Role of oblique whistler-mode waves in energetic electron precipitation at the lower edge of inner radiation belt

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Received 6 October 1998; revised 22 February 1999; accepted 9 March 1999

An attempt has been made to study the precipitation of energetic (keV and MeV) electrons at the lower edge of inner radiation belt in terms of their quasi-linear gyro-resonant interaction with oblique whistler-mode waves. The calculated average pitch angle scattering (10° degree) and lifetimes (days and hours only) indicate a significant electron precipitation by these waves into the lower latitude atmosphere. The low calculated lifetimes of energetic electrons and sufficiently high calculated flux of precipitated electrons caused due to gyro-resonant interaction involving oblique whistler-mode waves show that these non-ducted waves are likely to be an effective electron precipitator at the lower edge of inner radiation belt.

1 Introduction

Pitch angle scattering by gyro-resonant interaction between whistler-mode waves and counter-streaming energetic electrons is an important mechanism for the loss of trapped energetic electrons from radiation belts and their consequent precipitation into the lower ionosphere. Precipitation of energetic electrons from the inner radiation belt in terms of their gyro-resonant interaction with parallel propagating (ducted) whistler-mode waves has been frequently studied. However, hardly any attention has been paid so far on the study of energetic electron precipitation by oblique (i.e. non-ducted or propagating at an angle to the geomagnetic field) whistler-mode waves in the low latitude region, where almost all whistler-mode waves propagate in non-ducted mode with appreciably large wave normal angles because of the non-existence of suitable ducts in the low latitude ionosphere below \( L = 1.7 \) (Refs 1 and 2). Magnetospherically reflected (MR) whistlers, which have highly oblique propagation, have been found to resonantly interact with energetic (\( \sim 100 \) keV) electrons during multiple equatorial crossings over a wide range of \( L \)-shells having values\(^1\) between 1.5 and 4. Recently, Singh\(^4\) has interpreted the unusual relationship between the time intervals of discrete riser emissions (which occurred at time intervals increasing in geometrical progression) observed at low latitude ground station Bichpuri (\( L = 1.15 \)), Agra, in terms of their generation at \( L = 1.2 \) in the equatorial plane as a result of interactions between the trapped energetic electrons and the increasing hopes of a MR whistler having a dispersion of \( 19 \) s\(^{-1}\). Thus, MR whistlers are likely to exist at the lower edge of inner belt (\( L = 1.2 \)) also.

In the present paper, therefore, an attempt has been made to study the energetic (keV and MeV) electron precipitation at the lower edge of inner radiation belt (\( L = 1.2 \)) as a result of their quasi-linear gyro-resonant interaction with oblique whistler-mode waves. The results show that oblique (non-ducted) whistler-mode waves are likely to be an effective electron precipitator at the lower edge of inner radiation belt and may cause a significant flux of precipitated electrons there.

2 Theoretical formulations

The general first order gyro-resonance condition for the interaction of energetic electrons and oblique whistler-mode waves is expressed as\(^5\)

\[
\gamma \approx \gamma _v = (\Omega \gamma / \gamma - \alpha \omega ) / k
\]

... (1)

where, \( \gamma = (1 - v^2 / c^2 \cos ^2 \alpha )^{-1/2} \) is the relativistic factor, \( v \) the component of electron's velocity along the geomagnetic field, \( \alpha \) the particle's pitch angle, \( c \) the speed of light in vacuum, \( \gamma _v \) the resonance velocity, \( \Omega \), the electron angular gyro-frequency, \( \alpha \) the angular wave-frequency and \( k \) the wave number along the geomagnetic field. The wave number \( k \) is defined as\(^5\)
$k = k \cos \theta = \frac{1}{c} \left( \frac{\omega_p^2 \omega}{\Omega_e \cos \theta - \omega} \right)^{1/2} \cos \theta \quad \ldots (2)$

in which $\omega_p$ is the electron plasma frequency, $\theta$ the wave normal angle and $k$ the wave number in the direction $\theta$. Squaring Eq. (1) and rearranging the terms, we get

$$(k_r^2 + \Omega_e^2/c^2 \cos^2 \alpha) v_r^2 + 2 \omega k_r v_r (\Omega_e^2 - \omega^2) = 0 \quad \ldots (3)$$

which is a quadratic equation in $v_r$. For the given local values of $\Omega_e$ and $k$, and a specified value of $\alpha$, a positive solution of Eq. (3) may be written as

$$v_r = v_{ri} = \frac{P}{Q} \quad \ldots (4)$$

where,

$$P = -\omega k_r + \left[ \omega^2 k_r^2 + (\Omega_e^2 - \omega^2) Q \right]^{1/2}$$

and

$$Q = k_r^2 + \Omega_e^2/c^2 \cos^2 \alpha$$

Equation (4) is the only solution to satisfy Eq. (1) for $\mu \cos \alpha > 1$ (this condition is satisfied at $L = 1.2$ for the pitch angle range considered in the present paper), where $\mu$ is the refractive index. However, for $\mu \cos \alpha < 1$, two solutions are possible for Eq. (3) - one positive, given by Eq. (4), and the other negative, corresponding to $\gamma > \Omega_e/\omega$. But, negative $v_r$ represents electrons travelling in the same direction as the waves, while the general first order gyro-resonance mechanism requires energetic electrons travelling opposite to the waves. In gyro-resonance interaction between whistler-mode waves and counter-streaming energetic electrons near the equatorial plane, the Doppler-shifted wave frequency must be equal to the electron gyro-frequency, thus leading to the general first order gyro-resonance condition expressed by Eq. (1). Hence, we consider only the energetic electrons with $v_r$ as given by Eq. (4) in this paper.

The resonance energy of the energetic electrons is calculated using the relation

$$E_R = (\gamma - 1) m_e c^2 \quad \ldots (5)$$

in which $m_e c^2 = 511.88$ keV is the rest mass energy of the electrons. Equations (4) and (5) show that $v_r$ and $E_R$ are strong functions of pitch angle $\alpha$.

In order to study the pitch angle scattering and diffusion of energetic electrons resulting from resonant interaction with oblique whistler-mode waves, we can use quasi-linear theory presented by Kennel and Petschek. According to this theory, the change in pitch angle for a given resonant velocity due to interaction with oblique waves in a narrow wave number spread $\Delta k$ can be given by

$$\Delta \alpha = \frac{\Delta v_L}{v_L} = \frac{e v_L B_w}{mc} \frac{\Delta f}{v_L} = \frac{\Omega_e}{B_w} \frac{B_w}{B} \Delta t \quad \ldots (6)$$

and the average diffusion coefficient can be given by

$$D = \frac{\langle \Delta \alpha \rangle^2}{2 \Delta t} = \frac{\Omega_e^2}{2} \left( \frac{B_w}{B} \right)^2 \Delta t \quad \ldots (7)$$

with,

$$\Delta t = \frac{2}{\Delta k} \quad \ldots (8)$$

where, $B$ is the ambient magnetic field strength, $B_w$ the wave amplitude near resonance and $\Delta t$ the time in which an electron at a distance $2/\Delta k$, out of resonance changes its phase by one radian.

In order to obtain $\Delta \alpha$ and $D$ as functions of pitch angle $\alpha$, and wave normal angle $\theta$, we differentiate Eq. (2) w.r.t. $\alpha$ to get $\Delta k$. Now $\Delta k$ is substituted in Eq. (8) and the value of $\Delta t$ thus obtained is then substituted in Eqs (6) and (7) to finally obtain

$$\Delta \alpha = \frac{2 e \Omega_e^2}{\pi \omega_p} \frac{x^2}{\Delta f} \left( \frac{B_w}{B} \right) \quad \ldots (9)$$

and

$$D = \frac{e \Omega_e^2}{\pi \omega_p} \frac{x^2}{\Delta f} \left( \frac{B_w}{B} \right)^2 \frac{B_w^2}{B} \quad \ldots (10)$$

where, $x = \omega/\Omega_e$ is the normalized wave frequency, $\Delta f$ the wave frequency spread, and $B_w^2/\Delta f$ the wave power spectral density in T² Hz⁻¹.

For estimating the precipitated flux, $J_p$, we use the following relation

$$J_p = \left( \frac{T_m}{T_T} \right) J_T \quad \ldots (11)$$

where, $J_T$ is the trapped flux, $T_T$ the electron lifetime and $T_m$ the minimum lifetime corresponding to strong diffusion. The value of $T_m$ is obtained from the relation

$$T_m = 2T_e/\alpha_0^2 \quad \ldots (12)$$

where, $T_e$ is the electron's escape time (which is roughly equal to a quarter of bounce period) and $\alpha_0$, the equatorial loss cone angle. The lifetime $T_T$ is calculated by taking the inverse of the diffusion coefficient.
3 Experimental data

3.1 Wave magnetic field

Tsurutani et al. have reported the observation of intense ELF hiss emissions in the inner radiation belt at ~ 450 km during the periods of geomagnetic disturbances with the experiment aboard the OGO-6 satellite. The observed emissions have average peak power spectral density of $4 \times 10^{-7} \gamma^2$ Hz$^{-1}$ at 550 Hz with a bandwidth of 300 Hz. Since we intend to study the pitch angle scattering and the diffusion in the equatorial region at $L = 1.2$, this intensity is extrapolated to the equatorial plane assuming an enhancement of 5 dB. The extrapolated intensity at $L = 1.2$ is found to be $1.26 \times 10^{-18} \gamma^2$ Hz$^{-1}$ from which the wave field is estimated to be 19.44 mV. The wave field at the whistler frequency of 3.2 kHz is deduced from latitudinal profiles of magnetic field intensity observed by the ARIEL-3 satellite at 500 km in the ionosphere and reported by Hayakawa et al. The wave intensity is found to be $5.25 \times 10^{-19} \gamma^2$ Hz$^{-1}$ at 500 km in the ionosphere ($L = 1.2$). When extrapolated to the equatorial plane by considering an enhancement of 10 dB, this intensity becomes $5.25 \times 10^{-18} \gamma^2$ Hz$^{-1}$ which, in turn, yields a wave field of 2.29 mV on taking a bandwidth of 1 kHz. Recently, Nishino and Tanaka have reported the observation of intense 5 kHz VLF hiss emissions at Kagoshima ground station ($L = 1.2$) in Japan with a peak intensity of $10^{-3}$ W m$^{-2}$ Hz$^{-1}$ and a bandwidth of ~ 2 kHz. The ground-observed intensity is extrapolated to the equatorial plane by considering an enhancement of 15 dB. The equatorial intensity at $L = 1.2$ is found to be $3.16 \times 10^{-15}$ W m$^{-2}$ Hz$^{-1}$ ($\approx 2.14 \times 10^{-16} \gamma^2$ Hz$^{-1}$) which gives a wave field of 0.66 mV.

In the extrapolation of intensity to the equatorial plane, we have considered the enhancement of 5, 10 and 15 dB at 550 Hz, 3.2 kHz and 5 kHz, respectively, owing to the facts that (i) ELF/VLF waves are generated, most probably, in the equatorial plane, (ii) propagation loss increases with increasing wave frequency and (iii) 5 kHz waves suffer an additional propagation loss of ~ 5 dB in the lower region of ionosphere (height 50 - 500 km).

3.2 Energetic electron flux

In the present calculations, we consider an energetic electron flux of $1 \times 10^{9}$ el. cm$^{-2}$ s$^{-1}$ sterad$^{-1}$ ($L = 1.2$, energy > 1 MeV) which is deduced from the energy flux profiles reported by Katz from in situ measurements aboard HITCH-HIKER-I (1963-25B) satellite in the inner radiation zone.

4 Results and discussion

The calculations are done at the $L$-value of 1.2 in the equatorial plane for waves having different frequencies of 550 Hz, 3.2 kHz and 5 kHz. The value of plasma frequency ($\omega_p$) is determined by employing the relation $\omega_p/2\pi = 8.98 n_e^2$ and considering an electron density ($n_e$) of $8.13 \times 10^9$ el. m$^{-3}$ which is deduced from a low latitude diffusive equilibrium model used earlier by Singh. Since the loss cone angle at the $L$-value of 1.2 is ~ 46$^\circ$, we consider the pitch angles of the energetic electrons outside the loss cone, i.e. having values greater than 50$^\circ$. First, the values of wave number $k$, are determined from Eq. (2) at the considered $L$-value for the different frequency waves at different values of wave normal angle $\theta$. Then the values of $k_x$, $\omega$, $Q$, and $\alpha$ are substituted in Eq. (4) to obtain the resonance velocities of the energetic electrons resonating with different frequency waves having $\theta$ values of 60$^\circ$ and 80$^\circ$. The calculated equatorial resonance velocities of the electrons resonating with 550 Hz waves at $L = 1.2$ are presented in Table 1. Table 1 shows that the calculated resonance velocities, which are higher at lower values of $\alpha$, decrease rapidly with increasing $\alpha$. The resonance velocity slightly decreases with increasing wave frequency for the same value of $\theta$ and approaches a zero value at all values of $\theta$ as $\alpha$ approaches 90$^\circ$. It is to be pointed out here that the calculated resonance velocities, in the present study, are slightly higher than the corresponding values at higher latitudes.

The equatorial resonance energies of the energetic electrons at $L = 1.2$ are calculated by using Eq. (5). The values of the relativistic factor $\gamma$ appearing in Eq.

<table>
<thead>
<tr>
<th>Pitch angle (deg)</th>
<th>Resonance velocity (in 10$^6$ m/s) at L=1.2 and a wave frequency of 550 Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>1.9278</td>
</tr>
<tr>
<td>60</td>
<td>1.4997</td>
</tr>
<tr>
<td>70</td>
<td>1.0260</td>
</tr>
<tr>
<td>80</td>
<td>0.5209</td>
</tr>
<tr>
<td>85</td>
<td>0.2615</td>
</tr>
<tr>
<td>90</td>
<td>0.0</td>
</tr>
</tbody>
</table>
The pitch angle scattering and diffusion coefficients of relativistic energetic electrons resonating with oblique whistler-mode waves of different frequencies are then calculated by employing Eqs (9) and (10), respectively. In these calculations the values of $\omega_p$ and $\Omega$, at $L=1.2$ are taken to be $5.0875 \times 10^6$ rad s$^{-1}$ and $3.1765 \times 10^6$ rad s$^{-1}$, respectively, and the value of pitch angle is varied between $50^\circ$ and $85^\circ$, since the equatorial loss cone angle at this $L$-value is $46^\circ$. The calculated values of pitch angle scattering and diffusion coefficient of energetic electrons resonating with a whistler-mode wave of frequency $550$ Hz at wave normal angles of $60^\circ$ and $80^\circ$ are presented in Fig. 2 as a function of pitch angle $\alpha$.

The calculated values of average pitch angle scattering [Fig. 2 (a)] are found to be $5.78 \times 10^{-2}$ deg.
(θ=60°) and 9.70 × 10⁻² deg (θ=80°) which may be
considered to be significant near the loss cone. Thus,
the resonant electrons whose pitch angles are close to
the loss cone may be diffused into the loss cone and
precipitated into the lower ionosphere. The pitch an­
gle scattering is found to increase with pitch angle
(α) as well as with wave normal angle (θ). Calcula­
tions carried out at other wave frequencies (e.g. 3.2
kHz and 5 kHz) indicate that the pitch angle scatter­
ing decreases with increasing wave frequency. Like­
wise, the diffusion coefficient [Fig. 2 (b)] is also
found to increase with increasing α and θ. From Fig.
2 (b), the values of average diffusion coefficient D
are estimated to be 1.73 × 10⁻³ rad² s⁻¹ and 2.89 × 10⁻³
rad² s⁻¹ at the wave normal angles of 60° and 80°,
respectively. These values of D give the average
lifetime of energetic electrons resonantly inter­
acting with whistler-mode waves of frequency 3.2
kHz are estimated to be 16.22 h and 10.06 h and the
average lifetime of energetic electrons interacting
with the waves of frequency 5 kHz are calculated to
be 13.55 days and 2.28 days only at θ values of 60°
and 80°, respectively. Thus, the energetic electrons
which are close to the loss cone have very small lif­
times. Further, the lifetimes of energetic electrons are
significantly reduced in case of their interaction with
whistler-mode waves of larger wave normal angles
(i.e. more oblique waves). Hence, an interesting con­
sequence of the present study is that, owing to first
order gyro-resonance interaction with oblique whis­
tler-mode ELF-VLF waves, the relativistic electrons
of energy > 6 MeV may significantly be precipitated
at the lower edge of inner radiation belt (L=1.2). An­
other point which may be noted is that the resonance
energy of energetic electrons decreases with increas­
ing frequency of the resonating whistler-mode waves.
Therefore, whistler-mode waves of higher frequency
would be required for the precipitation of lower (keV)
energy electrons at the lower edge of inner radiation
belt.

From Fig. 2, it is observed that pitch angle scatter­
ing and diffusion increase with increasing equato­
rial pitch angles. It is contrary to our understanding
that farther the pitch angles from the loss cone, less
efficient will be the diffusion of electrons into the
loss cone. Again, at low L-shells, energetic electrons
with pitch angles ~ 90° are stably trapped. In order to
tackle this problem, theories similar to those given by
Lyons et al.⁴⁴ and Abel and Thorne⁴⁸ should be em­
ployed, because they have clearly shown a sharp and
rapid fall in the diffusion coefficient at higher pitch
angles near 90°. In their studies⁴⁻⁷, bounce averaged
diffusion coefficients with weighting functions in­
volving error function and Bessel function have been
computed. We intend to employ such type of theories
in our future work.

In order to estimate the precipitated flux of ener­
getic electrons, first the minimum lifetime corre­
sponding to strong diffusion Tm has been calculated by
using Eq. (12) which gives Tm = 0.1 s at L=1.2.
The values of Tm, Tl (=1/D) and Jp are then substi­
tuted into Eq. (11) to get the precipitated flux in el. cm⁻² s⁻¹
which is finally converted into ergs cm⁻² s⁻¹ by multi­
plying it by the corresponding resonance energy (in
ergs) of energetic electrons. The calculated precipi­
tated fluxes of energetic electrons resonating with
oblique whistler-mode waves of different frequencies
of 550 Hz, 3.2 kHz and 5 kHz at the lower edge of
inner radiation belt (L=1.2) are presented in Table 2.

Table 2 shows that the calculated fluxes of precipi­
tated electrons increase with the increase in the
pitch angle α. The asterisk mark indicates that the
gyro-resonance condition is not satisfied in this case.
From Table 2, the average precipitated flux of ener­
getic electrons resonantly interacting with oblique
whistler-mode waves of frequencies 550 Hz, 3.2 kHz
and 5 kHz are found out to be 2.70 × 10⁻³, 1.91 × 10⁻¹

Thus, it is felt that the loss of the lower energy (~
200 keV ≤ E ≤ 6 MeV) electrons at the lower edge of
inner radiation belt (L=1.2) may very well be ac­
counted for by their first order gyro-resonance inter­
action with non-ducted oblique whistler-mode waves
which are frequently found to prevail in the low lati­
tude region.

<table>
<thead>
<tr>
<th>Pitch angle deg</th>
<th>Precipitated flux Jp in ergs cm⁻² s⁻¹ at frequencies</th>
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<tbody>
<tr>
<td></td>
<td>550 Hz</td>
</tr>
<tr>
<td>50</td>
<td>6.18 × 10⁻⁵</td>
</tr>
<tr>
<td>60</td>
<td>9.66 × 10⁻⁵</td>
</tr>
<tr>
<td>70</td>
<td>1.97 × 10⁻⁴</td>
</tr>
<tr>
<td>80</td>
<td>7.25 × 10⁻⁴</td>
</tr>
<tr>
<td>85</td>
<td>*</td>
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</table>
and $2.49 \times 10^{-4}$ ergs cm$^{-2}$s$^{-1}$, respectively. These fluxes are comparable with the energy flux deposited in the lower ionosphere in the low latitude precipitation zone (energy <20 keV) during magnetically disturbed periods as a result of wave-particle interactions involving ELF-VLF emissions of natural origin\textsuperscript{19}. They are also consistent with the energy flux of $\sim 10^{-4}$-10$^{-3}$ ergs cm$^{-2}$s$^{-1}$ deposited in the lower ionosphere at $L=2.4$ caused due to lightning-induced precipitation\textsuperscript{22}. This result indicates that oblique (non-ducted) whistler-mode waves are likely to cause a significant precipitation of inner belt energetic electrons into the lower ionosphere.

Trapped energetic electrons mirror back and forth along the geomagnetic field line between conjugate points lying at a height of 100 km above the earth's surface. Since the centre of the geomagnetic dipole is offset from the geocentre, the equatorial pitch angle of electrons mirroring at a fixed altitude depends upon longitude. Electrons mirroring below 100 km are precipitated. As a result of their gyro-resonance interaction with whistler-mode waves, the energetic electrons with pitch angles close to the loss cone may be scattered into the loss cone and finally be precipitated into the lower ionosphere. The general precipitation mechanism is, in fact, the wave-particle interaction resulting in scattering into the drift loss cone (DLC) and subsequently eastward drift under the effects of gradient and curvature of the geomagnetic field. The DLC is a dominant factor in the mechanism which provides a repository of electrons closer to the top of the atmosphere from which scattering, directly into the atmosphere, can occur more easily than from the stably trapped electron population\textsuperscript{25}. Another method of describing a precipitating electron is to say that it is in the bounce loss cone (BLC). The edge of the BLC, defined as the equatorial pitch angle of a particle that has its mirror point at 100 km altitude (at $L=1.2$, BLC $\approx 46^\circ$), approximates a boundary between locally trapped and locally precipitating trajectories\textsuperscript{34}. The DLC (in terms of equatorial pitch angle) is defined as the region between the maximum value of the pitch angle at the edge of BLC for a given $L$-value/latitude and the local value at that $L$-value/latitude which varies with longitude. All eastward drifting electrons having pitch angles smaller than the maximum value are precipitated into the atmosphere. Thus, the maximum electron loss occurs in the region of minimum magnetic field intensity at the earth's surface, such as the South Atlantic Magnetic Anomaly (SAMA) region. Wave-particle interactions provide an important pitch angle scattering mechanism in the magnetosphere and, thus, play a role in filling DLC with electrons from the stably trapped population\textsuperscript{25}. In general, electrons which have pitch angles smaller than the maximum value are in DLC and will be precipitated into the atmosphere during their eastward drift cycle, while the electrons within the local BLC precipitate immediately.

**Acknowledgements**

The authors are thankful to the Head of Physics Department and the Principal of their College for providing them necessary research facilities. They also wish to thank the anonymous referees for helpful comments and suggestions.

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