussion.

References