Calculation of flux due to a non-uniform line radiation source and a uniform cylindrical volume source using Mathematica

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A non-uniform line source has a continuous spatial variation of activity along its length. It may not be possible to divide such a source into smaller sources of uniform activity. Such situation may arise in case of a pipe carrying radioactivity in liquid form. A Mathematica based code was developed for calculating uncollided flux due to a line source having Gaussian distribution of activity and also a line source with uniform activity placed in air or a non-attenuating medium for comparison. Uncollided flux due to a uniform cylindrical source behind a slab shield was also calculated by replacing it with an equivalent uniform line source using another Mathematica based code and it’s output was compared to a point kernel method based code GUI2QAD-3D. Both the codes employ numerical integration.

**Keywords:** Line source, Gaussian, Cylindrical source, Point kernel code, Numerical integration

1 Introduction

In the nuclear industry, various plants such as spent fuel reprocessing facility, waste immobilization plant etc. have to handle radioactive isotopes in solution form. During different stages of chemical or physical processing these solutions may have to pass through considerable length of pipes, which connect the various process vessels. In order to protect people working in the vicinity of sources of such geometry, the radiation doses (or flux) at various nearby locations have to be estimated as accurately as possible. Such pipes carrying radioactive liquid (uranyl nitrate solution for example) can be considered as line sources of constant specific source strength (i.e., photons emitted per unit time per unit length) because the distance at which the dose (or flux) is measured is much larger than the diameter of the pipe and the activity is distributed uniformly. The uncollided fluxes in air or a non attenuating medium can be known by already existing formulas for such a line source at various locations. The flux due to the uncollided radiation is the same as the total flux as there is only geometric attenuation and no scattering. The total dose can be obtained from this flux by multiplying the (energy dependent) flux to dose conversion factor. The assumption of constant activity per unit length may not be valid in all situations especially when the flow in pipes is non-uniform. One such case of non-uniform line source where the activity has a Gaussian spatial variation is considered in this paper and the difference caused in the uncollided flux by such an assumption is obtained. For mathematical simplicity, source strength is used instead of activity in further treatment. Also in a separate but similar study a uniform cylindrical volume source is considered. Uniform cylindrical volume source is the most common source geometry in nuclear facilities and can be approximated by an equivalent uniform line source for the purpose of uncollided flux calculations behind slab shield. The effect caused by such an approximation can be known by comparing the results with the output of a more accurate point kernel method based code like GUI2QAD-3D. Though not considered here this uncollided flux can also be converted to total dose by using flux to dose conversion factors followed by the appropriate dose build-up factors. Only single energies are considered in both the non-uniform line source and the uniform cylindrical volume source case but multiple energy calculations can be carried out by repeating the calculations for each energy separately as dose and flux are additive. Improved form of the codes may include flux to dose conversion factors and multiple energies within the codes in future.

2 Method

Consider a line source emitting photons of a single energy. The source lies with its length along z-axis as shown in Fig. 1. It’s source strength is given by:
\[ s(z) = s_0 \exp[-(z/z_0)^2] \quad \text{... (1)} \]

where \( s(z) \) is source strength at \( z \), \( s_0 \) is source strength at \( z = 0 \), \( z_0 \) is distance at which source strength drops by a factor 1/e. Let this line source be divided into infinitesimal point sources. The uncollided flux in air (neglecting any attenuation) or a non-attenuating medium due to such a point source is given by:

\[ d\Phi_u = ds/4\pi r^2 \quad \text{... (2)} \]

where \( d\Phi_u \) is uncollided flux due to infinitesimal source element, \( ds \) is source strength of infinitesimal source element, \( r \) is distance between the source element and the detector point.

Now considering Eq. (1)

\[ ds = -(2 s_0/z_0^2) z \exp[-(z/z_0)^2] \, dz \quad \text{... (3)} \]

the negative sign indicates that \( s(z) \) in Eq. (1) decreases as \( z \) increases hence is ignored in further treatment. Also,

\[ r^2 = (x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 \quad \text{... (4)} \]

where \((x, y, z)\) are the co-ordinates of source element and \((x_0, y_0, z_0)\) are the co-ordinates of the point detector. Using Eqs (3) and (4), \( \Phi_u \) the uncollided flux due to entire line source of length \( L \) is obtained by integrating \( d\Phi_u \) in Eq.(2):

\[ \Phi_u = (s_0/2\pi z_0^2) \int_0^L (z/r^2) \exp[-(z/z_0)^2] \, dz \quad \text{... (5)} \]

The integral in Eq. (5) cannot be expressed in analytical form and hence has to be subjected to numerical integration. The numerical integration is performed using a simple Mathematica 6.0 based code employing trapezoidal rule. Provisions were made in the code itself for evaluation at multiple detector points so as to obtain variation along the radial and \( z \) directions as well as plotting these variations. Also, for comparison, uniform line source calculations were included in the code using the formula:

\[ \Phi_u = (S_0/4\pi r_d)[\tan^{-1}(L_1/r_d)+\tan^{-1}(L_2/r_d)] \quad \text{... (6)} \]

For \( 0 \leq z_d \leq L \) where \( S_0 \) is the source strength per unit length, \( L_1 \) and \( L_2 \) are lengths of source portions into which the perpendicular from the detector divides the source, \( r_d \) is source to detector perpendicular distance and is same as \( [(x-x_0)^2 + (y-y_0)^2]^{1/2} \), \( z_d \) is the vertical location of the detector in the co-ordinate system as shown in Fig. 1. Similarly, a separate code was written for a cylindrical volume source as shown in Fig. 2 below. The source emits photons of a single energy and has a lateral shield of thickness \( t \). Uncollided flux measurements are considered at points P1 and P2 as shown in Fig. 2 below. The cylindrical source is replaced by an equivalent line source located at a distance \( z \) from source surface called the self absorption distance.

This distance is obtained from the \( \mu, z \) curves given in Ref. (1). The further analytical formulas are obtained by using this line source as given in Ref. (1). The uncollided flux at P1 is given by Eq. (7), if \( \theta_1 = 0 \) by Eq. (8) and at P2 by Eq. (9).

\[ \Phi_u = [S_v R_0^2/4(a+z)][F(\theta_1, b_2)+F(\theta_2, b_2)] \quad \text{... (7)} \]

**Fig. 1** — Co-ordinate system with source and source element at \((x, y, z)\) and detector at \((x_d, y_d, z_d)\), y-axis is directed into the page

**Fig. 2** — Source and shield placement with the detector points
\[ \Phi_b = \frac{S_o R_0^2}{2(a+z)} F(\theta, b_2) \] \hspace{1cm} \ldots(8)

\[ \Phi_a = \frac{S_o R_0}{4(a+z)} [F(\theta_2, b_2) - F(\theta_1, b_2)] \] \hspace{1cm} \ldots(9)

where \( S_o \) is source strength per unit volume, \( R_0 \) is source radius, \( a \) is source to detector distance, \( z \) is self absorption distance, \( b_2 \) is \( \mu_s z + \mu_a t \), \( \mu_s \) is source linear attenuation coefficient, \( \mu_a \) is shield linear attenuation coefficient, \( t \) is shield thickness and the sievert integral is given by:

\[ F(\theta, b) = \int_0^\theta \exp(-b \sec \theta') \, d\theta' \] \hspace{1cm} \ldots(10)

The sievert integrals occurring in Eqs(7-9) are not possible to evaluate analytically. Hence, they were numerically integrated using Nintegrate function in a Mathematica 6.0 code which evaluates the Eqs (7) or (8) or (9) depending on the location of detector point.

3 Results and Discussion

A set of computations were performed using the previously mentioned code so as to study the nature of radiation field around the uniform and Gaussian line sources. The two sources used in the computations are chosen to have same lengths (\( L = 5 \) m) and total source strengths. Therefore,

\[ S_L L = \int_0^L ds = \int_0^L -(2 \frac{s_0}{z_0^2})z \exp[-(z/z_0)^2]dz \] \hspace{1cm} \ldots(11)

and

\[ S_L = (s_0/L)(1-\exp[-(L/z_0)^2]) \] \hspace{1cm} \ldots(12)

The source strength at \( z = 0 \) is \( s_0 = 1000 \) photons/s and distance at which source strength is \( 1/e \) times \( s_0 \) i.e., \( z_0 \) is 3 m for the Gaussian source. The radial variation at every 10 cm for a plane at \( z = 2.5 \) m and the vertical variation at every 10 cm for a perpendicular distance of 1 m along the entire length of the source was plotted using the above mentioned code itself. The results obtained are shown in Fig. 3 below. In the azimuthal direction uncollided flux will be constant for a given radial distance and \( z \) value due to symmetry in this direction for both the uniform line source and the Gaussian line source. Hence, the nature of radiation flux variation or the radiation field is similar for both types of sources in this direction.

\[ \Phi_o = (s_o/2\pi z_0^2) \int_0^L (z/r^2)\exp[-(z/z_0)^2]\exp(-\mu t')dz \] \hspace{1cm} \ldots(13)

where \( \mu \) is the linear attenuation coefficient for the slab medium and \( t' \) is the thickness of the slab traveled by the radiation beam between source element and detector. Where \( t' = (rd)t, r \) is source element to detector distance, \( d \) is source to detector perpendicular distance and \( t \) is slab shield thickness. All other symbols are same as used in previous expressions.

To test the accuracy of the second code a simple example consisting of 0.1 \( \mu \)Ci/cm\(^2\) \( Cs^{17} \) in water in a cylindrical volume was considered. \( Cs^{17} \) emits 0.662 MeV gammas in 85% of the transformations. This corresponds to 3154 photons/s/cm\(^3\). The source radius was 25 cm and height was 79 cm. The detector was placed at a distance of 250 cm from the source surface and a concrete shield of 15 cm was placed at 100 cm from source surface. The shield is considered
The self absorption distance for \( \mu_s = 0.0891 \text{ cm}^{-1} \) of water in the above source was found out to be 16.835 cm from Ref. (1). The flux at detector locations corresponding to A (275, 0, 30) cms, B (275, 0, 39.5) cms, and C (275, 0, 89) cms was obtained using the code. The center of cylindrical source base is considered the origin of the coordinate system. These points correspond to a situation like in point P1, like in point P1 but \( \theta_1 = \theta_2 = \theta \) and like in point P2, respectively in Fig. 2. The results obtained are compared with the GUI2QAD-3D results which provide the total dose as well as the uncollided energy flux (in MeV/cm\(^2\)/s). This energy flux is converted to uncollided particle flux by dividing it with uncollided photon energy of 0.662 MeV. The results are shown in Table 1.

### Table 1 — Comparison of code outputs

<table>
<thead>
<tr>
<th>Detector Location</th>
<th>Code output (Photons/cm(^2)/s)</th>
<th>GUI2QAD–3D output (Photons/cm(^2)/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>7.255</td>
<td>9.337</td>
</tr>
<tr>
<td>B</td>
<td>7.282</td>
<td>9.369</td>
</tr>
<tr>
<td>C</td>
<td>6.583</td>
<td>8.615</td>
</tr>
</tbody>
</table>

**4 Conclusions**

Mathematica can be used to calculate flux due to non-uniform gamma radiation sources, also based on the code results for Gaussian line source it can be concluded that the Gaussian spatial activity of a line source does not affect significantly the nature of the radiation field in air or a non attenuating medium. The code based on the method of uniform line source approximation of a cylindrical volume source can be used to obtain an initial estimate of uncollided flux.

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**References**