Multiplicity dependent pion azimuthal fluctuation study with factorial correlator

Dipak Ghosh, Argha Deb, Subrata Biswas & Rittika Sarkar
Nuclear and Particle Physics Research Centre, Department of Physics, Jadavpur University, Kolkata 700 032, India
E-mail: dipakghosh_in@yahoo.com, deegee111@gmail.com
Received 12 March 2012; revised 29 December 2012; accepted 10 January 2013

The multiplicity dependence of fluctuation of pions produced in $^{32}$S-AgBr interactions at 200 AGeV in azimuthal angle ($\phi$) space in terms of a very rigorous technique (factorial correlator), has been studied in the present paper. The analysis shows that the strength of non-statistical fluctuations increases with the increase of multiplicity.

Keywords: Dynamical fluctuations, Multiplicity dependence of pions, Nucleus-nucleus interactions, Factorial correlator

1 Introduction
A number of experimental investigations over the last few decades have revealed non-statistical fluctuation in multiparticle production process in all type of interactions such as lepton-lepton, lepton-nucleus, hadron-hadron, hadron-nucleus and nucleus-nucleus interactions at a few GeV to few hundred GeV. Such large density fluctuations have been investigated by a powerful technique by Bialas and Peschanski$^1$ which involves the computation of scaled factorial moments (SFMs) as a function of the decreasing phase space interval size. According to them, the power law dependence of normalized scaled factorial moments with decreasing phase space cells characterizes the intermittent behaviour. This intermittent type of non-statistical fluctuations has been predicted to occur as a result of the transition from the quark-gluon-plasma to normal hadronic matter and all the interest was centred around self-similarity studies. Various experimental data sets$^{2-11}$ are there in support of this intermittent behaviour, but the results from the analyses of these various data are not enough for an unambiguous interpretation. Some alternative suggestions like conventional short-range correlations$^{12}$, formation of jets and mini-jets$^{13}$, self-similar random cascading mechanism$^{14}$ have also been proposed. There is also a strong feeling that the Bose-Einstein (BE) interference can play a role in producing self-similar dynamical fluctuations$^{15}$. BE correlations arise because the wave-functions of identical bosons are symmetric. This correlation between equal charged particles could produce rising of the factorial moments with decreasing bin size. It would make intermittency stronger for equal charged particles than for all charged particles. BE correlations are stronger for particles having a smaller difference of three-dimensional momenta. For NA22 data of hadron-hadron collision$^{16}$, a small increase of the slopes is indeed observed when restricting the analysis to identical charged particles. On the other hand, $e^+e^-$ data of TASSO Collaboration$^{17}$ show even smaller slopes for the same sign charge than for the full sample of $e^+e^-$ data. The $\mu$-p data$^{18}$ also supply evidence against the expectation. These observations lead to the conclusion that BE effect cannot be considered as the main source of intermittency, at least for $e^+e^-$, lepton-hadron and hadron-hadron collisions. Such data do not exist for nuclear reactions. However, the results from EMU01 data$^{19}$ exclude the BE correlation as a dominant source of intermittency in heavy ion interactions. So intermittent behaviour cannot be fully explained in terms of the conventional explanations. There is at least a small part of intermittent rise of factorial moment with bin size which remains unexplained. We want to stress the point that an effect which is small cannot be necessarily ignored. Hence, the intermittency should be studied more and more rigorously. Here, we study intermittency in terms of factorial correlator (FCs). This approach, as suggested by Bialas and Peschanski$^{1}$, is an important addition, since it provides extra information about bin-bin correlation. They measure not only the non-statistical local density fluctuation, but also give information about the correlation between these local density fluctuation. However, compared with analysis in terms of SFMs, analysis of data in terms of FCs is relatively scarce.
Recently, WA98 collaboration has performed a detailed event by event fluctuation study in the multiplicities of charged particles and photons and the total transverse energy in 158 AGeV Pb+Pb collisions\(^{20}\). The observed result shows that fluctuations increase with the increase in the impact parameter interval. The multiplicity fluctuation has been studied in terms of scaled variances for most central Pb+Pb collisions at 20 A, 30 A, 40 A, 80 A and 158 AGeV as measured by the NA49 experiment at CERN SPS. The scaled variance of multiplicity distribution was found to increase for decreasing rapidity and transverse momenta\(^ {21}\). The STAR experiment at RHIC has measured the dynamical net charge fluctuations in Au+Au collisions at 157 AGeV Pb+Pb collisions at

\[
\sqrt{S_{NN}} = 19.6, 62.4, 130, 200 \text{ GeV, Cu+Cu collisions at } \sqrt{S_{NN}} = 62.4, 200 \text{ GeV and P+P collisions at } \sqrt{S} = 200 \text{ GeV using the measure of a robust observable } \nu_{\text{max}}. \] \(^{22}\)

It was observed that the dynamical net charge fluctuations are non vanishing at all energies and exhibit a modest dependence on beam energy for Au+Au as well as Cu+Cu collisions. Recent investigations of NA49 data at CERN SPS have revealed that the behaviour of multiplicity fluctuation with collision centrality is non monotonic\(^ {23}\). Such tremendous enthusiasm in studying fluctuation effects stem from one major curiosity about the possible formation of quark-gluon-plasma (QGP) in nuclear collision. But later on, it is found that different data support different interpretations. So a more discriminative information is needed experimentally.

In recent years, a number of investigations\(^ {18,24-28}\) have been carried out with the help of ‘Factorial Correlators’. In these studies, fluctuations in pion production process have been probed with the help of factorial correlators. In all the cases, the results suggest self similar fluctuation pattern in hadronization process and support \( \alpha \)-model of particle production. Bialas and Peschanski\(^ {3}\) introduced the concept of this general mathematical model, namely the \( \alpha \)-model, which leads to the scaling law of moments, independently of the initial and final range of observation and for any rank of the moments. The \( \alpha \)-model describes each multiparticle event as a series of random cascading steps starting from some initial interval and dividing into smaller ones. The multiplicity dependence study of fluctuation pattern in terms of factorial correlators (FCs), for pions, emitted from \( ^{32} \)S-AgBr interactions at 200 AGeV, in \( \phi \)-space, has been the subject matter of investigation of this paper. Intermittency analysis can be performed in any variable whose distribution is smooth. Generally, analyses are done in rapidity space but it is equally important to probe azimuthal angle space. There are very few reporting of factorial moment analyses in azimuthal angle space\(^ {29-31}\). However, such studies in terms of factorial correlator are very scanty. Therefore, factorial correlator analysis in azimuthal angle space (\( \phi \)-space) has been studied. This study reveals self-similarity in pion production and a positive multiplicity dependence. It also supports \( \alpha \)-model of intermittency as indicated in the previous works\(^ {18,24} \).

2 Experimental Details

The data used in this present analysis were obtained by exposing G5 nuclear emulsion plates by \( ^{32} \)S beam with incident energy 200 AGeV at CERN SPS.

The scanning of the plates is carried out with the help of a high resolution Leitz metallopland microscope provided with semi-automatic scanning and measuring system. The scanning is done using objective 10X in conjunction with a 25X ocular lens. To increase the scanning efficiency, two independent observers scanned the plates independently. For measurement, 100X oil immersion objective is used in conjunction with 25X ocular lens. The measuring system fitted with it has 1 \( \mu \text{m} \) resolution along the X and Y axes and 0.5 \( \mu \text{m} \) resolution along the Z axis. The emulsion plate lies in the XY plane. Hence, X or Y axis is the beam axis and Z is the perpendicular axis to it.

Events are scanned according to the following criteria:

(a) The beam track should lie within 3\(^ {\circ} \) angles to the mean beam direction of the pellicle. (b) The events, which are within 20 \( \mu \text{m} \) thickness from the top or bottom surface of the plate, should be rejected. (c) After initial scanning, few events are thought to be the primary events. The supposed beam tracks of these events are traced back up to the edge of the emulsion plate for confirmation. If it is found that these tracks originate from some other events in its path, then it is concluded that the pre-assumed primary events are not the primary one but some secondary events of another interaction. These events should not be analyzed and are rejected.

According to the emulsion technique, the particles emitted after interactions are classified as:
(a) **Black particles** — Black particles consist of both single and multiple charged fragments. They are target fragments of various elements such as carbon, lithium, beryllium etc. with ionization greater than or equal to 10$I_0$, $I_0$ being the minimum ionization of a singly charged particle. These black particles having maximum ionizing power are less energetic and consequently, they are short ranged. Their range is less than 3 mm in emulsion medium. They have velocities less than 0.3$c$ and energy less than 30 MeV, $c$ is the velocity of light in vacuum. In the emulsion experiments, it is very difficult to measure the charge of the fragments. So identification of the exact nucleus is not possible.

(b) **Grey particles** — They are mainly fast target recoil protons with energy up to 400 MeV. They have ionization 1.4 $I_0$ ≤ $I$ < 10 $I_0$. These particles have range greater than 3mm in emulsion medium and having velocities 0.7$c$ ≥ $V$ ≥ 0.3$c$.

(c) **Shower particles** — The relativistic shower tracks with ionization $I$ less than or equal to 1.4$I_0$ are mainly produced by pions and are not generally confined within the emulsion pellicle. These shower particles have energy in the GeV range.

(d) **Projectile fragments** — Along with these tracks, there are a few projectile fragments. In high energy nuclear collisions, the projectile beam which collides with the target nucleus also undergoes fragmentation. These particles have constant ionization, long range and small emission angle. They generally lie within 3° with respect to the main beam direction. Great care should be taken to identify these projectile fragments.

We have chosen only the events with at least eight heavy ionizing tracks of (black+grey) particles so that targets chosen are silver or bromine. The events that have the number of heavy tracks less than eight, are due to the collision of the projectile beam with carbon, nitrogen and oxygen nucleus present in the emulsion. These types of events are called CNO events. Following the above selection procedure, we have chosen 140 events of $^{32}$S-AgBr interactions at 200 AGeV. In our data sample, the average number of pions is 95.8±3.68. The azimuthal angle φ is measured for each track by taking the readings of the coordinates ($X_0, Y_0, Z_0$) of the interaction point, the coordinates ($X_i, Y_i, Z_i$) at any point on each secondary track and the coordinates ($X, Y, Z$) of a point on the incident beam.

Nuclear emulsion covers $4\pi$ geometry and provides very good accuracy in the measurements of azimuthal angles of produced particles due to high spatial resolution and thus, is suitable as a detector for the study of factorial correlators in azimuthal angle space. However, like other detectors nuclear emulsion plates are not free from systematic errors. Systematic errors may be introduced in emulsion plates due to the presence of background events which may result from the cosmic rays during the exposure time. These background events can be eliminated by properly choosing the primary events due to incident beam. Systematic errors may also arise in the measurement of polar angles due to fading of tracks and variation of shrinkage factor with temperature. More important source of systematic error for this kind of hadron density fluctuation works may arise due to counting of electron pairs produced by Dalitz decay or photon-conversion as hadrons. Special care has been taken to avoid such contamination. However, Dalitz production alone is negligible and the systematic error due to combined error is, generally, lesser than the quoted statistical error for intermittency related works as the percentage of converted gamma is low.

3 **Factorial Correlator Analysis**

The factorial correlator study has been performed in the azimuthal angle space for pions. We have considered the azimuthal angle interval $\Delta \phi$ which is subdivided into $M$ bins of width $\delta \phi = \Delta \phi / M$. Two bins, the $m^{th}$ and $l^{th}$, having a separation $D$ are chosen so that $D = d \times \delta \phi$, where $d = ln - l$. $D$ is known as the correlation length. The factorial correlator $F_{ij}$ of order $i\times j$ between the $m^{th}$ and $l^{th}$ bins is defined as:

$$F_{ij}^{m,l}(\delta \phi) = \frac{\left\langle n_m (n_m - 1)(n_m - i + 1)n_l (n_l - j - 1)\right\rangle - \left\langle n_m (n_m - 1)(n_m - i + 1)\right\rangle \left\langle n_l (n_l - 1)(n_l - j + 1)\right\rangle}{\left\langle n_m (n_m - 1)(n_m - i + 1)\right\rangle \left\langle n_l (n_l - 1)(n_l - j + 1)\right\rangle}$$

where $n_m$ and $n_l$ are the number of particles in the $m^{th}$ and $l^{th}$ bins, respectively. $\langle \rangle$ denotes an averaging over whole sample of events.

In order to increase statistics, the average is calculated for all bin combinations with a given distance $D$. Thus:
Factorial correlators of order (i x j) with index (i, j) equals to (1,1); (2,1); (2,2); (3,1); (3,2) and (3,3) have been calculated by using Eq. (2) for each bin width for the above two cases. The power law dependence has been studied for different bin widths \( \delta \phi = 20^\circ, 13.3^\circ \) and \( 10^\circ \) using Eq. (3) and the variation of \( \ln C_{ij} \) as a function of \( -\ln D \) have shown in Fig. 1(a-c) for \( \delta \phi = 20^\circ, 13.3^\circ \) and \( 10^\circ \), respectively for pions for the lower multiplicity data. Similar plots have been shown in Fig. 2(a-c) for \( \delta \phi = 20^\circ, 13.3^\circ \) and \( 10^\circ \), respectively for pions for the higher multiplicity data set. If the relation between the two \( (\ln C_{ij} \text{ and } -\ln D) \) is linear then it suggests an intermittent type of fluctuation. The error bars shown in Fig. 2 are nothing but the statistical errors obtained from the dispersion of the \( F_{ij} \) values for different bin combinations. The increase in \( \ln C_{ij} \) with \( -\ln D \) is not linear in the full \( D \) range as predicted by the \( \alpha \) model of intermittency. It is seen in other studies\cite{24, 25, 26} also that the scaling behaviour does not hold beyond a particular \( 'D' \) value. It may be suggested that if the correlation length, \( D \) is greater than a critical value the fluctuation patterns do not show scaling behaviour that is they are not self-similar in nature any more. In higher order, the fits do not show systematic trend as the number of particles per beam range is very few. However, our analysis shows that the relationship between \( \ln C_{ij} \) and \( -\ln D \) is almost linear in restricted \( D \) region where \( D \leq 59^\circ \). The behaviour of correlated moments at large \( D \) is largely controlled by the long range correlations. The exponents \( \alpha_i \) are calculated by performing best linear fits in the selected regions.
20°≤D≤59°, 13.3°≤D≤53° and 10°≤D≤59° for bins 18, 27, and 36, respectively for lower multiplicity data. Similar calculations are done for higher multiplicity data by performing best linear fits in the selected regions 20°≤D≤59°, 13.3°≤D≤53° and 10°≤D≤50° for the three bin widths. The maximum and minimum D values depend on δφ. These D values are different for different bin widths for both multiplicity range. Tables 1 and 2 show the slope values (αij) for different bin widths for pions for events having lower and higher multiplicity, respectively. The positive slope values αij clearly indicate that the correlated moments increase with the decreasing correlation length, D. From Tables 1 and 2, it is clear that for both lower and higher multiplicities, the exponent values (αij) decrease with the decreasing bin size (δφ) for any particular order of moment (i×j). It is also evident from Tables 1 and 2 that for a given bin width, δφ, the slope values increase with the increase of order of moments (i×j). It is observed from Tables 1 and 2 that for all the bin widths (δφ=20°, 13.3° and 10°), the values of correlated moments in case of higher multiplicity are more than those of lower multiplicity for the order of moments (1,1), (2,1), (2,2), (3,1), (3,2) and (3,3). This result suggests that the strength of non-statistical fluctuations for all order of moments are more in case of higher multiplicity than those from lower multiplicity. 

The intermittency exponent values are expected to satisfy the following relation’:

\[ \alpha_i = ij \alpha_2 \]  \[ \cdots (4) \]

Fig. 2 — Dependence of lnCij on –lnD for bin widths (a) δφ=20° (b) δφ=13° and (c) δφ=10° for pions having multiplicity >100

<table>
<thead>
<tr>
<th>Order of correlator (ij)</th>
<th>δφ=20°</th>
<th>δφ=13°</th>
<th>δφ=10°</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.353 ± 0.017</td>
<td>0.220 ± 0.011</td>
<td>0.219 ± 0.010</td>
</tr>
<tr>
<td>21</td>
<td>0.431 ± 0.025</td>
<td>0.347 ± 0.020</td>
<td>0.327 ± 0.019</td>
</tr>
<tr>
<td>31</td>
<td>0.533 ± 0.026</td>
<td>0.408 ± 0.024</td>
<td>0.375 ± 0.021</td>
</tr>
<tr>
<td>22</td>
<td>0.687 ± 0.037</td>
<td>0.526 ± 0.032</td>
<td>0.443 ± 0.028</td>
</tr>
<tr>
<td>32</td>
<td>0.811 ± 0.041</td>
<td>0.626 ± 0.031</td>
<td>0.463 ± 0.033</td>
</tr>
<tr>
<td>33</td>
<td>0.942 ± 0.047</td>
<td>0.780 ± 0.044</td>
<td>0.501 ± 0.035</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Order of correlator (ij)</th>
<th>δφ=20°</th>
<th>δφ=13°</th>
<th>δφ=10°</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>0.419 ± 0.025</td>
<td>0.291 ± 0.017</td>
<td>0.251 ± 0.015</td>
</tr>
<tr>
<td>21</td>
<td>0.609 ± 0.030</td>
<td>0.503 ± 0.025</td>
<td>0.416 ± 0.020</td>
</tr>
<tr>
<td>31</td>
<td>0.771 ± 0.039</td>
<td>0.622 ± 0.035</td>
<td>0.494 ± 0.029</td>
</tr>
<tr>
<td>22</td>
<td>1.074 ± 0.055</td>
<td>0.857 ± 0.043</td>
<td>0.674 ± 0.033</td>
</tr>
<tr>
<td>32</td>
<td>1.260 ± 0.059</td>
<td>1.012 ± 0.050</td>
<td>0.792 ± 0.048</td>
</tr>
<tr>
<td>33</td>
<td>1.451 ± 0.067</td>
<td>1.264 ± 0.063</td>
<td>0.928 ± 0.047</td>
</tr>
</tbody>
</table>

where the equality sign is due to the lognormal approximation. To verify Eq. (1), we have plotted αij as a function of the product (i×j) for different bin widths. Figure 3(a-c) shows the plot of αij versus (i×j) for δφ=20°, 13.3° and 10°, respectively for lower multiplicity data. Similar plots for higher multiplicity data have shown in Figure 4(a-c) for δφ=20°, 13.3° and 10°, respectively. For each bin width, the plot is consistent with the linear growth of

Table 1 — Represents the slope values (αij) of the best-fitted points from the plot of lnCij versus –lnD in the region D ≤ 59° for different bin widths for pions having multiplicity <100

Table 2 — Represents the slope values (αij) of the best-fitted points from the plot of lnCij versus –lnD in the region D ≤ 59° for lower multiplicity data by performing best linear fits in the selected regions 20°≤D≤59°, 13.3°≤D≤53° and 10°≤D≤50° for the three bin widths. The maximum and minimum D values depend on δφ.
the exponent values as predicted by the lognormal approximation. Though the slope values of the curves i.e. $\Delta \alpha_{ij}/\Delta (i \times j)$ are not exactly equal to the intermittency exponent of second order ($\alpha_2$) but they are comparable. The plot of $\ln F_2$ against $-\ln \delta \phi$ have shown in Figure 5(a and b) for lower and higher multiplicity data, respectively. The slope values $\alpha_2$ are extracted by performing best linear fits of those plots.

$F_2$ values are obtained by using the following relation:

$$F_2 = \frac{1}{M} \sum_{m=1}^{M} \frac{\langle n_m (n_m - 1) \rangle}{\langle n_m \rangle^2} \quad \text{(5)}$$

where the total azimuthal angle space region $\Delta \phi$ is divided into $M$ equal bins of size $\delta \phi$, $n_m$ is the number of particles in the $m^{th}$ bin for a particular event. $\langle \rangle$ denotes the average over all the events. The values of $\Delta \alpha_{ij}/\Delta (i \times j)$ for different bin widths ($\delta \phi=20^\circ$, 13.3° and 10°) and $\alpha_2$ are presented in Tables 3 and 4 for lower and higher multiplicity data, respectively. The
Fig. 5 — Variation of $\ln F_2$ against $-\ln \delta \phi$ for pions having multiplicity (a) <100 and (b) >100

Fig. 6 — Plot of $\ln C_{ij}$ against $-\ln \delta \phi$ at $D=40^\circ$ for pions having multiplicity (a) <100 and (b) >100

Table 3 — Represents the comparison of $\alpha_2$ with slope values of $\alpha_i$ versus $(i \times j)$ plots for different bin widths for pions having multiplicity <100

<table>
<thead>
<tr>
<th>$\delta \phi \Delta \alpha_i / \Delta (i\times j)$</th>
<th>$\langle \Delta \alpha_i / \Delta (i\times j) \rangle$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20^\circ$</td>
<td>0.075 $\pm$ 0.008</td>
<td></td>
</tr>
<tr>
<td>$13.3^\circ$</td>
<td>0.031 $\pm$ 0.007</td>
<td>0.057 $\pm$ 0.007</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.067 $\pm$ 0.006</td>
<td>0.098 $\pm$ 0.002</td>
</tr>
</tbody>
</table>

Table 4 — Represents the comparison of $\alpha_2$ with slope values of $\alpha_i$ versus $(i \times j)$ plots for different bin widths for pions having multiplicity >100

<table>
<thead>
<tr>
<th>$\delta \phi \Delta \alpha_i / \Delta (i\times j)$</th>
<th>$\langle \Delta \alpha_i / \Delta (i\times j) \rangle$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$20^\circ$</td>
<td>0.130 $\pm$ 0.018</td>
<td></td>
</tr>
<tr>
<td>$13.3^\circ$</td>
<td>0.118 $\pm$ 0.012</td>
<td>0.110 $\pm$ 0.013</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>0.082 $\pm$ 0.011</td>
<td>0.142 $\pm$ 0.012</td>
</tr>
</tbody>
</table>
slope values of the curves $[\Delta \alpha_c/\Delta \langle j \delta \rangle]$ are different in different bin widths of the correlated moments but the average of these slopes $\langle \Delta \alpha_c/\Delta \langle j \delta \rangle \rangle$ is really comparable with the intermittency exponent of second order ($\alpha_2$). This analysis reveals a scale invariant property of the correlated non-statistical fluctuations in different regions of azimuthal angle space indicating the intermittent nature of particle production in both the multiplicity regions. From Tables 3 and 4, it is clear that the intermittency exponent of second order ($\alpha_2$) is more in case of higher multiplicity data than that in case of lower multiplicity data. According to the $\alpha$ model for fixed $D$, the correlators should be independent of the bin width $\delta \phi$. Figure 6(a and b) shows the variation of $\ln C_{ij}$ against $\ln \delta \phi$ for $D = 40^\circ$ for the two sets of data, respectively. The horizontal lines are drawn through the average values to facilitate observations.

5 Conclusions
One can conclude from the present in-depth investigation of pion fluctuations in the azimuthal angle phase space that the strength of non-statistical fluctuations increases with the increase of multiplicity.

Acknowledgement
The authors express their gratitude to Prof P L Jain, Buffalo State University, USA, for providing us with the exposed and developed emulsion plates used for this analysis.

References