Identification of interfacial heat transfer coefficient during casting solidification based on an inverse heat conduction model

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The interfacial heat transfer coefficients (IHTCs) are necessary for accurate simulation of the casting process. However, it is difficult to determine the values of heat transfer coefficient from experiments due to the influence of various factors, such as contacting pressure, oxides on surfaces, roughness of surfaces, coating material, coating thickness and gap formation caused by the deformation of casting and mold. In this paper, an inverse heat conduction model is introduced to calculate IHTCs from the measured temperatures in the mold. The model is proposed and established based on the least-squares technique and sequential function specification method. A typical profile of heat flux simulating practical conditions of casting solidification is used to investigate the accuracy and stability of the proposed inverse algorithm. In the test process, the effects of some calculation parameters in the inverse model are analyzed. Then, the inverse heat conduction model is applied to determine the heat fluxes and IHTCs during casting solidification and the results show that the proposed model is effective and reliable.

Keywords: Inverse heat conduction method, Casting/chill interface, Heat transfer coefficient, Heat flux

With the rapid development of numerical simulation technology in the last two decades, the solidification simulation of casting has been taken as an effective tool for designing the casting process and improving the quality of casting\textsuperscript{1,2}. However, some uncertainties must be eliminated before such simulations can be widely accepted as realistic descriptions of the process. The heat transfer at the casting-mold interface is one of these uncertainties. In general, it is difficult to determine the values of heat transfer coefficient from experiment due to the influence of various factors, such as contacting pressure, oxides on surfaces, roughness of surfaces, coating material, coating thickness and gap formation due to the deformation of casting and mold\textsuperscript{3-6}, etc, especially for castings in metal molds. Fortunately, in recent years, inverse heat conduction algorithms based on the time-temperature data measured during casting solidification have been developed and successfully applied to determine the IHTCs\textsuperscript{7,10}. This methodology was initially proposed by Beck for aerospace applications\textsuperscript{11}. Subsequently it has been developed by many researchers to solve inverse heat conduction problems for various applications, such as casting/mold IHTCs estimation\textsuperscript{12}, parameters identification of material characterization\textsuperscript{13}, heat flux calculation during impingement water cooling\textsuperscript{14} and in determining convection heat transfer coefficients\textsuperscript{15}.

In the present paper, an inverse heat transfer analysis procedure is developed based on the least-squares technique and sequential function specification method\textsuperscript{11} to determine the casting-chill heat fluxes and IHTCs. Before the proposed inverse model is used, its accuracy and stability is investigated by assuming a typical profile of heat flux simulating practical conditions of casting solidification to be known. Meanwhile, the effects of some calculation parameters in the inverse algorithm are analyzed to further improve the stability of the calculated results. In addition, the calculation results of heat flux values and IHTCs are compared with those available in literatures to validate the inverse methodology used for determination of the casting-chill IHTCs.

**Experimental Procedure**

In this work directionally solidified tapered cylinder castings were used to obtain the temperature data required for application of the mathematical
model. A schematic representation of the experimental setup including the position of the thermocouples is shown in Fig. 1. The mold sides were insulated by ceramic wool and the copper chill. Two K-type thermocouples with the diameter of 0.1 mm protected by a 2.3 mm diameter ceramic sheath were positioned within the mold and the chill at 1 mm from the interface respectively, as shown in Fig.1, to measure the evolution of temperature with time. The data from the thermocouples was recorded at a rate of 50 Hz. The thermocouples tips were twisted and spot welded. The thermocouple data was recorded using a Measurement Computing USB-2416 data acquisition system and PC. For the inverse method to be applied successfully, it is necessary to optimize the location of the thermocouples. It should not be too close to the interface because the temperature may not be representative and the inverse method might amplify the noise present in the sampled data (stochastic response). It should also not be too far from the interface because the inverse method could become unstable. The experiments were performed with an A356 alloy (Al-7Si-0.4Mg). The alloy was melted in a resistance furnace using a graphite crucible and was hand poured at 750°C into the fireclay mold. It took approximately 9 s to pour the mold. The lip of the crucible was approximately 150 mm above the mold top. The thermo-physical parameters for the alloy and chill are summarized in Table 1.

Mathematical Model

Forward heat conduction model

Forward heat conduction problems are defined as those in which the temperature in a body or domain is calculated by solving the heat transport equation subject to the appropriate boundary conditions and initial conditions (if transient). In the experimental set-up shown in Fig.1, the chill heat conduction belongs to a three-dimensional problem. However, it should be noticed that direct calculation of transient temperature field must be continuously solved for each of inverse iteration; thus, it will be very large if the temperature field is calculated as a three-dimensional problem. Therefore, the physical model during inverse calculation need be simplified. Barone and Caulk proposed a transient heat conduction model in 1993 and think that the temperature changes in the chill is violent near the interface at the casting-chill. Moreover, the heat flux in the chill close to the interface transfers vertically against the interface and may be considered as a one-dimensional problem.

In this paper, the heat transfer mainly exists at the interface between the casting and chill with adequate insulation of the chill and casting chamber, as shown in Fig. 1. Such surrounding heat isolation environment is designed so that the heat flux in the chill mainly transfers vertically against the interface. Meanwhile, the thermocouple was positioned within the chill at 1 mm very close to the interface. Therefore, for the convenience of calculation, the chill can be reasonably approximated from the interface to the location of the water-cooling channels as a 1-D domain. The general form of the 1-D direct heat conduction equation can be written as:

![Fig. 1—Schematic diagram of the experimental set-up (all dimensions in mm)](image)

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, kg·m⁻³</th>
<th>Specific heat, kJ·(kg·K)⁻¹</th>
<th>Latent heat, kJ·kg⁻¹</th>
<th>Thermal conductivity, W·(m·K)⁻¹</th>
<th>Solidus temperature, K</th>
<th>Liquidus temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Casting (A356)</td>
<td>2.685×10³</td>
<td>0.967</td>
<td>249</td>
<td>166.12</td>
<td>829</td>
<td>889</td>
</tr>
<tr>
<td>Chill (copper)</td>
<td>8.9×10³</td>
<td>0.385</td>
<td>-</td>
<td>383.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
\[\rho c_p \frac{\partial T}{\partial t} = k \left[ \frac{\partial^2 T}{\partial x^2} \right] + Q \quad \ldots (1)\]

Where \( \rho \) is the density of the melt (kgm\(^{-3}\)), \( C_p \) is the specific heat (kJkg\(^{-1}\)K\(^{-1}\)), \( T \) is the temperature (K), \( t \) is the time (s), \( k \) is the thermal conductivity (Wm\(^{-1}\)K\(^{-1}\)), and \( Q \) is a volumetric source term associated with the latent heat of solidification (Wm\(^{-3}\)). In certain cases the energy transport equations can be solved analytically, whereas for more complex cases numerical methods such as the finite difference or finite element methods must be used. In the paper, the finite element method was used to solve the transient temperature field within the casting and chill in the solidification process. The finite element solution is based on one of weighted residual method called Galerkin’s method, in which weight functions are set equal to shape function \( N_i \). So a weight function or so-called trial function for Eq. (1) is assumed:

\[\sum N_i T_i = [N]^T \{T\} \quad \ldots (2)\]

where \( T_i \) is the nodal temperature. \([N]\) and \([T]\) are the shape function matrix and the temperature vector for this element, respectively. By substituting Eq. (2) into Eq. (1) and requiring the residual to vanish, the following vector form of transient heat conduction equation can be achieved after a series of algebraic manipulations:

\[\int_0^L N_i \left[ \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) \right] dx + Q - \rho c_p \frac{d}{dt} \{N \{T\} \} = 0 \quad \ldots (3)\]

Eq. (3) was solved subject to the following boundary conditions and initial condition:

\[k \frac{\partial T}{\partial x} \bigg|_{x=0} = -q_{heating} \quad k \frac{\partial T}{\partial x} \bigg|_{x=L} = q_{heating} \quad \ldots (4)\]

\[k \frac{\partial T}{\partial x} \bigg|_{x=0} = h_{cooling} (T - T_c) \quad k \frac{\partial T}{\partial x} \bigg|_{x=L} = -h_{cooling} (T - T_c) \quad \ldots (5)\]

\[T = T_0(x) \bigg|_{t=0} \quad \ldots (6)\]

Eqs (4) and (5) are called Neumann boundary condition and Cauchy boundary condition, respectively.

**Inverse heat conduction model**

With adequate insulation of the casting at the mold wall, the heat flow through the casting can be reasonably approximated as a one dimensional heat transfer problem in this study. The interfacial heat fluxes during the solidification process are calculated by an inverse heat conduction method based on the measured temperature inside the chill. Then, the IHTCs can be determined by the following equation.

\[h = q / (T_{casting} - T_{chill}) \quad \ldots (7)\]

where \( h \) is the interfacial heat transfer coefficient, \( q \) is the interfacial heat flux, \( T_{casting} \) and \( T_{chill} \) are the surface temperatures of the casting and the chill, respectively. Based on sequential function specification method\(^{14,18}\), the heat flux, \( q(t) \), need be estimated as a series of discrete fluxes \( q_i = q_1, q_2, q_3 \ldots q_N \) with a time interval \( \Delta \theta \).

An algorithm flow chart which uses the inverse heat conduction method to solve an inverse heat conduction problem is shown in Fig. 2. An objective function

![Fig. 2–Flow chart of inverse heat conduction algorithm](image-url)
function in the least-squares method is established to find the value of \( q^i \) that minimizes the difference between the measured and calculated temperature in each time interval.

\[
F(q^i) = \frac{1}{M} \sum_{n=1}^{M} \left( T_{n}^{i+R\Delta\theta} - Y_{n}^{i+R\Delta\theta} \right)^2
\]  

(8)

where \( M \) is the number of the heat flux to be predicted (\( M \) is 1 in this paper). \( T_{n}^{i+R\Delta\theta} \) and \( Y_{n}^{i+R\Delta\theta} \) is the calculated and the measured temperature at time \( i + R\Delta\theta \) respectively where \( R \) is a number of future time steps to compensate for the heat difference caused from the lagging of temperature response at the sub-surface location. The function \( F \) is called the objective function and is minimized through iterations. In an iteration, the heat flux is increased by \( \Delta q \), which is calculated by:

\[
\Delta q = S^{-1} \times \Delta T^{i+R\Delta\theta}
\]  

(9)

where \( S \) is the sensitivity coefficient and defined as the temperature response at the measurements point with respect to a unit change in heat flux. It can be calculated by approximating the derivative using numerical differentiation. The calculation for each sensitivity coefficient in the matrix is performed using:

\[
S_i = \frac{\partial T}{\partial q^i} = \frac{T(q^i + \varepsilon q^i) - T(q^i)}{\varepsilon q^i}
\]  

(10)

where \( i = 1, 2, \ldots, N \). The value of \( \varepsilon \) is a small value used for numerical differentiation, i.e., 0.001.

Updating the current value for \( q^i \) is simply calculated by adding the iteration value, \( \Delta q \), to the current value. A damping factor, \( \mu \), can be used to avoid over-stepping and is usually set to a value close to but less than 1. The iteration equation is written as:

\[
q_{r+1}^i = q_r^i + \mu \Delta q
\]  

(11)

Once the difference between the measured and predicted temperatures satisfies the tolerance criteria, i.e., \( |\Delta T^{i+R\Delta\theta}| \leq T_{cr} \), an estimate of \( q^i \) can be obtained. At this point, the time is increased by \( \Delta \theta \), \( q^i \) is used as the initial heat flux guess for the next time step and the process is repeated for \( q^{i+1} \), yielding the full history of \( q(t) \). \( T_{cr} \) in this algorithm is called the criterion temperature. The use of a large value of a criterion temperature may reduce the accuracy of the calculation, but if it is too small, long calculation time is incurred without appreciably improving the accuracy.

**Results and Discussion**

**Inverse heat conduction model test**

The developed inverse heat conduction model in this paper was tested with verification cases that were designed to simulate the heating of chill in the casting process before application. In the test procedure, a typical value of heat flux was used as an input to obtain the corresponding temperature field by solving the direct heat conduction problem. The heat fluxes were then calculated using the inverse method from the known temperature data of 1 mm from the surface of the chill and compared with the assumed heat flux. The typical value of heat flux is assumed to change with time and is shown in Fig. 3.

Figure 4 shows the results of calculated heat flux with different parameters including the damping factor \( \mu \) and the tolerance criteria \( T_{cr} \). Figures 4a and 4b are the variation of calculated heat flux with different values of \( \mu \) when the value of \( T_{cr} \) are kept constant at 0.5°C. In Fig 4a, it can be clearly seen that the heat fluxes obtained through the inverse model are fluctuant at the initial calculation stage. After about 10 s, the calculated and real heat fluxes are in good agreement. Moreover, when the value of \( \mu \) is near 1, the heat flux fluctuations are the least and the better results can be obtained, as shown in Fig. 4b. The fluctuations at the beginning of calculation are also found in Fig.4c. Figure 4c shows the effect of the tolerance criteria \( T_{cr} \) on the calculated heat flux. In these cases, the value of \( \mu \) is kept at a constant value of 0.25. It can be clearly observed from Fig. 4c that the fluctuations of the calculated heat fluxes become worse with the increase of the value of \( T_{cr} \).

From the above discussion, it can be well known that the proposed inverse algorithm is generally stable.
and accurate for determining the interfacial heat fluxes during casting solidification. But the results of inverse calculation are very sensitive to some calculation parameters, such as the damping factor $\mu$ and the tolerance criteria $T_{cr}$. If these parameters are not selected as appropriate values, the inverse results would cause quite large fluctuation. It can be also well known from the above inverse algorithm that the inappropriate values of damping factor $\mu$ and tolerance criteria $T_{cr}$ will cause a very large error during iteration calculation.

Application to determination of interfacial heat fluxes and IHTCs during casting solidification

Experimental cooling curve

Figure 5 shows the temperature variation with time measured at the location T1 in the casting and T2 in the chill. The details of locations are described in Fig. 1. At the location T1 and T2, the distance to the interface between the casting and chill is both 1 mm. The temperature curve at the location T1 in Fig. 5 may be taken as a typical cooling curve during solidification of the alloy against copper chill. In the initial period, the temperatures drop rapidly down due to the cooling of chill. When the temperatures of casting decrease down to near solidus curve, about 556°C, the temperature gradients decrease slightly. This can be attributed to the latent heat released during solidification of molten metal. Then, at about 90 s, the gradients decrease by a second sharp drop in temperature. Figure 5 shows that the temperature of casting is under solidus curve in this time. This indicates that the chill cooling play a leading role to the temperature change of casting again after the
latent heat is completely released. In Fig. 5, it can still be obviously observed that the temperature variation in the chill has an inverse trend compared with the casting. This is reasonable because the heat transfer mainly exists at the interface between the casting and chill with adequate insulation of the chill and casting chamber, as shown in Fig. 1. Such surrounding heat insulation environment is designed so that the chill cooling may be considered as the main contribution to the decreasing of casting temperature, but the temperature variation in the chill is mainly attributed to the heat transferred out from the casting. Meanwhile, for the convenience of calculation, the heat flow through the casting can also be reasonably approximated as a one dimensional heat transfer problem.

**Interfacial heat fluxes and heat transfer coefficients**

It is well known that some calculation parameters have significant influence on the accuracy and stability of calculated heat fluxes. Therefore, the damping factor $\mu$ and the tolerance criteria $T_{cr}$ are selected as 0.9 and 0.05 respectively when the interfacial heat fluxes and IHTCs during casting solidification are determined using the proposed inverse model. For determining the IHTCs, among the cooling curves obtained above, the measured temperature data at the location T2 is used as the known temperature history for identifying the interfacial heat fluxes. Figure 6 shows the variations of the interfacial heat fluxes with time with copper chill. The temperature curve of the location T1 in the casting is also shown in Fig. 6 for the convenience of analyzing IHTC change. From Fig. 6, it can be clearly seen that at the beginning of casting solidification, the interfacial heat fluxes decrease rapidly down to its minimum value. Then, the interfacial heat fluxes keep constant at between 20 s and 60 s. When the temperatures decrease by a second sharp drop at about 90 s, the heat fluxes increase up to a local peak value. 

Afterward the interfacial heat fluxes decrease gradually with the temperatures in the location T1 decreasing. The variation trends of heat fluxes are very similar to those reported in literature.$^{19}$ Figure 7 illustrates the variation of IHTCs with time and the measured temperature curve for the casting. In Fig. 7, it is apparent that the IHTCs curve follows the same trend as that of heat fluxes as shown in Fig. 6 before the temperatures reach to solidus curve. At the beginning of casting solidification, IHTCs decrease sharply from its maximum value of 5000 Wm$^{-2}$K$^{-1}$ to minimum value of 250 Wm$^{-2}$K$^{-1}$ and increase gradually up to a local peak value after 60 s. Under solidus curve, the IHTCs decrease gradually down to about 1250 Wm$^{-2}$K$^{-1}$ and then remain almost unchanged. On the other hand, the maximum value of IHTCs is higher than those for steel chill reported in the literature$^{19-21}$. The difference is mainly due to a better thermal conductivity of the copper chill.

The rapid decreasing of interfacial heat fluxes and IHTCs can be attributed to the formation of gap at the interface between the solidified casting and the chill. When the gap is formed, the perfect contact between the chill and the solidified casting no longer exists because of contraction of the solidified casting. A thin gap is formed in the casting-mold interface. The heat is transferred through the gap at interface by means of convection, radiation and gas conduction. With the contraction of the solidified casting, the gap size continues increasing so that the heat fluxes and IHTCs decrease continuously. After the temperatures of the casting decrease down to the solidus curve, the

![Fig. 6–Calculated interfacial heat fluxes and measured temperatures in the casting versus time for copper chill](image1)

![Fig. 7–Variation of identified IHTCs and measured temperatures in the casting change with time for copper chill](image2)
stable gap is formed. At the moment the IHTCs reach to its minimum value. As for the local peak, it can be explained through a hypothesis proposed by Hwang et al. Hwang et al. thought that the possible exudation of eutectic liquid causes decreasing of the gap dimensions at the interface between casting and chill at later stages of solidification. The change of gap leads to increasing of heat transfers in the interface which increases the fluxes and IHTCs. After the IHTCs reach the local peak, it decreases slightly and almost remains a constant. It can be clearly observed from Figs 6 and 7 that the casting solidification has also been completed at this moment. The effect of volume contraction due to solidification might be balanced by the thermal expansion of the mold that might form a stable air gap between the casting and mold surfaces. This can contribute to a stable heat transfer through the interface from the casting to the mold and may be taken as a major factor for IHTCs to remain a constant.

**Verification of the identified heat transfer coefficients**

In order to further confirm the validity of the proposed model in the present study, the values of IHTCs obtained above are put into the forward heat conduction model with the same boundary condition. The temperature distribution at the location 1 in the casting are calculated and then compared with the measured temperatures. The results are shown in Fig. 8. It can be seen that a good agreement is obtained between the simulated and experimental temperatures. The relative error between the numerically calculation and measurements is lower than 3°C. Therefore, the feasibility of using the proposed inverse method to calculate the IHTCs can be verified.

**Conclusions**

In this paper, an inverse heat conduction model has been developed to determine the IHTCs between the casting and metal chill. The following conclusions can be drawn from this study:

(i) The effect of calculation parameters on the determined interfacial heat fluxes have been analyzed in the proposed inverse model. The analysis indicates that the fluctuation of determined heat fluxes in inverse calculation may be damped by selecting the appropriate value of parameters, such as the damping factor $\mu$ and the tolerance criteria $T_{cr}$.

(ii) The casting-metal mold IHTCs has been successfully determined by using the inverse heat conduction method based on the measured temperatures within the chill. The correctness of inverse algorithm has been verified through the comparison between numerically calculated and experimental results based on the identified IHTCs used in the finite element analysis. The results adequately demonstrate that the presented inverse algorithm and procedures are very simple, feasible and effective for determination of the casting-mold IHTCs.

(iii) The identified IHTCs vary with time during casting solidification and the values have varied in the range of about 250-5000 Wm$^{-2}$K$^{-1}$. The variation trends have shown good agreement with to those in the literature. Moreover, the variations of IHTCs with time are complex and local peak values exist when the casting temperatures decrease down to near the solidus curve. This may be attributed to the complex heat transfer during casting solidification.

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