Modelling for identification of stiffness parameters of multistorey frame structure from dynamic data

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Received: 9 July 2003; accepted: 05 November 2003

The paper deals with the procedure which systematically modifies and identifies the structural parameters, viz., the column stiffness of multistorey structures by using the prior known estimates of the parameters and the corresponding vibration characteristics. For having the accuracy of the identified parameters, an iteration algorithm with the identification procedure is proposed till the parameters converge. Various examples demonstrate the reliability and accuracy of the identification process. The effects of a range of experimental (hypothetical) data on the identified structural parameters for two storeyed frame structures are investigated as an illustration.

Keywords: Stiffness parameters, Structural dynamics, Mathematical modeling, Modelling

Introduction

Structural dynamics problems may be categorized as direct or inverse problems. The direct problem consists of finding the response for a specified input or excitation. In the inverse problem the response is known to develop a mathematical model of the system. The modelling problem may also be divided into two categories. In the first category the nature of the process is completely unknown. But in the second category, a considerable knowledge of the nature of the system may be available, whereas the particular values of the system parameters are unknown. In this paper the second category has been studied, where system equations are known or deducible from the physics of the system, with coefficients remaining to be estimated and modified as per the known initial dynamic characteristics.

In this context, various workers1-3 have reviewed the state of the art of system identification in structural dynamics. Developments and various methods for studying this important field are available in literature4. More recent methods and practical guidelines for linear systems may be found in the work of Schoukens and Pintelon4.

Although few researchers have studied the above issues but, at present also lot of efforts are being made to refine and develop the analytical models for the accurate results. Some representative works on the subject are available5-9. Recent work done is discussed subsequently.

Loh and Ton10 have studied a system identification approach to detect changes in structural dynamic characteristics on the basis of measurements. They used the recursive instrumental variable method and extended Kalman filter algorithm for the identification algorithm. The potential of using neural network to identify the internal forces of typical systems has been investigated recently by Chassios and Masri11. A localized identification of many degrees of freedom structures is investigated by Zhao et al.12 and a memory-matrix based identification methodology for structural and mechanical systems is studied by Udwadia and Proskurowski13. Notable recent studies in this field have also been done by other workers14-18.

In addition to the above literature there exist other research works in the present area of study. However the fundamental concepts are similar to those mentioned above. As regards the objective of the structural dynamic analysis related to identification is to develop an analytical model of a structure which can be verified and adjusted by actual test results. However, this adjustment is not easy and can be done by computer and convergence algorithm in terms of some iterative cycles.

The main aim, therefore, of the present study is to develop a systematic mathematical model for the identification of structure which can provide the vibration
CHAKRAVETY: MODELING FOR IDENTIFICATION OF STIFFNESS PARAMETERS OF FRAME STRUCTURE

Figure 1—Two storey frame structure

characteristics consistent with the experimental data. The method first uses the values of the structural parameters initially given to the structure by an engineer. It then modifies the original parameter values as per the observed values from test by an iteration process, giving convergent modified values of the parameters.

Mathematical Modelling and Method of Identification

To investigate the present method, a two-storeyed frame structure, as shown in Figure 1 is considered. This is investigated for the sake of demonstration of the procedure. The floor masses, \( m \) are assumed to be the same and the column stiffnesses \( k_1, k_2, k_3, k_4 \) (as labeled in Figure 1) are the structural parameters which are to be identified. The corresponding dynamic equation of motion in matrix form for this two degrees of freedom system is well known and is written as:

\[
[M] \ddot{x} + [K]x = \{0\},
\]

where \([M] = 2x2\) mass matrix of the structure,

\[
[M] = \begin{bmatrix}
m & 0 \\
0 & m
\end{bmatrix},
\]

\([K] = 2x2\) stiffness matrix of the structure,

\[
[K] = \begin{bmatrix}
(k_1 + k_2 + k_3 + k_4) & -(k_1 + k_4) \\
-(k_1 + k_4) & (k_1 + k_2)
\end{bmatrix},
\]

\(\{x\} = 2x1\) vector of displacements.

Now, if the structural parameters to be identified are denoted by \( p_1, p_2, p_3, p_4 \) then for simple harmonic motion, putting:

\(\{x\} = \{\phi\}e^{i\omega t}\),

in Eq. (1) the equation of motion can be written as functions of \( p_i, i=1,2,3,4 \), as

\[
([K] - \lambda_i(p)\{M\})\{\phi(p)\} = \{0\}
\]

... (2)

where \(\lambda_i(p) = \{\omega(p)\}^2\) are eigen values (frequencies) of the structure and \(\{\phi(p)\}\) are mode shapes of the structure.

As mentioned earlier the values of the structural parameters of the original structure are given initially as, \(\bar{p}_i, i=1..4\), and the corresponding eigen values and eigen vectors are symbolized as,

\(\lambda_i(\bar{p})\) and \(\{\phi(\bar{p})\}\).

Now, the well known Taylor's series expansion of the model parameters about the initial estimates of the parameters gives:

\[
\begin{bmatrix}
\lambda(p) \\
\phi(p)
\end{bmatrix} =
\begin{bmatrix}
\lambda(\bar{p}) \\
\phi(\bar{p})
\end{bmatrix} + [S]\{(p) - \{\bar{p}\}\}
\]

... (3)

where

\[
\{p\} = \begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
p_4
\end{bmatrix}, \quad \{\bar{p}\} = \begin{bmatrix}
\bar{p}_1 \\
\bar{p}_2 \\
\bar{p}_3 \\
\bar{p}_4
\end{bmatrix},
\]

\([S]\) is eigen value-eigen vector partial derivative matrix, \(\partial(\lambda,\phi)/\partial(p)\).

Let us now denote experimentally measured values of eigen values and eigen vectors by \(\{\lambda_e\}\) and \(\{\phi_e\}\), respectively.

It is interesting to note here that if the values of the parameters from the initial values and experiment are equal, then no modification is done. If the values are different then these are denoted by,

\[
\begin{bmatrix}
\delta\lambda \\
\delta\phi
\end{bmatrix} =
\begin{bmatrix}
\lambda_e \\
\phi_e
\end{bmatrix} -
\begin{bmatrix}
\lambda \\
\phi
\end{bmatrix}
\]

... (4)

The modified parameters are written as:
\{\hat{p}\} = \{\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4\}^T \tag{5}

and, in general, for \(n\) degrees of freedom system the expression for the modified parameters from Eq. (3) can be written as :

\{\delta \lambda\} = \{\delta \lambda_1, \delta \lambda_2, \ldots, \delta \lambda_n\}^T = ([S]^{-1} [P]) \{\delta \phi\} \tag{6}

where \([P]\) = \(( [S]^{-1} [S] \) 

In order to have the desired accuracy of the identified parameters, here it is proposed to iterate for the revised parameters. After finding the modified parameters from Eq. (6), these are substituted in Eq. (2) to get revised analytical vibration characteristics denoted by,

\{\hat{\lambda}, \hat{\phi}\} = \{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_n\} \equiv \{\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_n\}

The new eigen value – eigen vector partial derivative matrix, \([\hat{S}]\) is then obtained, using the current values of :

\{\hat{p}\} and \{\delta \phi\}

From Eq. (6), the modified parameters are again found using the above values and denoted by,

\{\hat{p}_i\} \tag{7}

Then the new (revised) values of analytical vibration characteristics are obtained as :

\{\hat{\lambda}_i, \hat{\phi}_i\} = \{\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_n\} = \{\hat{\phi}_1, \hat{\phi}_2, \ldots, \hat{\phi}_n\}

If the vector norm of :

\{\hat{\lambda}, \hat{\phi}\} and \{\hat{\lambda}_i, \hat{\phi}_i\}

is less than some specified accuracy then the procedure is stopped and the revised parameter is identified which is given by Eq. (7), otherwise the next iteration is to be computed.

Results and Discussion

As mentioned earlier the procedure is demonstrated for a two storeyed frame structure. Implementing the above procedure with the proposed iterative cycle for the revised frequencies and parameters, computer programs have been written and tested for variety of example problems. But only two examples are provided here for the two degrees of freedom system (Figure 1).

In the first example, equal floor masses, \(m=1800x2\) kg and the column stiffnesses \(k_1 = k_2 = 1800x2\) N/m, \(k_3 = k_4 = 1800x2\) N/m have been taken. The analytical vibration characteristics from these prior mass and stiffness parameters can be found from Eq. (2). Accordingly the analytical frequencies are given by \(\lambda_1=1.00, \lambda_2=6.00\).

Using the above sets of initial data of the parameters with different experimental (hypothetical) test data for the frequencies, viz. \(\lambda_{1E}\) and \(\lambda_{2E}\) (i.e. first and second experimental eigen values of the system) the stiffness parameters of the structure have been identified. These are reported in Table 1. Various experimental test data (hypothetical) have been incorporated to have numerical experiment of the methodology and for checking the convergence procedure. The identified parameters are reported in Table 2 by taking the values of \(\lambda_{1E}\) as \(5.5, 6.0, 6.5, 7.0\) and \(7.5\) for each values of \(\lambda_{1E} = 0.1, 0.3, 0.5, 0.7, 0.9\). Let us denote the revised (convergent) frequencies as \(\lambda_{1R}\) and \(\lambda_{2R}\) as per the iterative cycle. These are reported in third and fourth columns of Table 1 for various combinations of \(\lambda_{1E}\) and \(\lambda_{2E}\). In the last row of Table 1 the exact identified results of the stiffness parameters are obtained, taking the experimental data exactly same as the initial analytical frequency values \(\lambda_{1E} = 1.0, \lambda_{2E} = 6.0\). However, this is obvious and is given here, to have completeness of the results from the computer run. Similar results are given in Table 2 for the second example, where floor masses, \(m=1800x2\) kg and stiffnesses, \(k_1 = k_2 = 35950\) N/m, \(k_3 = k_4 = 53950\) N/m have been taken. The initial analytical frequency data for this example are given by \(\lambda_1=0.9989, \lambda_2=5.9928\). Here also the last row of Table 2 gives the exact identified results of the stiffness parameters by taking the experimental data exactly same as the initial analytical frequency values \(\lambda_{1E} = 0.9989, \lambda_{2E} = 5.9928\).

It may be seen from Table 1 and 2 that when we increase the values of \(\lambda_{1E}\) from 5.5 to 7.5 for all the values of \(\lambda_{1E}\) (viz. 0.1, 0.3, 0.5, 0.7, 0.9) the revised frequency for the first mode is maximum for \(\lambda_{1E} = 6.0\). These revised frequencies (first mode) start increasing as the value of \(\lambda_{1E}\) is changed from 5.5 to 6.0 and then start decreasing as
$\lambda_{2e}$ is increased from 6.0 to 7.5. However, the revised frequency for second mode always shows increasing tendency as $\lambda_{2e}$ increases from 5.5 to 7.5. It is interesting to note that the identified stiffness parameters $k_{1}$ and $k_{2}$ continuously increase as $\lambda_{2e}$ is increased for each values of $\lambda_{2e}$ (0.1, 0.3, 0.5, 0.7, 0.9). On the other hand the stiffness parameters $k_{1}$ and $k_{2}$ increase as $\lambda_{2e}$ increases from 5.5 to 6.0 and then start decreasing as $\lambda_{2e}$ is increased from 6.0 to 7.5. The identified stiffness parameters $k_{1}$ and $k_{2}$ are maximum for $\lambda_{2e} = 6.0$. It may be observed that the pattern for the revised frequency for second mode is same as stiffness parameters $k_{1}$ and $k_{2}$ designating the second level of the frame structure. Similarly the tendency of the revised frequency for first mode is similar as the stiffness parameters $k_{1}$ and $k_{2}$ designate the first level of the structure. However, this is evident as in the present analysis of two degrees of freedom system the first and second frequencies are nomination for first and second level of the structure, respectively. Further, this study shows that for experimental value of the second frequency data equal to the second prior analytical frequency the maximum value of the stiffness parameters can be attained for the first level of the frame structure for any value of the experimental first frequency data.

<table>
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<th>$\lambda_{2e}$</th>
<th>$\lambda_{2e}$</th>
<th>$\lambda_{2e}$</th>
<th>$\lambda_{2e}$</th>
<th>$k_{1} = k_{2}$ (N/m)</th>
<th>$k_{1} = k_{2}$ (N/m)</th>
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<td>0.1</td>
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<td>0.1321</td>
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<td>4871.959</td>
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<td>0.3</td>
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<td>4723.002</td>
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Table 2 — Identified stiffness parameters for example 2

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<th>$\lambda_{1E}$</th>
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<th>$\lambda_{1R}$</th>
<th>$\lambda_{2R}$</th>
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<td>7.0</td>
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<tr>
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The above effects can be seen from the Figure 2 and 3 also. Here, however, results are depicted in Figure 2 for variation of $k_j$ and $k_j$ for different values of $\lambda_{1E}$ taking various values of $\lambda_{1E} = 0.2, 0.4, 0.6$ and 0.8. In Figure 3 the variation of $k_j$ and $k_j$ is shown for different values of $\lambda_{1E}$ with the values of $\lambda_{1E} = 0.2, 0.4, 0.6$ and 0.8. The pattern shown for the stiffness parameters in Figure 2 and 3 are the same, as discussed in Table 1.

Now if the value of $\lambda_{1E}$ is kept constant (say, 5.5, 5.8, 6.0, 6.2, 6.5, 7.0, 7.5, etc.) then the variation of $k_j$ and $k_j$ for different values of $\lambda_{1E}$ will be as shown in Figure 4. Figure 4 shows that the values of the stiffness parameters ($k_j$ and $k_j$) decrease as the values of $\lambda_{1E}$ are increased for the considered values of $\lambda_{1E}$. Similar variation for the stiffness parameters $k_j$ and $k_j$ for different values of $\lambda_{1E}$ is seen in Figure 5. In this case, it is evident that the values of $k_j$ and $k_j$ increase as the values of $\lambda_{1E}$ are increased. It may be noted here that the value of experimental test data (hypothetical) of the first mode ($\lambda_{1R}$) is not considered.
larger than the initial (prior) analytical first frequency because structures (here the frame structure) after its design would not, in general, have more frequency values than the prior one in the case of first mode. And the experimental test data for the second mode has been taken as both less and more than the initial (prior) analytical second frequency, as this may happen depending upon some complicated situations. It is seen from Table 1 and Figures 2-5, when the experimental test data of the frequencies are in the neighbourhood of the prior analytical frequencies then the identified stiffness parameters are close to the prior values of the parameters, as expected. On the other hand the identified stiffness parameters are found to be very different from the prior values when the experimental test data are far from the prior analytical frequencies.

The discussions made for the example 1 hold good in the case of the second example too. In this regard, only Figure 6 has been given as an illustration for the second example which illustrates the variation of $k_i$ and $k_e$ for different values of $\lambda_{m}$ with particular values of $\lambda_{m}$, as shown in Figure 6. It is worth mentioning here that the noise effects in the test data here not been considered it being a modelling (deterministic) study and the data is supposed to be filtered for the noise. Thus, it is shown here that if the variation in the experimental test data that occurs actually in practice, as has been considered here, then how the method would converge to the revised frequency parameters of the system (Tables 1 and 2). The corresponding converged stiffness parameters and their various effects, depending upon the influencing parameters have also been investigated.
Conclusion
A procedure is presented in this paper which systematically modifies and identifies the structural parameters, viz. the column stiffnesses, for a two storeyed frame structure by using the prior known estimates of the parameters with the corresponding vibration characteristics and the known dynamic data from some experiments. The convergence of the estimates is achieved by proposing iterative cycles mentioned in the description of the model. The numerical procedure is tested by incorporating different sets of data. Result of the example problems show that the method is accurate enough and reliable. This study investigates the various effects of the range of experimental data on the identified stiffness parameters and revised frequencies. A numerically efficient technique for system identification of structure is provided and the corresponding model may easily be extended for frame structures with many stories, in general.

Acknowledgements
The financial help by the HRD Division of CSIR for the present work is duly acknowledged by the author.

References