An estimate of kinetic energy maxima of cyclotron-resonant electrons interacting with whistler mode waves

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Active experiments to probe hot plasma in the magnetosphere were conducted at Siple Station, Antarctica. The multi-stations measurements of VLF Siple signals on the ground were also conducted in Canada. The results from the Siple active experiments at L=5.1, the relativistic treatments, and the results obtained from the multi-stations measurements at L=4.1 showed almost similar results. The relativistic treatment of separatrix at L=5.1 theoretically indicates the total energy of trapped electron, which was estimated to be 0.315 to 16 keV (at maximum of $V_\perp$) by estimates of parallel resonance velocity and wave magnetic field amplitude of 11 PT, and the effective instability depending on the weak relativistic electrons with kinetic energy 10.9 keV in the perfect resonance condition. The kinetic energy 10.9 keV is almost coincident with some results of Sonwalkar et al. [Properties of the magnetospheric hot plasma distribution deduced from whistler mode wave injection at 2400 Hz: Ground-based detection of azimuthal structure in magnetospheric hot plasmas, J Geophys Res (USA), 102 (1997) 14363].

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1 Introduction

Active experiments to probe hot plasma in the Earth’s magnetosphere were conducted a number of times between 1973 and 1988 at Siple Station, Antarctica. In these active experiments, whistler mode radio waves amplified in the magnetosphere were received in the Northern Hemisphere conjugate regions, Roberval and Lake Mistissini, Canada. The data obtained near Roberval showed the effective propagation and intensification of Siple signals with constant frequency, moreover, the existences of whistler mode sideband waves around the frequencies of signals transmitted from Siple Station in the dynamic spectra. Furthermore, according to Sonwalkar et al. and Ikeda, the Siple station VLF wave injection experiments found some properties of the hot plasma in the magnetosphere, by analyzing the transmitted Siple signals and their sideband waves amplified in whistler mode.

These results were obtained from in situ observations by satellites and further from observation on the ground near Lake Mistissini, Canada in 1988. For examples of the hot plasmas at L=5.1, known by some analyses of Sonwalkar et al., the kinetic energies, $E_\perp$, perpendicular to the geomagnetic field were estimated to be between 0.6 and 11.0 keV, and the resonant energy, $E_z$, parallel to the geomagnetic field was estimated to be between 0.3 and 1.0 keV. In this paper, these data are re-examined.

About 12 keV of the electron kinetic energy resonant with Siple signal was estimated from the multi-stations observation of VLF Siple signals on the ground in 1979. These multi-station measurements of L=4.1 at Roberval, Canada were conducted by Tsuruda et al. They represented that the spatial distribution of wave intensity varied severely with time. One of the reasons might be because an energetic electron cloud pass through the ducts located to the south from the observation station network. As a result, Ikeda et al. estimated that the velocity of the observed electron cloud seen on the ground might be about 20 km min$^{-1}$ obtained from the drift velocity of gradient-B in the region of L=4.1. This estimation means that the kinetic energy for electron was about 12 keV.

The resonant electrons had been treated as non-relativistic ones in relation to the wave-particle interaction with whistler mode signal, but it can be analyzed if the relativistic treatment is necessary for
this interaction, and the relativistic treatment can be
performed even in the low energy phenomena.
Further, this expectation is verified by using the
observed data of Siple transmitting experiments.
Accordingly, the relativistic resonance condition
with whistler mode signal can be derived in detail.
The electron energy of 0.315 to 16.0 keV (at
maximum of $V_{\perp}$) at L=5.1 was roughly determined by
the estimated parallel resonance velocity of $1.05 \times 10^7$
m s\(^{-1}\) and the wave magnetic field of 11 PT (Ref 9).
The basic objective of the present paper is to state that
the energy of 16 keV is higher than 11 keV inside the
separatrix.

2 Ground estimation of resonant electron energy

The exit areas of Antarctic Siple VLF signals on
the ionospheric lower boundary were estimated by the
multi-stations measurements, which were conducted
in Quebec, Canada in 1979 (Refs 11,12). It was
inferred\(^ {13}\) that there were at least three exit areas from
which VLF waves were radiated in the earth
ionosphere wave guide modes. The whistler duct is
the window of VLF waves intensified from Siple
transmitted signals. The drift velocity of energetic
electrons resonant with the Siple transmitted signals
in the inner magnetosphere was estimated on the
ground at L=4.1, considering the turning-on and off of
VLF wave exit areas. The drift of energetic electron
cloud was due to only the gradient B drift of
longitudinal characteristics. From some data
expressing wave activities in the magnetosphere, the
velocity of the electron cloud was estimated as the
gradient-B drift of about 20 km min\(^{-1}\) seen on the
ground\(^ {10}\) and is shown in Fig. 1 as the moving one of
shaded circles.

These analyses show that the interesting exit area
was within 200 km from the station network (A to G,
and SI), and this distance was within a valid region
for the reasonable multi-station analyses\(^ {13}\). Actually,
the elliptical shape of wave exit area approximately
showed the ionospheric lower boundary projection of
the duct along which the whistler mode waves
propagated. These results were determined from the
data such as the ones of Fig. 2, where the time
variations of the Siple signals were shown in linear
amplitude, and the upper curves in each panel
represent peak amplitude in the one second pulse
duration, while the shaded curves represent mean
amplitude during the same time interval.

The gradient-B drift velocity of $V_{eq}$ of resonant
electrons on the equatorial plane in the magnetosphere
is expressed. First, the bounce-averaged $\nabla B$ drift
angular velocity $\left< \frac{d\phi}{dt} \right>$ of electron cloud on the
equatorial plane is expressed as:

$$\left< \frac{d\phi}{dt} \right> = \frac{3}{eB_0a^2} Lg(a) \left( \frac{1}{2} m_e V^2 \right)$$

where, $a$ is the earth radius ($6.4 \times 10^6$ m); $B_0$ ($3.12 \times$
$10^{-5}$ Wb m\(^{-2}\), geomagnetic field strength on the
ground at the Earth’s equator; $m_e$ electron rest mass
($9.11 \times 10^{-31}$ kg); L value is 4.1; and $e$, the electron
charge of $1.6 \times 10^{-19}$ C. Furthermore, the function
$g(\alpha)$ with pitch angle $\alpha$ is assumed to be given as
follows\(^ {14}\) in the non-relativistic rough expression:

Fig. 1 — Exit areas of Antarctic Siple VLF signals on the
ionospheric lower boundary

Fig. 2 — Amplitude of Siple waves observed at the four stations
D, E, F, and G during 11:30-12:00 hrs UT (Source: Tsuruda et al.\(^ {11}\) )
Accordingly, the gradient-B drift velocity $V_{eq}$ on the equatorial plane is given as follows:

$$V_{eq} = La \left( \frac{d\phi}{dt} \right)$$ ...

(3)

If the geomagnetic field is assumed to be a magnetic dipole field and $L = \frac{1}{\cos^2 \theta}$, the drift velocity $V_g$ at magnetic latitude $\theta = 60.4^\circ (L=4.1)$ on the ground is expressed as:

$$V_g = a \cos \theta \left( \frac{d\phi}{dt} \right)$$ ...

(4)

If the drift velocity on the ground, which shows the value of $V_g = 20 \text{ km min}^{-1}$ estimated by Ikeda et al., is used for a result from Eq. (4):

$$\frac{dV_g}{dt} / a \cos \theta \approx 1.1 \times 10^{-4} / \text{sec}$$ ...

(5)

Furthermore, the pitch angle of resonant electron is assumed to be $70^\circ$. This value was derived by approximately weight-averaging the pitch angles between $40^\circ$ and $80^\circ$. The kinetic energy $E$ of resonant electron in non-relativistic expression is roughly given from Eq. (1):

$$E = \frac{1}{2} m_0 V^2 = \frac{eB_0 a^2}{3Lg(a)} \left\langle \frac{d\phi}{dt} \right\rangle \approx 12 \text{keV}$$ ...

(6)

On the other hand, according to whistler mode cyclotron resonance, Sonwalkar et al. obtained the results that the kinetic energies $E$ of electrons resonant with whistler mode signals were between 0.6 and 11.0 keV and correspondingly their pitch angles were between $40^\circ$ and $80^\circ$ from the ground observation at $L=5.1$. In these cases, the small Lorentz factors, $\gamma = 1 + \frac{E}{m_0c^2}$ are only 1.01-1.03; where, $m_0$, is the rest mass of electron; and c, the light speed. There may be the possibility that the relativistic effects severely appear in the processes of scattering, diffusion, acceleration and precipitation due to whistler mode waves amplified by the Siple-transmitted signals, because the electrons near the separatrix are very sensitive to the influences of noises. As a result, the relativistic effects may influence the frequency gaps of whistler mode sideband waves generated by electrons outside the separatrix. The energy range from 100 keV to 10 MeV corresponds to the severe relativistic treatment, and one of this paper’s purposes is to check this relativistic treatment.

3 Derivation of relativistic motion equations

Resonant interactions between relativistic electrons and whistler mode waves have been already described for instability analyses. Furthermore, pitch angle diffusion and energy diffusion of relativistic electrons by turbulent whistler mode waves, were also examined in order to determine the distribution function of electrons by Sonwalkar et al., Albert and Singh & Singh. The relativistic electrons resonant with whistler mode waves may exist near the plasmapause, and it is imagined that these electrons with relativistic characteristics are possible to generate waves of not only whistler mode but also other modes.

Relativistic motion equations of complex number expression in the electron cyclotron coordinate system resonant with whistler mode signals are derived at first, and then using the unified electric and magnetic field tensor $F_{ij}$ in the expression of relativistic motion equations are shown in the rest coordinate system fixed at the Earth. It is believed that the initial interactions between resonant electrons and whistler mode waves with constant frequency can occur near the equatorial plane. Therefore, the effects of inhomogeneity and so on can be neglected.

$$\frac{dP_i}{d \tau} \approx -eF_{ij}v_j$$ ...

(7)

where, $i$ and $\tau$, respectively mean the imaginary unit and the proper time measured in the object’s rest frame. Further, it is approximately assumed that the electric and magnetic fields are composed of the geomagnetic static field, and the wave magnetic and electric fields. The definition of four-momentum $P_j$, four-velocity $v_j$, and Lorentz factor $\gamma$ is given as follows:

$$P_j = (im_0\gamma c, m_0\gamma V_x, m_0\gamma V_y, m_0\gamma V_z)$$ ...

(8)

$$v_j = (i\gamma c, \gamma V_x, \gamma V_y, \gamma V_z)$$ ...

(9)

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$ ...

(10)
\[ \beta = \frac{1}{c} \sqrt{V_x^2 + V_y^2 + V_z^2} \]  
\[ \ldots (11) \]

where, \( V_x \), \( V_y \) and \( V_z \) are three components of velocity in the frame fixed at the Earth, the coordinate system of which is shown in Fig. 3, and where the plane perpendicular to the geomagnetic field is shown in the \( x - y \) coordinate system. The coordinate system in this rest frame is termed as the rest coordinate system, and the cyclotron resonance condition of the special relativity in this frame fixed at the Earth, is expressed as:

\[ \omega - k V_z - \frac{1}{\gamma} \Omega_0 = 0 \]  
\[ \ldots (12) \]

Using Eq. (12), the definition of cyclotron resonance frame in the special relativity is that both of whistler mode signal and resonant electron make the same rotation in the frame of the cyclotron resonance, given by Sonwalkar \textit{et al.} \textsuperscript{1} and Singh & Singh \textsuperscript{19}. As a result, the resonant velocity \( V_R \) is determined in the special relativity as:

\[ V_R = \frac{\gamma}{k} \left( \frac{1}{\Omega_0} - \omega \right) \]  
\[ \ldots (13) \]

The resonant velocity, \( V_R \), forwards to the direction opposed to the wave vector of whistler mode signal, when \( \gamma \) is near 1. The relations among some velocities are shown in Fig. 4. There are three kinds of the coordinate systems: (a) guiding center frame moving with velocity \( V_z \); (b) resonance frame moving with velocity \( V_R \); and (c) frame fixed at the Earth or Laboratory system. The wave vector indicated by \( k \) arrow, \( V_z \), and the external magnetic field \( B_0 \) are all towards \( z \) direction; while \( V_R \) is towards \(-z\) direction.

The Lorentz transformation to the guiding center frame (\( g\)-frame) moving with velocity \( V_z \) in the frame fixed at the Earth, which is namely termed as the rest frame, is expressed as:

\[ L_g = \begin{pmatrix} \gamma_g & 0 & 0 & -i\beta_g \gamma_g \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ i\beta_g \gamma_g & 0 & 0 & \gamma_g \end{pmatrix} \]  
\[ \ldots (14) \]

where, \( \beta_g \) and \( \gamma_g \) are:

\[ \beta_g = \frac{V_z}{c} \]  
\[ \ldots (15) \]

\[ \gamma_g = \frac{1}{\sqrt{1 - \beta_g^2}} \]  
\[ \ldots (16) \]

Using \( V_R \) of Eq. (13), the resonant velocity \( V_{gR} \) seen in the \( g\)-frame is expressed by:

\[ V_{gR} = \frac{V_z + V_R}{1 + \frac{V_z V_R}{c^2}} \]  
\[ \ldots (17) \]

Furthermore, the Lorentz transformation \( L_{gR} \) from the \( g\)-frame to the frame (\( R\)-frame) moving with the resonant velocity \(-V_{gR}\) is expressed as:

\[ L_{gR} = \begin{pmatrix} \gamma_{gR} & 0 & 0 & i\beta_{gR} \gamma_{gR} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta_{gR} \gamma_{gR} & 0 & 0 & \gamma_{gR} \end{pmatrix} \]  
\[ \ldots (18) \]

where, \( \beta_{gR} \) and \( \gamma_{gR} \), are respectively expressed as:

\[ \gamma_{gR} = \frac{1}{\sqrt{1 - \beta_{gR}^2}} \]  
\[ \ldots (19) \]
\[ \beta_{gr} = \frac{V_{gr}}{c} = \frac{1}{1 + \frac{V_{gr}}{c^2}} \]  \hspace{2cm} \text{... (19)}

\[ \gamma_{gr} = \frac{1}{\sqrt{1 - \beta_{gr}^2}} \]  \hspace{2cm} \text{... (20)}

Accordingly, the kinetic motion Eq. (7) is rewritten as:

\[ \frac{d}{d\tau} P_j = -eF_y L_y^j L_{gr} L_{g\bar{r}g} v_j \]  \hspace{2cm} \text{... (21)}

The relativistic four-velocity \( U_j \) seen in the frame (R-frame) moving with the resonant velocity \(-V_{gr}\) in the g-frame is shown for a four-velocity \( v_j \) as:

\[ U_j = L_{gr} L_g v_j \]  \hspace{2cm} \text{... (22)}

Furthermore, the equation for time dilation is obtained by using the Lorentz factor \( \gamma \) of Eq. (10) in the rest frame:

\[ cd\tau = \gamma cd\tau \]  \hspace{2cm} \text{... (23)}

Unified electric and magnetic fields expressed in the rest frame fixed at the Earth are shown as (this expression is not just exact but approximate for relativistic resonance condition with whistler mode signal):

\[ F_{ij} \approx \begin{pmatrix} 0 & i \frac{E_z}{c} & i \frac{E_z}{c} & i \frac{E_z}{c} \\ -i \frac{E_x}{c} & 0 & B_z & -B_y \\ -i \frac{E_y}{c} & -B_z & 0 & B_x \\ -i \frac{E_z}{c} & B_y & -B_x & 0 \end{pmatrix} \]  \hspace{2cm} \text{... (24)}

Furthermore, using the Lorentz factor, \( \gamma_{\mu} \), newly defined in the R-frame, the relativistic four-velocity \( U_j \) seen in the R-frame is again shown as:

\[ U_j = (i\gamma_{\mu} \gamma_{ij}, \gamma_{\mu} U_x, \gamma_{\mu} U_y, \gamma_{\mu} U_z) \]  \hspace{2cm} \text{... (25)}

Furthermore, the electric and magnetic fields \( E_k \) and \( B_k \) \((k = x, y, z)\) seen in the rest frame fixed at the Earth are expressed as a whistler mode wave with right-handed circular polarization, propagating along the external geomagnetic field, \( B_0 \), using vector symbols of \( \vec{k} \) and \( \vec{r} \):

\[ E_x = \frac{\omega}{k} b \sin(\omega t - \vec{k} \cdot \vec{r}) \]  \hspace{2cm} \text{... (26)}

\[ E_y = -\frac{\omega}{k} b \cos(\omega t - \vec{k} \cdot \vec{r}) \]  \hspace{2cm} \text{... (27)}

\[ E_z = 0 \]  \hspace{2cm} \text{... (28)}

\[ B_x = b \cos(\omega t - \vec{k} \cdot \vec{r}) \]  \hspace{2cm} \text{... (29)}

\[ B_y = b \sin(\omega t - \vec{k} \cdot \vec{r}) \]  \hspace{2cm} \text{... (30)}

\[ B_z = B_0 \]  \hspace{2cm} \text{... (31)}

Furthermore, \( \phi \) is defined by the phase of electron velocity in the rest frame fixed at the Earth,

\[ \Omega_{\omega} = \frac{eb}{m_0} \]  \hspace{2cm} \text{is the cyclotron angular frequency in relation to the magnetic field of whistler mode wave, and}

\[ \Omega_{0} = \frac{eB_0}{m_0} \]  \hspace{2cm} \text{is the cyclotron angular frequency in relation to the geomagnetic field. The electron rest mass, } m_0, \text{ is used in the definitions of } \Omega_{\omega} \text{ and } \Omega_{0}. \]

After all, the velocity relation is shown as:

\[ V_x = V_\perp \cos \phi \]  \hspace{2cm} \text{... (32)}

\[ V_y = V_\perp \sin \phi \]  \hspace{2cm} \text{... (33)}

\[ U_\perp = \frac{\gamma_{g} V_\perp}{\gamma_{gr}} \]  \hspace{2cm} \text{... (34)}

The phase of resonant electron changes from \( \phi \) to \( \xi \).

\[ \xi = \phi - (\omega t - \vec{k} \cdot \vec{r}) + \pi \]  \hspace{2cm} \text{... (35)}

Then, Eq. (21) changes as follows, but the synchrotron radiation loss, any dissipation and acceleration are not considered in this system for the simplicity. According to the approximation of the special relativity:

\[ \frac{d}{dt} V_x \approx \frac{\Omega_{\omega}}{\gamma} (V_x - \frac{\omega}{k}) \sin \xi + \frac{\Omega_{0}}{\gamma} \frac{\omega}{ck} U \frac{d}{dt} V_\perp \sin \xi \]  \hspace{2cm} \text{... (36)}

\[ \frac{d}{dt} V_\perp \approx -\Omega_{\omega} \frac{\gamma_{g} U_\perp}{\gamma_{gr}} \sin \xi + \Omega_{0} \frac{\gamma_{g} \omega}{ck} \frac{d}{dt} V_\perp \sin \xi \]  \hspace{2cm} \text{... (37)}
These equations are necessary for the relativistic analyses of whistler mode sideband generation near the separatrix, because the electron inertia effects in special relativity become a little stronger in the relativistic treatment than in the non-relativistic treatment, and the scattering and diffusion are a little difficult to generate whistler mode sideband gap in frequency space. Comparatively, if all of the Lorentz factors are unity and the terms including \( \frac{U_{\perp}}{c} \) and \( \frac{V_{R} V_{B}}{c^3} \) are neglected, the equations described above result in the non-relativistic equations seen in the whistler mode and electron resonance coordinate system, namely in R-frame\(^2,12,22\). Then, the right-hand side in the Eq. (39) becomes almost 0, and \( \gamma \) or the energy change is neglected. On the other hand, in the relativistic treatment, the electron inertia effects in the R-frame become a little more important than those in the non-relativistic treatment.

4 Velocity relation

Both \( \gamma \) and \( V_{R} \) are functions of arbitrary velocities \( V_{z} \) and \( V_{\perp} \), as shown in Eqs (10-13) and \( V_{R}^2 = V_{z}^2 + V_{\perp}^2 \). In Fig. 5, graphs of \( V_{R} \) are shown as function of arbitrary and independent variables, \( V_{z} \) and \( V_{\perp} \). Furthermore, the perfect resonance condition of the R-frame means \( V_{gr} = V_{R} = 0 \) in Eq. (17), \( V_{z} + V_{R} = 0 \). As a result, the following equation is given from Eq. (17):

\[
V_{h}(V_{z},V_{\perp}) = -V_{z} \quad \text{... (40)}
\]

Generally, the arbitrary \( V_{z} \) described above cannot make the perfect resonance condition (40). The parameters of \( f = 2.4 \text{ kHz} \), cold electron number density = 280 per cc and \( L = 5.1 \) are used in Figs 5-14 and Table 1. The Eq. (40) can also give the relation between \( V_{z} \) and \( V_{\perp} \). Accordingly, \( V_{h} \) can be derived from \( V_{\perp} \) as:

\[
V_{h} = \pm \Omega_{0}\sqrt{(1 + \frac{V_{R}^2}{c^2})(1 - \frac{V_{z}^2}{c^2}) - \frac{\omega^2}{c^2k^2} - \omega^2}\quad \text{... (41)}
\]

If \( V_{\perp} \) is solved from Eq. (41), the solution is given by:

\[
V_{\perp} = c \sqrt{1 - \left(\frac{k^2V_{z}^2}{\Omega_{0}^2 + c^2} + \frac{V_{R}^2}{k^2} - \frac{\omega^2 + 2\omega kV_{R}}{\Omega_{0}^2 + c^2}\right)} \geq 0 \quad \text{... (42)}
\]

The graph of Eq. (41) is shown in Figs 6 (a and b). The perfect resonance condition means the centre of the two curves is where the function is null.

| Table 1 — Parameters of calculation in the rest-frame fixed at the Earth for special-relativistic approximation scale of Separatrix \( \Delta V_{R} = 2\sqrt{\frac{V_{R}\Omega_{0}}{\gamma k}} \) |
|---|---|
| Frequency of carrier signal, \( f = \frac{\omega}{2\pi} \) | 2.4 kHz |
| Wave length of carrier signal, \( \lambda \) | 2.8 km |
| Saturated amplitude of carrier signal, \( b \) | 11 PT |
| L value of geomagnetic field line | 5.1 |
| Cold electron number density | 280 cc\(^{-1}\) |
| Perpendicular velocity of resonant electron, \( V_{\perp} \) | \( 6.0 \times 10^7 \text{ m s}^{-1} \) |
| Parallel velocity of resonant electron, \( V_{z} \) | (-1.05 to -1.10) \times 10^7 \text{ m s}^{-1} |
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of separatrix. Figure 6(a) indicates the backward interaction of Doppler effect \( \omega + kV_R = \frac{1}{\gamma} \Omega_o \)
derived by Eq. (13), if + is selected from the symbol ± of Eq. (41). Then, in the case where \( V_R \) becomes minus, the Fig. 4 shows that the signals resonating with the relativistic electrons may correspond to the VLF and ELF emissions interacted by Landau resonance. Lyons et al.\(^{23}\) has already presented the possibility of the Landau resonance between relativistic electrons and whistler mode wave. These resonances can connect to the general Cyclotron resonance. When \( V_R \) is plus, the direction of resonance velocity, \( V_R \), is opposite to the wave vector. However, \( V_R \) changes from plus to minus, and then the direction of \( V_R \) smoothly changes so as to be same with the direction of wave vector owing to much greater Lorentz factor. In the case where \( V_R \) is under and near zero, the frequency of whistler mode wave may become within a range of ELF even if \( V_R \) has a plus value or a minus value. Then, \( V_R = -6.207 \times 10^6 \) ms\(^{-1}\) is given at the peak of \( V_\perp = 2.99728 \times 10^8 \) ms\(^{-1}\), which is smaller than the light speed \( = 2.9972 \times 10^8 \) ms\(^{-1}\).

The relativistic effects are very interesting even in the stage of weak relativistic condition, therefore, the inhomogeneous effects of background magnetic field were neglected for simple analyses of the whistler mode resonant interaction. However, Fig. 6(b) shows the detailed graph of \( V_R - V_\perp \), where the horizontal and vertical axes are both magnified so as to clearly show the detail near the peak of the perfect resonance line. For the sake of smooth connection, the graph of the perfect resonance line under the horizontal peak is described by the minus sign selected from the symbol ± of Eq. (41). The end point of \( V_R = -6.22661 \times 10^6 \) ms\(^{-1}\) means that the electron velocity resonant with whistler mode wave can attain to the light speed there [Fig. 6(b)]. At this end point, the kinetic energy and the Lorentz factor both become infinite.

The energy corresponding to \( V_\perp \) of the relativistic treatment is shown in Fig. 7, and the resonance velocity \( V_R = 1.05 \times 10^7 \) ms\(^{-1}\) is shown as a maximum in Fig. 6(a), consequently, the lowest energy \( m_0(\gamma - 1)c^2 \) is correspondingly about 0.315 keV for \( V_\perp = 0.0 \) ms\(^{-1}\). The relativistic value, \( V_R = 1.05 \times 10^7 \) ms\(^{-1}\) given by using \( V_\perp = 0.0 \) ms\(^{-1}\) in Eq. (41) is almost same with the value of Table 2, i.e. \( V_\perp = 1.05 \times 10^7 \) ms\(^{-1}\).

The resonance velocity \( V_R = -6.207 \times 10^6 \) ms\(^{-1}\) is shown at the peak in Fig. 6(a), consequently, the high energy \( m_0(\gamma - 1)c^2 \) at the peak is about 44.11 \times 10^6 keV (441.1 MeV) for \( V_\perp = 2.99728 \times 10^8 \) ms\(^{-1}\) in Fig. 7. These results are owing to the relativistic perfect resonance condition and confined to the region of single value function of \( V_\perp \). It becomes evident that this energy range between 0.315 keV and 441.1 MeV includes 2-6 MeV relativistic electrons observed in the slot region\(^{24}\). Then, the Lorentz factor \( \gamma \) corresponding to \( V_\perp \) for the relativistic treatment is

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**Fig. 6 (a) — Perfect resonance condition which means the center of separatrix**

**Fig. 6 (b) — Detailed graph of \( V_\perp - V_\perp \), where horizontal and vertical axes are both magnified so as to clearly show the detail near the peak of the perfect resonance line**
also shown in Fig. 8, where the minimum of $\gamma$ is 1.00062 for $V_\perp = 0.0$ ms$^{-1}$ and $\gamma$ is correspondingly 864.3 for the peak of $V_\perp = 2.99728 \times 10^8$ ms$^{-1}$.

Further, as shown in Fig. 9(a), the gyro-radius at the peak corresponds to about 2500 $\lambda$, where $\lambda$ corresponds to the wave length of whistler mode wave 2.8 km near the equatorial plane at $L = 5.1$. If the gyro-radius of electron almost coincides with the wave length $\lambda$ of resonant whistler mode wave, the electron total energy is about 33.4 keV shown in Fig. 9(a and b). It is indicated that the electrons with energy of lower than 33.4 keV at $L = 5.1$ can interact through the pitch angle diffusion with the whistler mode waves in the quasi-linear approximation. On the other hand, electrons of over 33.4 keV at $L = 5.1$ may be related to other types of electron-wave interactions.

5 Solutions of relativistic equations

Some results obtained by solving Eqs (36 – 39) in the resonance frame (R-frame) are shown in Fig. 10 as a test particle calculation. The $V_\perp$ and $V_z$ values of separatrix are known by checking the trajectories of resonant electrons calculated from the relativistic resonance equations and the observed values. The resonance electron moving along the field line of $L = 5.1$ interacts with the whistler mode signal of frequency $f = 2.4$ KHz. The phase $\xi$, the parallel velocity $U_z$, the perpendicular velocity $U_\perp$, and the Lorentz factors of $\gamma$ and $\gamma_\parallel$ in the R-frame are respectively shown from the top graph to the fourth graph in Fig. 10. The initial conditions in Fig. 10 are $V_{z0} = 6.0 \times 10^7$ ms$^{-1}$ and $V_{\perp 0} = -1.05 \times 10^7$ ms$^{-1}$ at $t = 0.0$ sec as shown from Table 1. The perfect resonance velocity shown in Fig. 6(a) indicates $V_R = -V_z = 1.017767 \times 10^7$ ms$^{-1}$, then the change of $V_R$ approximately is neglected owing to the principle of special relativity. Further, $V_{z0} = 6.0 \times 10^7$ ms$^{-1}$ was decided from the reasonable pitch angle $\alpha = 80^\circ$ in Table 2.

<table>
<thead>
<tr>
<th>Frequency, kHz</th>
<th>Property</th>
<th>Pitch angle 75°</th>
<th>Pitch angle 78°</th>
<th>Pitch angle 80°</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>$E_{\text{total}}, \text{KeV}$</td>
<td>4.7</td>
<td>7.8</td>
<td>10.4</td>
</tr>
<tr>
<td>From observation</td>
<td>$V_{\text{res}}, \text{m s}^{-1}$</td>
<td>$0.105 \times 10^8$</td>
<td>$0.105 \times 10^8$</td>
<td>$0.105 \times 10^8$</td>
</tr>
<tr>
<td>Non-relativistic estimation</td>
<td>$V_{\perp}, \text{m s}^{-1}$</td>
<td>$0.392 \times 10^8$</td>
<td>$0.494 \times 10^8$</td>
<td>$0.595 \times 10^8$</td>
</tr>
<tr>
<td>Non-relativistic estimation</td>
<td>$E_{\text{res}}, \text{KeV}$</td>
<td>0.315</td>
<td>0.315</td>
<td>0.315</td>
</tr>
<tr>
<td>Non-relativistic estimation</td>
<td>$E_{\perp}, \text{KeV}$</td>
<td>4.4</td>
<td>7.5</td>
<td>10.1</td>
</tr>
</tbody>
</table>

Fig. 7 — Kinetic energy for perfect resonance condition of $V_R$ [kinetic energy corresponding to $V_\perp$ in the rest frame fixed at the Earth is shown on a log scale of KeV]

Fig. 8 — Lorentz factor for perfect resonance condition [Lorentz factor $\gamma$ corresponding to $V_\perp$ in the rest frame fixed at the Earth is shown on a log scale]
The phase $\xi$, the parallel velocity $U_z$, the perpendicular velocity $U_\perp$, and the Lorentz factors of $\gamma$ and $\gamma_u$ similarly show repetitive changes with same period in their graphs because of no energy loss, no diffusion and no scattering of model trapped electron in the R-frame of Fig. 10. It is shown that the approximation of special relativistic principle is almost valid, because the Lorentz factors of $\gamma$ and $\gamma_u$ are almost constant. Further, the relativistic treatment of $\gamma$ make the inertia effects more important than the non-relativistic treatment of $\gamma = 1$.

Figure 11(a) shows the trajectories of resonant electrons in the phase diagram of $U_z$-$\xi$ (XI). The points along each trajectory indicate the electron locations in the phase diagram, in which the time intervals are all 0.1 millisecond in the time frame fixed at the Earth. The initial parameters of the trajectory calculations are shown in Table 1, in which the initial parallel velocities are in order from $V_{z0} = -1.05 \times 10^7$ ms$^{-1}$ to $V_{z0} = -1.10 \times 10^7$ ms$^{-1}$ and the initial perpendicular velocity is $V_{\perp 0} = 6.0 \times 10^7$ ms$^{-1}$. In relation to the un-trapped electrons, the intervals of $\xi$ along the trajectory of $V_{z0} = -1.10 \times 10^7$ ms$^{-1}$ are all larger than those of $V_{z0} = -1.05 \times 10^7$ ms$^{-1}$. The meanings of un-trapped electron, trapped electron, and separatrix are all shown in Fig. 11(c). At the stationary phase of -180 degree, the motion for un-trapped electrons becomes slow and the electron stays here for a while. However, at this stationary point of -180 degree in the R-frame, the scattering of electrons is easily attained by some noisy signals owing to smaller velocity, because the right hand sides in the Eqs (36–39) show the disappearance of force at this stationary point.

Furthermore, the phase diagram in Fig. 11(a) shows the relations between $U_z$ and $\xi$ of the R-frame, where the phase bunching is very outstanding around the phase of -180 degree, because the diamond shapes are arranged in a line with the equal interval of 0.1 millisecond measured at the rest frame fixed at the Earth. The $U_z = 0$ means the center line of resonance in the separatrix, namely the perfect resonance condition. The sideband wave generation mechanism that the bunching electrons with initial velocity of $V_{z0} = -1.07 \times 10^7$ ms$^{-1}$ can collect around -180° and can generate the sideband wave current, which has already been presented by Ikeda$^9$. On the other hand, the trajectory of electron with the velocity of $V_{z0} = -1.05 \times 10^7$ ms$^{-1}$ is inside the separatrix. The definition $V_{\perp 0}$ of the separatrix described above is shown in Table 1 and defined by the function of $V_{\perp}$ and $\gamma(V_{\perp})$. The graphs of $V_R \pm \Delta V_z$ approximately showing the separatrix and the perfect resonance velocity $V_R$, respectively are shown by the dotted lines and the bold solid line in Fig. 11(b). The calculation of trajectories shown in Fig. 11(a) were done from the initial condition of $V_{\perp 0} = 6.0 \times 10^7$ ms$^{-1}$, and it was imagined that the separatrix indicated by the upper dotted line in Fig. 11(b) was located between $V_{z0} = -1.06 \times 10^7$ and $V_{z0} = -1.07 \times 10^7$ ms$^{-1}$ in Fig. 11(a). Correspondingly, it is verified that the upper dotted line of separatrix is between $V_R = 1.06 \times 10^7$ and $V_R = 1.07 \times 10^7$ ms$^{-1}$ at $V_{z0} = -6.0 \times 10^7$ ms$^{-1}$ in Fig. 11(b). The resonance electrons trapped in the wave-potential well is contained between these dotted lines.

6 Results from relativistic resonance conditions

Whistler mode resonant velocities of electrons moving along the geomagnetic field are possible to be
estimated from various kinds of observations, for example, whistler analyses, multi-stations observations and direction finding techniques of Siple signals. As shown in Table 2, Ikeda got the data in which the resonance electron velocity along the geomagnetic field was $71.05 \times 10^3$ m/s by the propagation time and observation location ($L = 5.1$) obtained from Siple Station Wave Injection Experiments conducted in 1988, and this velocity corresponds to the parallel kinetic energy of 0.315 keV. The analyses of Siple signals presented in this paper are based on the detail examinations of various frequency-time variations. The resonance velocity $V_{res} = 1.05 \times 10^7$ m/s estimated from Siple signals in Table 2 is almost same with the resonance velocity $V_r = 1.05 \times 10^7$ m/s as the maximum in Fig. 6(a) and estimation of Fig. 11(b). However, the region of actual resonance velocity should be within the both side extents of the separatrix, namely within $\Delta V_z$ shown in the lower part of Table 1, but the lack of electron population due to precipitation through loss cone angle may appear near the resonance velocity of $V_r = 1.05 \times 10^7$ m/s.
In relation to whistler mode sideband problem, Ikeda\(^9\) showed the observed frequency gap of about 30 Hz from the carrier signal; and as one of the sideband causes, suggested the linear amplification just outside the separatrix. On the other hand, the coherent wave-electron interaction can generate the non-linear wave amplification of carrier signal. This frequency gap of about 30 Hz could give the amplitude of the Siple transmitted carrier, by using the phase resonance condition of the first perturbation. The detail of the separatrix is shown in Fig. 11(c). In Fig. 11(a), the difference among the electron trajectories clearly shows the meanings of electron bunching and a wave linear growth due to the current generation by the bunching. Generally, the bunching electrons collectively radiate whistler mode waves and lose any energy; as a result, any diffusion of electron may occur near the separatrix at that time. Collectively, the trapped electrons can also radiate whistler mode waves. The same physical processes may occur at -180 degrees of phase for un-trapped electrons too. In addition, the observed amplitudes of Siple signals in the magnetosphere were also referred\(^{26,27}\).

When the resonance velocity is obtained in detail from Fig. 11(b), Fig. 12 shows that the interval determined by the intersections of \(V_R = 1.05 \times 10^7\) ms\(^{-1}\) can give the resonance region determined by the reasonable estimates of \(V_{\perp}\). The reason is chiefly because the electrons trapped within the separatrix are related to the amplification of the observed Siple signals. As shown in Fig. 12, the black and bold line of \(V_R = 1.05 \times 10^7\) ms\(^{-1}\) intersects at the outer limit of the separatrix, namely at \(V_{\perp} = 0.0\) and \(7.0 \times 10^7\) ms\(^{-1}\) (at maximum of \(V_{\perp}\)). Thus, this estimation gives the extent of perpendicular velocity in relation to the resonant electrons, \(V_R = 1.05 \times 10^7\) and \(V_{\perp} = 0.0-7.0 \times 10^7\) ms\(^{-1}\) (at maximum of \(V_{\perp}\)). As a result, this interval corresponds to the extent for the resonance energy 0.315 – 16 keV (at maximum of \(V_{\perp}\)) at \(L = 5.1\), as shown in Fig. 13 magnifying Fig. 7. This means that we can get the tool by which the resonance energy is given exactly.

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**Fig. 11(a)** — Phase diagram in relativistic resonance frame showing relations between \(U_z\) and \(\xi(x)\) in the R-frame, where black diamonds are arranged in a line with equal time duration.

**Fig. 11(b)** — Perfect resonance velocity with estimation of separatrix [black and bold line corresponds to perfect resonance velocity; and upper and lower dotted lines correspond to the outer limits of separatrix].

**Fig. 11(c)** — Phase \(U_z - \xi(x)\) diagram of electron resonant with a monochromatic whistler signal given by the pendulum equations.

**Fig. 12** — Resonance energy with estimation and separatrix.
The constant $V_R > 0$ is significant for Kennel and Petschek type amplification. Then $V_R$ does not depend on $V_\perp$ owing to energy transfer to wave. Generally, in the case of non-relativity, if the energy change $\Delta W$ of trapped electron is minus, namely, $\Delta W < 0$, then the whistler mode wave grows in amplitude, following the Brice theory of $\Delta W_z = \frac{k V_R}{\omega} < 0$, $\Delta W_z > 0$, and $V_R$ is conversely increased in amplitude. On the other hand, $\frac{\Delta W_z}{\Delta W} = \frac{\Omega_e}{\omega} > 1 > 0$ is also verified, as a result of diffusion, $\Delta W_\perp < 0$ and the $V_\perp$ is reduced in amplitude. Therefore, it is verified that the approximation of constant $V_R > 0$ becomes valid for maximum wave growth. As shown in Fig. 6(a), if $V_R$ is minus, fully relativistic electrons resonant with whistler mode waves may behave via an interaction different from the Kennel and Petschek type amplification.

Furthermore, according to whistler mode sideband analyses, Ikeda showed that the perpendicular energy of 13.1 keV obtained from the data ($L = 4.23$) of Park was roughly estimated as a result of perpendicular acceleration from 7.5 keV ($L = 5.1$) of Sonwalkar et al., which is similar to the estimation of about 12 keV at $L = 4.1$ by the multi-station measurements of Ikeda et al. The location of $L = 4.23$ was used from the analyses of whistler mode sideband amplification presented by Park. Actually, 13.1 keV is obtained from the perpendicular acceleration (Betatron acceleration) of $E_\perp = 7.5$ keV at $L = 5.1$ and the pitch angle $\alpha = 78^\circ$ as shown in Table 2.

The range of the resonance energy $0.315 – 16$ keV (at maximum of $V_\perp$) derived from the data of $L = 5.1$ is very wide, but it is also related to the amplification of whistler mode waves, because these data were observed at Lake Mistissini in the conjugated hemisphere of Siple Station, Antarctica, as shown by Sonwalkar et al. According to Kennel & Petschek, the stable trapped electron fluxes creates a pitch angle distribution that is unstable to wave growth, then the electrons are continuously pitch-angle scattered and the electron distribution reaches the limiting flux, in which the anisotropy $A = \frac{T_\parallel - T_\perp}{T_z}$ is typically 1/6.

This means that the average pitch angle derived from the temperature $(T_\parallel, T_z)$ anisotropy $A = \frac{T_\parallel - T_\perp}{T_z}$ at $L = 5.1$ is included within the range of $40^\circ$ to $80^\circ$ of $V_\perp = 6.0 \times 10^7$ ms$^{-1}$ for resonant electrons at $L = 5.1$, as presented by Sonwalkar et al. The whistler mode wave growth is generally attained in the case of sufficient anisotropy $A$ or sufficient pitch angle $\alpha$, accordingly the velocity of $V_\perp = 6.0 \times 10^7$ ms$^{-1}$ within the separatrix is selected and it is appropriate to the wave growth on the base of whistler mode instability theory.

However, the electrons with velocity of $V_\perp = 7.0 \times 10^7$ ms$^{-1}$ (at maximum of $V_\perp$) located near the separatrix may also contribute to wave amplification, because of any scattering effects and unstable conditions. This may mean there are few electrons near the separatrix. Furthermore, the fact that the electrons of resonance energy lower than 33.4 keV can precipitate toward the ionosphere, should be necessarily emphasized. The total energy of 33.4 keV was derived in this paper, and this value corresponds to the fact that the Larmor radius of electron almost coincides with the wavelength $\lambda = 2.8$ km of resonant whistler mode wave, and then the Lorentz factor is about 1.07.

When Fig. 9(a) is examined in detail, it is clearly verified that there are various combinations of electron velocities $V_\parallel$ and $V_\perp$ in relation to relativistic resonances between whistler mode waves such as plasmaspheric hiss and energetic electrons. In other researches executed so far and included in the analyses of Sonwalkar et al., weak relativistic treatments are applicable to probe hot plasma in the Earth’s magnetosphere, further Figs. 9(a and b).
clearly show that fully relativistic resonances can also exist in the interactions between whistler mode waves and high energy electrons, which have not been taken into account in this paper. This high energy interaction (e.g. Landau resonance and Fermi acceleration) may indicate that there are any possibilities of whistler mode resonance, acceleration, deceleration, diffusion and precipitation mechanisms related to fully relativistic electrons in the magnetosphere. Actually, Katoh et al. investigated resonant diffusion, scattering and acceleration processes in order to explain distribution variations of fully relativistic electrons existing in the radiation belts, by using the numerical simulation.

7 Summary and Conclusions
Recently, Singh & Singh showed that gyro-resonance between transmitted whistler mode VLF signal and energetic electron played an important role in relation to electron diffusion near the separatrix in the whistler mode signal, as a result, pitch angle diffusion and radial diffusion. This study was done on the base of giving pitch-angle of resonant electron, but if the relativistic resonance condition is taken into account, the pitch-angle may not be necessary because Eqs (41 and 42) show that \( V_\perp \) and \( V_R \) are dependent on each other, and this fact can give the pitch angle by the function of arctan \( (V_R / V_\perp) \). Further, it is very probable that the sideband-wave generation of whistler mode around a monochromatic VLF signal is also related to electron near the separatrix severely.

Although the Siple Station Wave Injection Experiments haven’t been planned at present, the power line radiations, the experiments of VLF signals transmitted from satellites, and the 3.6 MW HAARP ionospheric heater can become important objects instead of the Siple Station Wave Injection Experiments. Power line radiations (PLR) are also observed at many locations on the ground, and these radiations, which are similar to Siple signals, show discrete and monochromatic characteristics and harmonics of 50 Hz and 60 Hz. However, their active frequencies are unknown and complex, owing to nonlinear characters and variable parameters at the equatorial plane in the magnetosphere.

For example, Park presented the ground-based observation of magnetospheric wave activity at Eights, Antarctica during 0000-1200 hrs UT, 12-26 June 1965, in which the wave intensity in the 0.5- to 10-kHz range showed clear association with geomagnetic activity, and the received wave frequencies also changed with it severely in the range 0.5-7-kHz.

The kinetic energy graph in Fig. 7 was derived by the calculation on the base of the fixed frequency of 2.4 KHz. Conversely, if the angular frequency \( \omega \) and wave number \( k = k(\omega) \) are both variable in Eq. (41) and \( V_\perp \) is fixed at \( V_\perp = 6.0 \times 10^7 \text{ ms}^{-1} \), the resonance energy with velocities of \( V_z \) and \( V_\perp \) is given by Eq. (41). Then, the relativistic kinetic energy is described by \( E = m_0 (\gamma(V_\perp, V_z) - 1)c^2 \) shown as a dark solid line in Fig. 14. This dark solid line shows that the wave frequency of 2.4 KHz corresponds to the relativistic kinetic energy of about 10.9 keV, which is included in the energy range 0.315–16 keV (maximum of \( V_\perp = 7.0 \times 10^7 \text{ ms}^{-1} \)) described before. \( V_\perp = 7.0 \times 10^7 \text{ ms}^{-1} \) was determined by the separatrix derived from the sideband wave generation. The electrons with velocity of \( V_\perp = 7.0 \times 10^7 \text{ ms}^{-1} \) (at maximum of \( V_\perp \)) located near the separatrix may also contribute to wave amplification, because of any scattering effects and unstable conditions. This may mean there are few electrons near the separatrix.

On the other hand, the non-relativistic kinetic energy is described by \( E = \frac{1}{2} m_0 (V_\perp^2 + V_z^2) \) and shown as a bright solid line in Fig. 14. This bright solid line shows that the wave frequency of 2.4 KHz corresponds to the non-relativistic kinetic energy of about 10.4 keV (Table 2) and is also included in the energy range of 0.315–16 keV (at maximum of \( V_\perp = 7.0 \times 10^7 \text{ ms}^{-1} \)). However, on the base of the
perpendicular velocity fixed at \( V_p = 6.0 \times 10^7 \) ms\(^{-1} \), this non-relativistic kinetic energy of about 10.4 keV is clearly smaller than and different from the relativistic kinetic energy of about 10.9 keV, which is similar to 11 keV of Sonwalkar et al. \(^1\)

Thus, if the estimated resonance velocity, \( V_R \), and wave frequency, \( f \), are both adjusted with each another and further the separatrix is taken into account, the detail and right estimation of relativistic kinetic energy is obtained from Fig. 6(a), Fig. 11(b), Fig. 12 and Fig. 14. Actually, it is essential for the detail estimation to obtain a lot of observed and comparative data of energy and frequency \(^{44}\). As a result, if active frequencies of VLF activity are known by some techniques and observations, the kinetic energies of resonant electrons in the magnetosphere can be found out at many places on the ground, especially inside the plasmapause with propagation to the ground, thus, this method may connect to one of the Space Weather Research items.

Lastly, it is further emphasized that the relativistic treatment becomes important in relation to electron acceleration, deceleration, diffusion and precipitation problems and further wave generation problems, because the electrons with fully relativistic \( \gamma \) of over about 1.1 are able to interact with whistler mode waves.

Thus, the resonance condition derived in the relativistic expression is applicable in a number of cases, because different measurements give almost same and real results. However, in relation to man-made discrete emissions, there are number of unsolved problems, for example, generation of sidbands emission, wave–electron interaction with multi-frequency discrete emissions like PLR, chaos and diffusion problems near the separatrix, influences of relativistic treatment, acceleration, deceleration and diffusion of relativistic electrons, and the generation of VLF and ELF emissions like forward Landau Resonance generated by the relativistic electrons. These problems will be examined in future, and these methods may connect to one of the Space Weather Research items.

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References

3. Tsuruda K, Ikeda M & Hayashi K, Propagation characteristics of whistler-mode VLF waves injected into the magnetosphere from the Antarctic station ‘Siple’ (L ~ 4.0), Institute of Space and Aeronautical Science (ISAS) Research note 42 (ISAS, Japan), 1977, 1.
13. Ikeda M, Nagano I, Shimbo T & Carpenter D L, Intensities and polarization rates of whistler mode VLF signals observed
IKEDA: ESTIMATE OF KINETIC ENERGY MAXIMA OF CYCLOTRON-RESONANT ELECTRONS

from a ground network near L = 4, J Geophys Res (USA), 100 (1995) 5691.

14 Roederer J G, Dynamics of Geomagnetically Trapped Radiation (Springer-Verlag, Germany), 1970.


22 Matsumoto H, Wave Instabilities in space plasmas (Reidel, Dordrecht, The Netherlands), 1979, 163.


26 Inan U S, Bell T F, Carpenter D L & Anderson R R, Explorer 45 and Imp 6 observations in the magnetosphere of injected waves from the Siple station VLF transmitter, J Geophys Res (USA), 82 (1977) 1177.


28 Brice N, Fundamentals of very low frequency emission generation mechanisms, J Geophys Res (USA), 69 (1964) 4515.


30 Schulz M & Lanzerotti L J, Particle diffusion in the radiation belts (Springer-Verlag, Germany), 1974.

31 Matsumoto H & Kimura I, Linear and nonlinear cyclotron instability and VLF emissions in the magnetosphere, Planet Space Sci (UK), 19 (1971) 567.


35 Hashimoto K & Matsumoto H, Temperature anisotropy and beam type whistler instabilities, Phys Fluids (USA), 19 (1976) 1507.


43 Park, C G, VLF wave activity during a magnetic storm: A case study of the role of power line radiation, J Geophys Res (USA), 82 (1977) 3251.