A new wind-wave spectrum model for deep water

Yongcun Cheng¹,² Yuguang Liu¹* & Qing Xu¹

¹Physical Oceanography Laboratory, Ocean University of China, Qingdao, 266003, China
²Institute of Meteorology, PLA University of Science and Technology, Nanjing, 211101, China

*{E-mail: yugliu@mail.ouc.edu.cn}

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With the statistical relationships and equations, a new wind wave spectrum model for deep water is proposed in this study. In the new model, the total spectral energy level, the location of the maximum spectral energy (represented by location of spectral peak), and the width character of energy distribution (represented by the spectral steepness) at high frequencies located to the right of spectral peak of wind waves are all determined by the two basic parameters, i.e., the wind speed and the inverse wave age. The statistical relationships also show that with the same wind speed and wave age, the steepness of the wind wave spectrum at high frequencies located to the right of the spectral peak for field case is different from that for laboratory case. This is the main difference between field wind wave spectrum and laboratory spectrum. With the inverse spectral width, the new model is more appropriate to describe the real wind wave status. Compared with measurements in the Black Sea, the model calculated zeroth spectral moment $m_0$ and the spectral width are in both good agreements with measured data. Furthermore, the new model can match elevation spectrum data obtained by four-frequency microwave radar and other field measurements fairly well. The new model can describe better and explain the influence of the wind speed and wave age on the energy distribution of developing wind waves generated in open ocean, also it plays a significant role in the study of oceanic microwave remote sensing, especially for understanding the uncertainty of retrieved ocean environment variables.

[Key words: Wind wave spectrum, wind speed, spectral width, wave age, microwave remote sensing]

1 Introduction

The wind wave spectrum models are widely used in various fields of ocean investigations. They in particular play an important role in the study of the remote sensing theory concerning microwave scatterometers and microwave radiometers. Within the full wavenumber range, the wind wave spectrum can be divided into long wave spectrum (gravity spectrum) and short wave spectrum (gravity-capillary spectrum). In the study of the remote sensing mechanism of microwave radiometers and scatterometers, the influence of both long and short waves must be considered. Investigations indicate that the energy distribution of wind waves plays an important role in microwave remote sensing, and four wind wave spectrum models¹⁴ are often used to retrieve wind speed from altimeter and scatterometer observations. However, the differences among the four models¹⁴ have a large influence on the calculation of “brightness temperature” of sea surface, which brings difficulty in the application of theoretical model of microwave remote sensing, and also presents a higher requirement for wind wave spectrum models.

A wind wave spectrum model proposed by Liu et al.⁵⁻⁹ can be used to describe wind wave energy distribution of both deep-water wind waves and gravity-capillary waves. The model for deep water contains three parameters: the wind speed, the inverse wave age and the inverse spectral width. In this study, based on this wind wave spectrum model and measured data, statistical relationships among wind speed, inverse wave age and inverse spectral width, are derived for field case and laboratory case, respectively. The significance of the study is, firstly, we find that the inverse spectral width that affects the energy distribution of wind waves is not an independent parameter. It is determined by the wind speed and the inverse wave age. Secondly, with the establishment of the statistical relationships, the wind wave spectrum model proposed by Liu et al.⁵⁻⁹ is improved. The new model with the statistical relationships not only decreases the number of parameters that determine the

__* Corresponding author; Phone: +86-532-82031629 Fax: +86-532-82032471__
wave spectrum, but also plays a significant role in the microwave remote sensing, especially in the study of the uncertainty of remote sensing of environment variables.

2 Models

2.1 The inverse wave age and inverse spectral width

In wind wave spectrum models, the wave age denotes the developing state of wind waves. In the wind wave model proposed by Liu et al.\(^5\)\(^-\)\(^9\) the parameter “inverse wave age” was used to describe the influence of wave age on spectral-peak frequency and total spectral energy level, and “inverse spectral width” was used to describe width character of wind wave spectrum. When correct value of “inverse spectral width” is selected, the model can be used to describe energy distribution of wind waves at wavelength ranging from 10 cm to 100 cm.

For long waves at lower frequency, Liu et al.\(^5\)\(^-\)\(^9\) proposed a model of directional frequency spectrum

\[
S(\omega, \phi) = 0.0093\alpha_s\alpha_w D(\phi,k)\left(\frac{\omega_p}{\omega}\right)^{2\xi-4} \frac{g^2}{\omega^2} \exp\left\{-2\xi + \frac{1}{4} \left[\frac{b\omega_p + (1-b)\omega_p^3}{\omega^3}\right]\right\} \left\langle \eta \right\rangle^p \quad \text{(1)}
\]

For wind induced gravity-capillary waves, the spectrum is of the form\(^5\)\(^-\)\(^9\)

\[
S(k, \phi) = \frac{1}{320} k^{-4} \left(\frac{u_* - \delta}{c}\right)^2 D(\phi)D_d D_e \epsilon_p^r \quad \text{(2)}
\]

Compared with the spectra proposed by Pierson & Moscowitz\(^10\), Hasselmann et al.\(^11\) and Donelan et al.\(^12\), only one parameter “inverse spectral width \(\xi\)” was added in Eq. (1).

In wind wave spectrum model proposed by Liu et al.\(^5\)\(^-\)\(^9\), the spectral-peak angular frequency for fully-developed waves is denoted as \(\omega_o\), which is determined by \(U_{10}\) through \(\omega_o = g/1.2U_{10}\), where \(g\) is the gravitational acceleration, and \(U_{10}\) is the wind speed at a height of 10 m in a neutrally stable atmosphere. \(\omega_o\) is the spectral-peak angular frequency for wind waves, also the angular frequency of dominant waves. The relation of \(\omega_o\) with phase speed of waves at spectral-peak frequency, \(\omega_p\), is \(\omega_p = 2\pi f_p = g/C_p\), which can be derived from basic relations of waves and the dispersion relation of gravity waves expressed by \(C_p = \sqrt{g/k_p}\). The inverse wave age \(\bar{\omega}_p\) is defined as the ratio of \(\omega_p\) to \(\omega_o\) [Liu et al.\(^8\), Eq. (8)]

\[
\frac{\omega_p}{\omega_o} = \frac{1.2U_{10}}{g} \frac{C_p}{U_{10}} = 1.2\Omega \quad \text{(3)}
\]

where \(\Omega = U_{10}/C_p\) is the “inverse wave age” proposed by Donelan et al.\(^12\). Thus \(\bar{\omega}_p = 1.0\) or \(\Omega = 0.84\) corresponds to the fully-developed wind waves, and \(\bar{\omega}_p > 1.0\) or \(\Omega > 0.84\) corresponds to the developing wind waves.

In Eq. (1), the exponent of \(\omega\) is \(-2\xi - 4 + 5\), therefore, \(\xi\) (inverse spectral width) can be used to describe the spectral steepness at high frequencies located to the right of spectral peak of wind wave spectrum. The theoretical relationship of \(\xi\) and spectral width \(\varepsilon\) proposed by Cartwright & Longuet-Higgins\(^13\) is

\[
\varepsilon = \sqrt{\frac{1 - \frac{m_2}{m_0 m_4}}{1 - \left(\frac{\Gamma(\xi)}{\Gamma(2)}\right)^2}} \quad \text{(4)}
\]

where \(m_n\) is the spectral moment of variance spectrum; \(\Gamma(z) = \int_0^\infty e^{-t} t^{z-1} dt\) is Gamma function. The relationship between \(\xi\) and the significant slope in Wallops spectrum model\(^14\) is

\[
\xi = \log(\pi\delta)/\log 2 \quad \text{(5)}
\]

where \(\delta\) is the significant slope, and \(\xi\) increases with the significant slope.

Figure 1 shows the relationship between inverse spectral width \(\xi\) proposed by Liu et al.\(^5\)\(^-\)\(^9\) and spectral width \(\varepsilon\) proposed by Cartwright & Longuet-Higgins\(^13\). One can see that \(\xi\) decreases with the increase of \(\varepsilon\). \(\xi \rightarrow 2.0\) when \(\varepsilon \rightarrow 1.0\), and \(\xi \rightarrow \infty\) when \(\varepsilon \rightarrow 0\). As \(\xi\) increases, the spectrum becomes narrower and the spectrum slope becomes steeper vice versa. Hence, \(\xi\) is called the inverse spectral width. Since the other wind wave spectrum models did not consider the influence of environmental factors on wind wave spectral steepness, the model proposed by Liu et al.\(^5\)\(^-\)\(^9\) is an improvement in describing the wind wave spectral steepness. However, Liu et al.\(^5\)\(^-\)\(^9\) did not search the
causality between the inverse spectral width and inverse wave age, which leads to an increase of the number of model parameters.

2.2 The statistical relationship among inverse wave age, inverse spectral width and wind speed for field data

Observations show that wind wave spectrum has a close relation with wind friction velocity, and wind duration or fetch. Wind duration and fetch are equivalent factors. The influence of wind duration or fetch on wind waves is described by the wave age or inverse wave age. Usually, investigators need to use various data obtained from both open ocean and laboratories with different instruments to validate their models. However, it is impossible in laboratories to re-create the large-scale turbulence of wind which plays the same role for wind wave growth as that in the open ocean. Short fetch of wave tank restricts wind wave growth, too. Consequently, measured data from ocean and laboratories should be separately dealt with to find the statistical relationship among inverse wave age, inverse spectral width and wind speed.

In this section, measured data from ocean are matched with model proposed by Liu et al. Based on the match of model with measurements, the model parameters are estimated, and then the statistical relationship among these parameters is proposed.

Table 1 lists the three parameters of wind wave spectrum based on the measurements. Investigators of field observations, wind speed ($U_{10}$), inverse wave age ($\bar{\omega}_p$), and inverse spectral width ($\xi$) are ranked from left to right. In Table 1, values of parameters are obtained in the following way: (1) substitute the wind friction velocity $u_*$ into the spectrum model proposed by Liu et al. to calculate the wind speed $U_{10}$, (2) substitute $U_{10}$ and the spectral-peak angular frequency $\omega_p$ into the spectrum model proposed by Liu et al. to calculate the inverse wave age $\bar{\omega}_p$, (3) match the field measurements with the model proposed by Liu et al. and select a right value of the inverse spectral width by comparing the spectral steepness to the right of the spectral peak.

For the study of integral and spectral wave parameter variations, systematic observations were carried out by the Ukrainian National Academy of Sciences (UNAS). The observed data in the Black Sea, such as wind speed ($U_{10}$), spectral-peak frequency ($f_p$), dimensionless spectral-peak frequency ($f_p^*=U_{10} f_p / g$) and zeroth spectral moment ($m_0$), were given by Babanin & Soloviev. All available data (with $f_p^* \leq 0.23$, corresponding to undeveloped conditions) are listed in Appendix A, and the first 20 groups of them, with wind speeds covering the range of 4–11 m/s except one group (17.8 m/s), are used to obtain statistical relationship between wind speed, inverse wave age and inverse spectral width.

With Eqs. (1) and (2), the omni-directional elevation spectrum, $S(\omega)$, is defined as
\[
S(\omega) = \int_{-\pi}^{\pi} S(\omega, \phi) d\phi 
\]  \hspace{1cm} \text{(6)}

The dependence of \(m_0/\omega_p S(\omega_p)\) on dimensionless frequency \(f_p\) (it is related with the inverse wave age via \(\overline{\omega}_p = 1.2\Omega = 2.4\pi f_p^m\)) as a parameter of wave development stage was obtained by Krivinskii\(^{20}\):

\[
\frac{m_0}{\omega_p S(\omega_p)} = (1.05 \pm 0.02) - (2.49 \pm 0.13) f_p^0 
\]  \hspace{1cm} \text{(7)}

where "±" denotes the range of positive and negative standard error, \(m_0 = \int_{0}^{\infty} S(\omega) d\omega\) is zeroth spectral moment, and \(\omega_p = 2\pi f_p\) is the spectral-peak angular frequency. The suitable inverse spectral width \(\xi\) is selected by making the difference between \(m_0/\omega_p S(\omega_p)\) obtained from the wind wave spectrum model proposed by Liu \textit{et al.}\(^{5-9}\) and experiential Eq. (7) as small as possible. The first 20 groups of data are listed in Table 2.

Based on data in Tables 1 and 2, a statistical relationship among the inverse wave age, inverse spectral width and wind speed obtained as:

\[
\begin{align*}
\xi &= 3.03(\overline{\omega}_p c_m / U_{10})^{-0.07} \quad \overline{\omega}_p c_m / U_{10} > 0.02 \\
\xi &= 2.0 \quad \overline{\omega}_p c_m / U_{10} \leq 0.02
\end{align*}
\]

\hspace{1cm} \text{(8)}

where \(c_m = 0.23\) m/s is the minimum phase speed of gravity-capillary waves; \(\overline{\omega}_p c_m / U_{10}\) is a dimensionless parameter. In Eq. (8), \(\xi\) denotes the estimated value from inverse wave age and wind speed. For real ocean status, because of the random variety of wind speed and wind direction caused by turbulence and gust, and the indirect influence of swell and ocean current, the randomness of wind wave energy distribution is quite strong. Therefore, Eq. (8) just denotes the statistical relationship under a neutrally stable atmosphere and the mean statistical status of wind waves. The statistical relationship [Eq. (8)] suggests that the inverse spectral width should be determined by only two basic parameters, i.e., the wind speed and the inverse wave age. With the statistical relationship and equations given by Liu \textit{et al.}\(^{5-9}\), a new wind wave spectrum model for deep water is proposed here. In the new model, the spectral energy level, the spectral-peak frequency and the energy distribution at high frequencies located to the right of spectral peak of wind waves are all determined by the inverse wave age and the wind speed.

Figure 2 shows the relationship among the wind speed, inverse wave age and inverse spectral width based on field measurements listed in Tables 1 and 2.

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Table 2 — The wind speed, inverse wave age and inverse spectral width obtained from field observations: The inverse spectral width is estimated by matching the model proposed by Liu \textit{et al.}\(^{5-9}\) with experiential relationship [Eq. (7)] and measurements in the Black Sea.

<table>
<thead>
<tr>
<th>(U_{10}) (m/s)</th>
<th>(\overline{\omega}_p)</th>
<th>(\xi)</th>
<th>(U_{10}) (m/s)</th>
<th>(\overline{\omega}_p)</th>
<th>(\xi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 10.60</td>
<td>1.58</td>
<td>2.22</td>
<td>11 6.50</td>
<td>1.13</td>
<td>2.30</td>
</tr>
<tr>
<td>2 17.80</td>
<td>1.73</td>
<td>2.00</td>
<td>12 6.90</td>
<td>1.21</td>
<td>2.36</td>
</tr>
<tr>
<td>3 10.00</td>
<td>1.51</td>
<td>2.30</td>
<td>13 7.10</td>
<td>1.28</td>
<td>2.38</td>
</tr>
<tr>
<td>4 9.30</td>
<td>1.58</td>
<td>2.20</td>
<td>14 6.70</td>
<td>1.28</td>
<td>2.38</td>
</tr>
<tr>
<td>5 9.80</td>
<td>1.43</td>
<td>2.26</td>
<td>15 7.50</td>
<td>1.36</td>
<td>2.38</td>
</tr>
<tr>
<td>6 9.00</td>
<td>1.58</td>
<td>2.22</td>
<td>16 7.80</td>
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<td>7 8.70</td>
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<td>17 6.80</td>
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<tr>
<td>8 6.70</td>
<td>1.43</td>
<td>2.36</td>
<td>18 4.80</td>
<td>1.21</td>
<td>2.44</td>
</tr>
<tr>
<td>9 7.90</td>
<td>1.06</td>
<td>2.18</td>
<td>19 5.00</td>
<td>1.36</td>
<td>2.44</td>
</tr>
<tr>
<td>10 6.60</td>
<td>1.36</td>
<td>2.38</td>
<td>20 5.20</td>
<td>1.28</td>
<td>2.40</td>
</tr>
</tbody>
</table>

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Fig. 2 — The statistical relationship [Eq. (8)] among the wind speed, inverse wave age and inverse spectral width based on field measurements ("*" denote data in Table 1; "+" denote data in Table 2).
The abscissa denotes dimensionless parameter \((\overline{c}_a/\overline{U}_{10})\); the ordinate denotes the inverse wave age. The solid line and dashed line denote the relationship calculated from the first equation and second equation of statistical relationship [Eq. (8)], respectively, and the second equation is not obtained from fitting the data in Tables 1 and 2, but from other measurements\(^{11,17}\).

The spectrum of young wind waves with a value of \(\xi\) larger than 2.45 is quite difficult to measure because of the influence of “Doppler Effect”. For very young waves under slow winds, the orbital speed of dominant waves is comparable to the phase speed of shorter gravity-capillary waves riding on the dominant waves, the Doppler Effect makes it very difficult to measure the right side of wind wave spectrum, which is located in higher-frequency range. In order to overcome the Doppler Effect, special instruments, such as laser slope gauge\(^{16,18}\) and four-band microwave radar\(^{15}\), are needed to measure the right side of wind wave spectrum in high frequency range, which will be used to determine the inverse spectral width. Therefore, the majority of data in Fig. 2 are distributed in the range of inverse spectral width less than 2.45, the data with inverse spectral width larger than 2.45, in Fig. 2, are very sparse\(^{19}\).

3 Results and Discussion

3.1 Comparison with the measurements in the Black Sea

3.1.1 Comparison of \(m_0\)

To verify the proposed spectral model with the statistical relationship [Eq. (8)], the measurements and some empirical relations of wind wave spectrum of the deep water from previous investigators will be used for comparisons. The data measured in the Black Sea\(^{19}\) include wind speed, wave age and zeroth spectral moment simultaneously, and they are useful for verification.

Figure 3 shows the comparison of the measured data with \(m_0\) obtained from the new proposed model, consisting of Eqs. (1), (2) and statistical relationship [Eq. (8)]. All the 72 groups of data \((\overline{U}_{10}, \overline{f}, m_0)\) measured in the Black Sea are used to verify the new wind wave spectrum model proposed in this study (see Appendix A). The abscissa of scattered points is \(m_0\) obtained from the new proposed model. The ordinate is \(m_0\) measured in the Black Sea\(^{19}\). The error of root mean square is 0.08 m\(^2\), and the mean bias is 0.01 m\(^2\). This suggests that \(m_0\) from the new proposed model has a good agreement with measurements. Also, the comparisons with other models\(^{10-12}\) of wind waves show big differences. Therefore, the proposed model is verified to be reasonable.

3.1.2 Comparison of \(m_0/\omega_p S(\omega_p)\)

Based on the observed data given by Krivinskii\(^{20}\), an experiential formula is proposed by Babanin & Soloviev\(^{19}\) as

\[
\frac{m_0}{\omega_p S(\omega_p)} = (0.118 \pm 0.056) \bar{f}_{pp}^{911-0.30} f_{pp}^{0.23} \leq 0.23 \quad \ldots(9)
\]

Figure 4 shows the comparison of \(m_0/\omega_p S(\omega_p)\) obtained from the new proposed model with that...
obtained from experiential formula [Eq. (9)] based on measurements in the Black Sea. The abscissa of scattered points is \( m_0/\omega_p S(\omega_p) \) obtained from the new proposed model based on measured \( U_{10} \) and \( \tilde{f}_p \). The ordinate is \( m_0/\omega_p S(\omega_p) \) obtained from experiential formula [Eq. (9)] based on measured \( f_p \). The correlation coefficient between the two groups of calculated results is 97.9%, the error of root mean square is 0.01 and the bias is 0.004. This close relationship reflects reliability of the new proposed model.

3.2 Comparison of omni-directional elevation spectrum

The omni-directional elevation spectrum, \( S(\omega) \), is of the form of Eq. (6). Figure 5 gives the omni-directional elevation spectrum of fully developed \((\omega_p = 1.0)\) wind waves based on the new wind wave spectrum model. The solid lines from lower to upper are corresponding to wind speed \( U_{10} = 3, 5, 7, 11, 15, 21 m/s \), respectively. The dashed line denotes \((0.0046 \, \omega^2 \omega^{-5})\) and it represents the equilibrium spectrum of fully developed wind waves proposed by Phillips. The equilibrium range of wind wave spectrum is within a higher frequency range located to the right of \( 2 \omega_p \). As shown in Fig. 5, for fully developed wind waves, the wind wave spectral steepness is \( \omega^{-5} \) in equilibrium range. This is consistent with measurements given by Kitaigorodskii & Alesksandrovich.

3.3 Comparison of dimensionless omni-directional elevation spectrum

According to Eq. (8), for fully developed wind waves \((\omega_p = 1.0)\), when the wind speed \( U_{10} > 10 m/s \), the inverse wave age is \( \xi = 2.0 \). When \( U_{10} \) is in intermediate range from 6 to 10 m/s, the value of \( \xi \) is between 2.0 and 2.29. When \( U_{10} \) is in a low range from 2 to 6 m/s, the value of \( \xi \) is between 2.29 and 2.57. Equation (1) shows that the exponent of angular frequency \( \omega \) is \( -2(\xi-4+5) \). Consequently, inverse spectral width \( \xi \) in the model can be used to describe spectral steepness of the spectrum at higher frequencies located to the right of spectral peak of elevation spectrum. Substituting \( \xi = 2.0 \) into Eq. (1), we find that the wind wave spectrum has a spectral steepness of \( \omega^{-5} \). In the same way, by substituting \( \xi = 2.29 \) and \( \xi = 2.57 \) into Eq. (1), the wind wave spectrum has a spectral steepness of \( \omega^{-5.58} \) and \( \omega^{-6.14} \) respectively. According to observations of Liu, for fully developed wind waves, the spectral steepness with low wind speed is \( \omega^{-5} \), and it is \( \omega^{-6} \) with high wind speed. So, the statistical relationship [Eq. (8)] is consistent with the reported measurements.

Figure 6 shows the dimensionless omni-directional elevation spectrum obtained from the new spectrum wind wave model. The abscissa of Fig.6 denotes angular frequency normalized by spectral-peak angular frequency \( (\omega_p/\omega_p) \), where \( \omega_p \) is spectral-peak angular frequency; the ordinate denotes the dimensionless omni-directional elevation spectrum \( S(\omega)g^3/U_{10}^5 \). The solid lines from lower to upper are dimensionless omni-directional elevation spectra of full developed wind waves obtained from the new proposed model corresponding to wind speed \( U_{10} = 3, 5, 7, 11, 15, 21 m/s \), respectively. The line A, B, C denote that the spectral steepness is \( \omega^{-4} \), \( \omega^{-5} \) and \( \omega^{-6} \) respectively. From Eq. (8), for
developed wind waves, when the wind speed is larger than 10 m/s, the spectral steepness is \( \omega^{-5} \), hence, the dimensionless omni-directional elevation spectra are overlapped with each other when \( U_{10} = 11, 15, 21 \) m/s. It also can be seen from Fig. 6 that for low wind speed \( (U_{10} = 3, 5 \) m/s), the spectral steepness of developed spectrum is \( \omega^{-6} \) in range of \( \omega > 2\omega_p \). For intermediate and high wind speed \( (U_{10} = 7 \) m/s), the spectral steepness of developed spectrum has a transition from \( \omega^{-4} \) to \( \omega^{-5} \) in range of \( 1.5\omega_p < \omega < 2.5\omega_p \), which is consistent with measurements of Forristall23, and Leykin & Rozenberg24. So, with the parameter inverse spectral width \( \xi \), the new wind wave spectrum model is appropriate to be used to describe the real energy distribution of wind waves.

3.4 Comparison with other field measurements

3.4.1 Comparison of the elevation spectrum of wind waves with observations

Figure 7A shows a comparison of the elevation spectrum calculated from the new model with the field data of the four-frequency radar 15. Since the four-frequency radar cannot distinguish the wind waves at upwind direction with those at downwind direction, the field measurements of the four-frequency radar represent \( S(k,0^\circ) + S(k,180^\circ) \), i.e., the sum of upwind direction spectrum and downwind direction spectrum. In Fig. 7A, the solid line denotes \( S(k,0^\circ) + S(k,180^\circ) \) calculated from Eqs. (1) and (8), and the dashed line denotes that calculated from Eq. (2). The inverse wave age of \( \omega_p > 1.0 \) represents the developing wind waves, rather than the developed wind waves. For \( U_{10} = 2.5 \) m/s, the inverse wave age \( \omega_p = 1.2 \); for \( U_{10} = 11, 13 \) m/s, \( \omega_p = 1.5 \). According to the statistical relationship (8), the corresponding inverse spectral width \( \xi \) are 2.56, 2.21 and 2.13, respectively.

Obviously, there is a perfect agreement between the developing elevation spectrum data 15 and the new proposed model. Consequently, the new model can simulate spectral steepness fairly well. It should be noted that other models do not include a factor, which can influence spectral steepness, thus they cannot explain the data measured by Valenzuela15.

3.4.2 Comparison of wave-slope spectrum with measurements

The new proposed model is also consistent with slope spectrum of wind waves measured by laser-optical sensor. The directional frequency spectrum of wind wave slopes is defined by

\[
\Phi(f, \phi) = S(\omega, \phi)k^2 \frac{d\omega}{df} = 2\pi k^2 S(\omega, \phi) \quad (10)
\]

where \( f \) is the frequency in Hz. The frequency spectrum of wind wave slopes can be calculated from

\[
\Phi_{down}(f) = \int_{-\pi}^{\pi} \Phi(f, \phi) \cos^2(\phi - \theta) d\phi \quad (11)
\]

where \( \theta \) is the angle between dominant wave direction and wind direction. Because of the change of wind direction, the dominant wave direction may be inconsistent with the wind direction.

To avoid the influence of Doppler Effect, Tang & Shemdin16 measured a wave-slope frequency spectrum using a laser-optical sensor mounted on a wave
follower. Figure 7B shows a comparison of the wave-slope frequency spectrum calculated from the new proposed model with the filed measurements of Tang & Shemdin. The solid lines denote the long wave spectra calculated from Eqs. (1) and (8) with $U_{10} = 7.5 \text{m/s}$, $\bar{U}/p = 2.1$, $\xi = 2.44$, $\theta = 39^\circ$ and $U_{10} = 7.7 \text{m/s}$, $\bar{U}/p = 2.8$, $\xi = 2.50$, $\theta = 38^\circ$, respectively. The dashed lines denote the short wave spectra calculated from Eq. (2) with $U_{10} = 7.5 \text{m/s}$ and $U_{10} = 7.7 \text{m/s}$, respectively. Except for the data at low frequency, which were caused by the influence of swell, the new model can keep a good consistency with measured data.

3.5 Application of the new proposed wind wave spectrum model

3.5.1 Mean square slope of sea surface

The mean square slope of sea surface is very important for understanding the physical processes at the air-sea interface, and for interpreting altimeter and scatterometer radar backscatter measurements. Ebuchi & Kizu indicated that the sea surface mean square slope measured by Cox & Munk were obtained under developing wind wave status, and same should not be compared under the same wave age. Figure 8 gives comparison of the integrated mean square slope obtained from the new wind wave spectrum model under different wave age conditions with the optical observations of Cox & Munk. The solid lines from lower to upper, the inverse wave age is 1.0, 2.0, 3.0 and 4.0, respectively. It can be seen that under different wave age conditions, the new wind wave spectrum model, consisting of Eqs (1), (2) and statistical relationship [Eq. (8)], can well cover the majority of measurements.

3.5.2 Importance of wind wave spectrum model in microwave remote sensing

The directional wavenumber spectrum can be calculated from the directional frequency spectrum as follows

$$S(k, \phi) = S(\omega, \phi)k^{-1} \frac{d\omega}{dk}$$

where $k$ is the wavenumber, $\phi$ is the directional angle, and $\omega$ is the angular frequency. For gravity waves, the dispersion relation is

$$\frac{d\omega}{dk} = \frac{g}{2\omega}$$

and for gravity-capillary waves, it is

$$\frac{d\omega}{dk} = \frac{1}{2\omega} \left( g + \frac{3\tau}{\rho} k^2 \right)$$

where $\tau$ is the sea surface tension, $\rho$ is the sea water density, and $g$ is the acceleration of gravity. To fully display the feature of gravity-capillary wave spectrum, the curvature spectrum, also called the degree of saturation by Phillips, is often used. Sea surface slope is the first-order derivation of sea surface elevation, and sea surface curvature is the second-order derivation of sea surface elevation. The omni-directional curvature spectrum for full range can be calculated by

$$B(k) = \int_{-\pi}^{\pi} B(k, \phi)d\phi = \int_{-\pi}^{\pi} k^4 S(k, \phi)d\phi$$

Figure 9 shows the comparison of omni-directional curvature spectrum for fully developed wind waves in full wavenumber range based on the new wind wave spectrum model with existing wind wave spectrum models. The comparison shows that these spectrum models are in good agreements with each other in the range of wavenumber $<1 \text{ rad/m}$. However, they are different in the range of wavenumber over 1rad/m. After the spectral level arrives at the secondary gravity-capillary peak, it decreases sharply with the increase of wavenumber, except for the model proposed by Durden & Vesecky.
When the incidence angle is about 30°, the electromagnetic (EM) waves of C-band scatterometer have Bragg-resonance scattering with wind waves at wavelength ranging from 5 cm to 10 cm, and the EM waves of Ku-band scatterometer have Bragg-Resonance scattering with wind waves at wavelength ranging from 0.5 cm to 2 cm. What should be noted is that the wavenumber ranging from 6 rad/m to 80 rad/m (Fig. 9E) corresponds to the wavelength ranging from 8 cm to 100 cm. This range is just corresponding to the most powerful signals received by the L-band microwave radiometer for remote sensing of sea surface salinity.

Figure 9 shows the omni-directional curvature spectrum for developed wind waves. Ten curves denote the wind speed range is from 3 to 21 m/s with a step of 2 m/s, and these models are quite different within the wave-number range between 6 rad/m and 80 rad/m. Therefore, it is quite important to select a better model of wind wave spectrum for salinity remote sensing of microwave radiometer.

As shown in Fig. 9E, firstly, for fully developed wind waves with low wind speed, the wind wave spectrum located to the right of spectral peak is

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Fig. 9 — The omni-directional curvature spectrum for developed wind waves: (A) Durden & Vesecky\(^1\); (B) Apel\(^2\); (C) Elfouhaily \( et \) al.\(^3\); (D) Kudryavtsev \( et \) al.\(^4\); (E) the new wind wave spectrum model. Ten curves denote the wind speed range is from 3 to 21 m/s with a step of 2 m/s. The abscissa denotes wavenumber, and the ordinate denotes omni-directional curvature spectrum.
steeper, and this is consistent with measurements of Liu\textsuperscript{17}. Secondly, when the wind speed is low, the long wave spectrum with a spectral steepness of $\omega^{-6}$ will decrease with the increase of wavenumber, and it will connect with short wind wave spectrum with low energy level, naturally. A smooth connection makes it possible that the model be used more conveniently in the studies of oceanic microwave remote sensing. Consequently, the new wind wave spectrum model consisting of model proposed by Liu\textit{ et al.}\textsuperscript{5-9} and statistical relationship [Eq. (8)] is able to describe the real energy distribution of wind waves.

The parameter “inverse spectral width” in the new model can describe the steepness of wind waves. Thus, the new model can be used to describe the variation of wind wave spectral steepness with the change of inverse wave age and wind speed. From section 3, it can be seen the new model can describe the real energy distribution of wind waves. Also, based on the study of Liu\textit{ et al.}\textsuperscript{6,7}, the 5.3GHz C-band scatterometer empirical model supports spectrum shown in Fig. 9E. Therefore, the new proposed model provides a powerful tool for studying the uncertainty of retrieved ocean environment variables and the mechanism of microwave remote sensing of sea surface salinity.

3.6 Compare with the statistical relationship among the three parameters for laboratory condition

Considering that laboratory experiments are not representative of wave spectral structure because it is impossible to re-create the large-scale turbulence of wind that plays a large role in wind wave growth. There are other important disadvantages of laboratory measurements, such as limited directional structure or involvement of capillary effects that restrict the resemblance of natural and laboratory waves too\textsuperscript{19}. Hence, measured data from laboratories and the ocean should be separately dealt with to study the statistical relationship among the wind speed, inverse wave age and inverse spectral width.

3.6.1 The statistical relationship among the three parameters for laboratory condition

Same as for field case, statistical relationship among the inverse wave age, the inverse spectral width and the wind speed are obtained as follows

\[
\begin{align*}
\xi &= 4.90(\overline{\omega}_p c_m / U_{10}) - 0.05 & 0.16 < \overline{\omega}_p c_m / U_{10} < 1.75 \\
\xi &= 2.0 & \overline{\omega}_p c_m / U_{10} \leq 0.16
\end{align*}
\]

where $c_m = 0.23 \text{m/s}$ is minimum phase speed of gravity-capillary waves, $\overline{\omega}_p c_m / U_{10}$ is a dimensionless parameter, and $\xi$ denotes the estimated value from the inverse wave age and the wind speed.

Figure 10 shows the relationship among the wind speed, inverse wave age and inverse width based on laboratory measurements. The solid line and dashed line in Fig. 10 denote the relationship calculated from the first equation and the second equation of the statistical relationship (16), respectively. The first Table 3 — The wind speed, inverse wave age and inverse spectral width obtained from laboratory observations

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Fig. 10 — The statistical relationship among the wind speed, the inverse wave age and the inverse width obtained based on laboratory measurements (\textsuperscript{*} denote data from Table 3).
equation in formula (16) is obtained by simulating scattered data listed in Table 3, and the second equation is obtained from other experiments\textsuperscript{11}.

With the statistical relationships (8) and (16), the influence of the wind speed or the wave age on the wind wave spectrum can be studied. With the same inverse wave age ($\frac{1}{\omega}$), the inverse spectral width ($\xi$) decreases with the increase of the wind speed ($U_{10}$). Substituting $\xi$ into Eq. (1), through the term $\omega^{-\frac{2}{5} - \frac{4}{5}}$ we can find that the spectral steepness at high frequencies located to the right of the spectral peak of the wind wave spectrum decreases either. With the same wind speed, the inverse spectral width increases with the increase of the inverse wave age, so does the spectral steepness at high frequency range. Therefore, the new proposed model composed of the statistical relationship (8), (16) and Eqs. (1) and (2), is suitable for describing the wind waves in developing stage and representing the width character of energy distribution of wind waves.

3.6.1.1 Comparison with laboratory measurements

The statistical relationship (16) also suggests that the inverse spectral width, which has an influence on the energy distribution of wind waves, is determined by only two basic parameters, i.e., the wind speed and the inverse wave age. The two basic parameters determine the total spectral energy level, the location of the maximum spectral energy, and the width character of the energy distribution at high frequencies located to the right of the spectral peak of the wind waves.

Because the gravity-capillary waves ride on the long waves, to avoid the influence from the Doppler Effect, special techniques are needed to measure gravity-capillary waves, for example, “laser slope gauge” and “imaging optical technique and digital image processing”. Figure 11 gives a comparison of the omni-directional curvature spectrum for full frequency range obtained from Eqs. (1), (2) and (16) with the laboratory measurements of Jähne & Riemer\textsuperscript{27} at a 90 m fetch using the “laser slope gauge”. The solid lines denote the long wave curvature spectra calculated from Eqs. (1) and (16), and the dashed lines denote the short wave curvature spectra calculated from Eq. (2). The comparison shows that the new wind wave spectrum model consisting of the model proposed by Liu et al.\textsuperscript{5,9} and the statistical relationship (16) can accurately describe the location and magnitude of the spectral peak, and also it can describe the spectral steepness at high frequencies located to the right of the spectral peak. In statistical meaning, the inverse spectral width is used to describe the spectral steepness. Obviously, the new wind wave spectrum model can well describe the variation of spectral steepness when the wind speed and the inverse wave age change.

3.6.1.2 Comparison of measurements by “imaging optical technique and digital image processing”

Figure 12A gives a comparison of the omni-directional curvature spectrum for full wavenumber range obtained based on the new proposed model with laboratory measurements of Jähne & Riemer\textsuperscript{27} at a 90 m fetch using the “imaging optical technique and digital image processing” (different with data listed in Table 2). The lines in Fig. 12A denote the omni-directional curvature spectra calculated from Eqs. (1), (2) and (16). Figure 12B gives a comparison of the omni-directional curvature spectrum obtained based on the model proposed by Elfouhaily et al.\textsuperscript{3} with laboratory measurements of Jähne & Riemer\textsuperscript{27}. The lines in Fig. 12B denote the omni-directional curvature spectra calculated from the model proposed by Elfouhaily et al.\textsuperscript{3}.

Compared with the wind wave spectrum model proposed by Elfouhaily et al.\textsuperscript{3}, the wind wave spectrum model consisting of Eqs. (1), (2) and (16) is in better agreement with the laboratory measurements. In particular, the new model can describe the spectral steepness at high frequencies located to the right of the spectral peak fairly well within the range of wavelength larger than 5 cm (corresponding to
wavenumber less than 125 rad/m). And there is a large difference between measurements and the wind wave spectrum model proposed by Elfouhaily et al.\textsuperscript{3} in the range mentioned above, which decreases the practicability of the model in oceanic remote sensing.

3.6.2 Comparison of the wind wave spectrum for field and laboratory conditions

Figure 13 shows the difference between field wind wave spectrum and laboratory wind wave spectrum with the same wind speed and wave age. The solid line denotes the spectrum of the wind wave for full frequency range in developing stage generated in the open ocean, which is plotted based on Eqs. (1), (2) and (8). The dashed line denotes that generated in the wind wave tank, which is plotted based on Eqs. (1), (2) and (16). The solid line corresponds to $U_{10}=6.0\text{ m/s}$, $\bar{\omega}_p=6.0$, and $\xi=2.72$ that is obtained from Eq. (8). Substituting $\xi=2.72$ into the term $\omega^{-\left(2\xi-4+5\right)}$ in (1), we can find that the spectral steepness at high frequencies to the right of the spectral peak is $\omega^{-0.44}$. The dashed line corresponds to $U_{10}=6.0\text{ m/s}$, $\bar{\omega}_p=6.0$, and $\xi=2.47$ that is obtained from Eq. (16). In this case the spectral steepness of the wind wave spectrum is $\omega^{-0.54}$ for laboratory condition.

From Fig. 13, it is can be seen that, to the right side of the spectral-peak frequency, the steepness of spectrum generated in the open ocean is different from that generated in the wind wave tank. This is the major difference between the two kinds of spectrum, which was also shown in measurements of Lu\textsuperscript{29} (see Ref. 29, Fig. 1). Although the spectral-peak frequencies of the wind wave spectrum are the same for laboratory and field cases, the energy dissipation rates of wind waves at the spectral-peak frequency are different, which leads to the difference of the width character of energy distribution at high frequencies located to the right of the spectral peak of wind waves.

3.6.3 Background of statistical relationships

Compared with existing models of wind wave spectrum for developing waves, only one parameter “inverse spectral width” is added in the new wind wave spectrum model. With the parameter, the width character of energy distribution (represented by the spectral steepness) at high frequencies located to the right of spectral peak of wind waves can be described\textsuperscript{13}. The new model can be used to describe the wind wave spectrum with different spectral steepness. This is the difference between the new model and other models.

Both field wind waves and laboratory wind waves have the same process in energy input and transmission, only difference is in extent of dissipation. The energy inputs from wind into gravity-capillary waves through wind stress, especially those waves with slower phase speed and wavelength of about 0.5-2 cm. Due to the modulation of dominant
wind waves which the gravity-capillary waves ride on, the energy is transmitted from the gravity-capillary waves into dominant waves with lower frequency. Then, with wave-wave interaction, the energy is transmitted to shorter gravity wind waves, which are between the dominant waves and the gravity-capillary waves, from the dominant waves, and this process is common for both wind waves in the sea and laboratory tank. The only difference is that the energy dissipation differs from each other. The effect of dissipation of wind waves can just be described by the inverse spectral width. Therefore, the wind wave spectrum model proposed by Liu et al.5-9, combined with Eqs. (8) and (16), can be used to describe both field and laboratory measurements. Except for different dissipation, there is no sufficient evidence indicating the generation mechanism of field wind waves is different from laboratory wind waves. So, as an argument, whether or not a model of wind wave spectrum with different relationship of spectral width can describe wind waves from field and laboratory tank should be checked out by readers. With more measurements, the question may be well solved.

4 Conclusion
Based on the wind wave spectrum model for deep water proposed by Liu et al.5-9 and statistical relationships (8) and (16) derived in this paper, a new omni-directional spectrum model for full wavenumber range is established. The comparison with field measurements shows that the statistical relationship (8) proposed in this study with the spectrum model given by Liu et al.5-9 can better describe and explain the field measurements of wind waves in developing stage. The comparison confirms that spectral width calculated based on the new model shows excellent agreement with measurements in the Black Sea. On the other hand, the new model can better simulate the field data of the four-frequency radar, and other field data. Especially, the wind wave spectrum calculated from the new model is in a good agreement with the measurements not only in the location of spectral peak and the spectral level, but also in spectral width character of wind waves. Other wind wave models cannot clearly describe this character.

The inverse spectral width can be used to describe spectral width character of wind waves, i.e., spectral steepness of long wind waves. The spectral steepness of long wind waves influences the connection between long wave spectrum and short wave spectrum. The wind wave energy at the joining location with wavelength ranging from 8 cm to 100 cm is just corresponding to the most powerful signals received by the L-band microwave radiometer for remote sensing of sea surface salinity. The change of this location also influences the spectral description of gravity-capillary waves. The energy of gravity-capillary waves is just the target of C-band, X-band and K-band microwave radiometers. Therefore, the new model provides a powerful tool for studying the uncertainty of microwave remote sensing30,31.

5 Acknowledgement
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6 References


### Appendix A

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75 groups of data measured in the Black Sea