Extracting nuclear level density of \(^{56}\)Fe using a microscopic model with inclusion of pairing interaction

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The nuclear level density and thermal properties of \(^{56}\)Fe have been extracted using the BCS Hamiltonian with inclusion of interacting fermions. Single particle levels for Nilsson potential have been used in the calculations. The shape of the energy and entropy curves has been interpreted as indication of a double phase excitation region and proposed as signature of the pairing phase transition. The results are compared to the experimental data obtained by the Oslo group.

Keywords: Nuclear level density, BCS calculation, Pairing energy, Thermal properties, \(^{56}\)Fe

1 Introduction

An important goal in nuclear physics is to trace thermodynamic quantities as functions of excitation energy. The level density as a function of excitation energy is the starting point to extract thermodynamic quantities such as entropy and temperature. Much of the research in the area of nuclear level densities has been based on the Fermi gas model first introduced by Bethe\(^1\). In an attempt to reproduce the experimental data, various phenomenological modifications to the original analytical formulation of Bethe have been suggested, in particular to allow for shell and pairing effects. This has led first to the constant temperature formula, then to the shifted Fermi gas model and later to the popular back shifted Fermi gas model\(^2-7\).

Pairing correlations are one of the fundamental properties of nuclei and have been successfully described by Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity\(^8\).

Experimentalists recently developed a new method to extract level densities from measured \(\gamma\) spectra at the Oslo cyclotron laboratory. This method has been applied to extract level densities in \(^{56}\)Fe using \((^3\text{He}, ^4\text{He})\) reactions\(^9,10\).

In this work, the nuclear level density and entropy have been computed for \(^{56}\)Fe nucleus using superconducting theory with the inclusion of pairing effects. The results are compared with experimental values.

2 Microscopic Calculations

For a complete derivation of the formulas given in this section, see our previous publications\(^11,12\). For a system of interacting fermions, for example that of neutrons the logarithm of grand partition function is given by:

\[
\Omega = -\beta \sum_k (\varepsilon_k - \lambda - E_k) + 2 \sum_k \ln[1 + \exp(-\beta E_k)] - \beta \xi^2 / G \quad \text{... (1)}
\]

where \(T=1/\beta\) is the statistical temperature, \(\lambda = \alpha/\beta\) is the chemical potential, \(E_k\) is the quasi particle energy where \(\varepsilon_k\) is the single particle fermion energy level, \(\Delta\) is the gap parameter and it is a measure of nuclear pairing, while \(G\) is the strength of pairing interaction.

The BCS equations determine the gap parameter and the chemical potential as a function of pairing strength through the following equations:

\[
N = \sum_k \left( 1 - \frac{\varepsilon_k - \lambda}{E_k} \tanh \frac{\beta E_k}{2} \right) \quad \text{... (2)}
\]

\[
\frac{2}{G} = \sum_k \left( 1 - \frac{1}{E_k} \tanh \frac{\beta E_k}{2} \right) \quad \text{... (3)}
\]

The state density which is the inverse Laplace transform of the grand partition function can be evaluated, the result is:
\[ \omega(N, Z, U) = \frac{\exp(S)}{(2\pi)^{3/2} D^{1/2}} \] \quad \ldots (4)

$D$ is determinant of the second derivatives of the grand partition function taken at the saddle point. Here the entropy $S$ can be written as:

\[ S = 2\sum_k \ln[1 + \exp(-\beta E_k)] + 2\beta \sum_k \frac{E_k}{1 + \exp(\beta E_k)} \] \quad \ldots (5)

For a system of $N$ neutrons and $Z$ protons at excitation energy $U$ is:

\[ \rho(N, Z, U) = \omega(N, Z, U) / (2\pi\sigma^2)^{3/2} \] \quad \ldots (6)

where $\sigma^2$ is the spin cut-off factor.

### 3 Theory

The numerical calculations are done in the following way. (i) For a specified nucleon number $N$ the ground state energy $E_n^0$ is calculated by setting $T=0$ and solving Eqs (2 and 3). (ii) The critical temperature $T_c$ are evaluated by setting $\Delta=0$. (iii) The quantities $\lambda(T)$ and $\Delta(T)$ are evaluated for a given value of $T$.

The values of level density parameters $\lambda(T)$ and $\Delta(T)$ are used to compute $E_n$, $S_n$, $\omega(N,U)$. The excitation energy was then determined by subtracting the energy at $T=0$. The steps (i) to (iii) above are repeated for protons. The total excitation energy $U = U_n + U_p$.

The nuclear level density is evaluated from Eq. (6), Fig. 1 shows the logarithm of the level density as a function of excitation energy for $^{56}$Fe nucleus. Calculations have been based on the modified harmonic oscillator potential according Nilsson et al.\(^\text{13}\) It is seen from this Fig.1 that the agreement between the experimental and theoretical data with inclusion of pairing correlations is very good.

In Fig. 2, the evaluated temperature dependence of excitation energy for $^{56}$Fe nucleus is shown. The entropy of the neutron and proton system is evaluated from Eq. (5) at temperature $T$ from the values of $\lambda(T)$ and $\Delta(T)$. The total entropy is obtained as $S = S_n + S_p$. The entropy is plotted as function of temperature in Fig. 3 for $^{56}$Fe nucleus. The critical temperature in the combined system of both neutrons and protons is\(^\text{14}\):

\[ T_c = (T_c^n + T_c^p) / 2 \] \quad \ldots (7)

The deduced critical temperature is $T_c = 0.69$ MeV. The excitation energy and entropy curves have been interpreted as indication of a double phase excitation region, a phase with strong pair correlations and a phase with quenched pair correlations.
4 Conclusions

The level density of \( ^{56}\text{Fe} \) was determined from microscopic theory of interacting fermions. Examination of the Fig. 1 shows good agreement between theoretical and experimental data. However, at low energy also energy between 2.5 to 3 MeV, the agreement is rather poor. This may be due to uncertainty in the gap parameter.

Thermal properties are studied within the statistical canonical ensemble. It is found that the thermodynamic properties as the excitation energy and the entropy show structure reflecting the nuclear phase transition. The thermal breaking of cooper pairs leads to increasing entropy and level density. We found quenching of pairing gap near the critical temperature \( T_c = 0.69 \text{ MeV} \).

References