Novel methodology for broken-rotor-bar and bearing faults detection through SVD and information entropy

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Received 23 April 2012; revised 07 August 2012; accepted 08 August 2012

This study introduces a novel methodology for early detection of broken-rotor-bars and bearing faults on induction motors by using singular value decomposition and information entropy estimation to detect motor operational condition. Proposed technique could detect these faults with a certainty greater than 99.7%. A field programmable gate array implementation is developed for online application of proposed approach in real-time.

Keywords: Broken-rotor-bars (BRB), Faulty bearings (FBR), Field programmable gate arrays (FPGA), Induction motor, Information entropy, Singular value decomposition (SVD)

Introduction

Squirrel-cage induction motors are the most popular motors in industry\(^1\)\(^-\)\(^2\). Studies\(^3\)\(^-\)\(^5\) are available on induction motor failure that produces unexpected interruptions on production lines. At least, 50% of faults in rotating machines are bearing related, and 10% are rotor faults\(^6\). Fault identification of induction motors still represents a big challenge for condition monitoring (CM)\(^7\)\(^-\)\(^10\). Common signal processing techniques for CM are fast Fourier transform (FFT)\(^11\)\(^-\)\(^13\), and discrete wavelet transform (DWT)\(^14\)\(^-\)\(^16\). Singular value decomposition (SVD) and information entropy are techniques for extracting significant information from different datasets, but rarely used for induction motor CM\(^17\)\(^-\)\(^19\). This study presents a CM technique for induction motor that combines SVD with information entropy to detect broken-rotor-bars (BRB) and faulty bearings (FBR).

Proposed Methodology

Proposed methodology (Fig. 1) consists of a data acquisition system (DAS), matrix generation with acquired data in onboard memory, and a hardware processing unit implemented into an FPGA device for SVD and information entropy estimation to determine quantitatively the operational condition of induction motor as healthy (HLT), BRB and FBR.

Singular Value Decomposition (SVD)

SVD of an \(m \times n\) matrix \(A\) is defined as \(A = UV^T\), where \(U\) and \(V\) are \(m \times m\) and \(n \times n\) orthogonal matrices, respectively. \(UU^T = I_m\), and \(VV^T = I_n\), where \(I\) is identity matrix. \(\Sigma\) is a diagonal matrix such that \(\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r)\), where \(\sigma_1, \sigma_2, \ldots, \sigma_r\) are singular values of \(A\). For \(m>n\), at least \(m-n\) singular values are zero. If \(r=\text{rank}(A)\) and \(r<n\), \(r\) of singular values are non-zero. \(U\) contains \(m\) left singular vectors, and \(V\) contains \(n\) right singular vectors. Most SVD algorithms are based on diagonalizing rotations to form orthogonal transformations that preserve angles and lengths.

Jacoby Rotations

Usually Jacobi rotation matrix \(J\) is used for diagonalization through rotations\(^2^1\). For instance, when applying \(J\) in Eq. (1) to 2x2 symmetric matrix \(A_s\) in Eq. (2), rotation angle \(\theta\) is chosen such that off-diagonal elements \(a_{ij} = a_{ji}\) of \(A_s\) become zero, thus obtaining diagonal matrix \(A_d\) in Eq. (3) as

\[
J = \begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\]
\[ A_S = \begin{pmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{pmatrix} \quad \text{(2)} \]

\[ A_D = J^T A_S J = \begin{pmatrix} a_{ii} & 0 \\ 0 & a_{jj} \end{pmatrix} \quad \text{(3)} \]

If \( A \) is not symmetric, off-diagonal elements can still be annihilated by applying a symmetrizing rotation as \( A_S = J^T A \) and a diagonalization rotation as \( A_D = J^T A_S J \).

**Hestenes-Jacobi Method**

Hestenes\(^{22}\) observed equivalence between annihilation of a matrix element and orthogonalization of two vectors applying Jacobi rotations. If an \( m \times n \) non-symmetric matrix \( A \) is multiplied by an orthogonal matrix \( U \), a matrix \( B \) whose rows are orthogonal is obtained. \( B \) is normalized by calculating squares of row norms, then \( B = SS^T \), \( AB = SV \), where \( S \) is a diagonal matrix containing square norm of each row in \( B \), and \( V \) is an orthonormal matrix obtained by dividing each row in \( B \) by its corresponding square norm. From this, and because \( U \) is an orthogonal matrix, \( B = UA = SV \) and \( A = U^T SV \), where \( S = \text{diag}(s_1, s_2, \ldots, s_n) \) are squares of singular values \( (\sigma_1, \sigma_2, \ldots, \sigma_n) \) of \( A \).

**Information Entropy**

Shannon\(^{23}\) defined entropy as a measure of average information contents associated with a random outcome. Considering a random event \( X \) with \( n \) possible outcomes \( x_1, x_2, x_3, \ldots, x_n \) and every \( x_i \) \((i = 1, \ldots, n)\) with a probability \( p(x_i) \), then information entropy \( H(X) \) of a random event \( X \) is given as

\[ H(X) = -\sum_{i=1}^{n} p(x_i) \log_2 [p(x_i)] \quad \text{(4)} \]

If total number of outcomes in random event \( X \) is \( N \), probability \( p(x_i) \) is given as \( p(x_i) = \frac{r_i}{N} \), where \( r_i \) represents incidence rate of each possible outcome \( x_i \), and total number of outcomes \( N \) is given as \( N = \sum_{i=1}^{n} r_i \).

For entropy calculation, Mitchell algorithm\(^{24}\) is considered in this study because of its advantages during hardware implementation.

**Experimental Setup and Instrumentation System**

Proposed methodology based on SVD computation and information entropy estimation for BRB and FBR identification in real-time is used to classify induction motor condition online. Under experiment setup, different 1-hp three-phase induction motors (model WEG 00136APE48T) are used. Tested motors have 2 poles, 28 bars, and receive a power supply of 220 V AC, at 60 Hz. Applied mechanical load is that of an ordinary alternator. One phase of startup transient current is acquired using an AC current clamp (model Fluke i200s). A 16-bit serial-output analog-to-digital converter (ADC) (ADS7809)\(^{25}\) is used in data acquisition system (DAS). A sampling frequency \( f_0 = 1.5 \text{ kHz} \) is used to obtain 4096 samples during induction motor startup transient, acquiring up to the tenth harmonic of fundamental frequency, and beyond. Start of motor is controlled by a relay in order to synchronize the data acquisition with the motor switch on. Sampled data are arranged into a 32×128 matrix. Obtained matrix is processed by SVD computation algorithm implemented in FPGA device along with information entropy unit, which provides induction motor condition based on the information contents from obtained singular values.
Case Study 1: Broken Rotor Bars (BRB)

BRB condition is produced artificially by drilling one hole (diam 7.938 mm) without harming rotor shaft (Fig. 3a).

Case Study 2: Faulty Bearing (FBR)

FBR is produced artificially by drilling a hole (diam 1.191 mm) on its outer race using a tungsten drill bit (Fig. 3b).

Hardware Implementation

Proposed methodology was implemented in a low cost FPGA device Cyclone- II EP2C35F672C6 from Altera, embedded on DE2 development board from Terasic with maximum operational frequency of 65.928 MHz. FPGA implementation figures (used and available, respectively) in Altera device for various resources were as follows: MULT 18x18, 21 of 35; logic elements, 7632 of 33216; and RAM bits, 434,643 of 483,840.

Results and Discussion

Utilizing FPGA implementation of proposed methodology, startup transient current supply to a healthy motor (Fig. 3a), a motor with one BRB (Fig. 3b), and a motor with a FBR (Fig. 3c) is processed by computing SVD of generated matrix. Then, information entropy from obtained singular values is estimated. This process is applied on 20 different trials for each motor condition. Obtained entropy values on each condition are sent to a PC for presentation purposes. Mean $\mu(H)$ and standard deviation $\sigma(H)$ are computed. A $3\sigma$ rule that ensures a 99.7% of certainty on detecting motor condition is used (Fig. 4). Detectability zones defined by proposed
Table 1—Statistical study on obtained current signal and singular values from generated 32×128 matrix

<table>
<thead>
<tr>
<th>Statistical parameter</th>
<th>Motor condition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>HLT</td>
</tr>
<tr>
<td>Current signal</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0233</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>10.7055</td>
</tr>
<tr>
<td>RMS</td>
<td>10.7042</td>
</tr>
<tr>
<td>Generated-matrix</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>22.4587</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>120.1740</td>
</tr>
<tr>
<td>RMS</td>
<td>121.7812</td>
</tr>
<tr>
<td>Entropy</td>
<td>1.6276</td>
</tr>
</tbody>
</table>

Table 2—Comparison chart of advantages and disadvantages of proposed methodology against earlier studies for induction motor fault detection

<table>
<thead>
<tr>
<th>Method (ref)</th>
<th>Detected faults</th>
<th>Applied analyses</th>
<th>Implementation</th>
<th>On/Off-line</th>
</tr>
</thead>
<tbody>
<tr>
<td>[7]</td>
<td>Faulty bearings, 3 broken bars</td>
<td>Noise statistics, bias analysis, FFT, statistical reliability test, signature identification</td>
<td>Qualitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>[8]</td>
<td>Shaft defect, faulty bearings</td>
<td>Analysis of a laser beam of reflection through installation a robust optical sensor Infrared thermography, 2D</td>
<td>Qualitative analysis by voltage signal observation</td>
<td>Off-line</td>
</tr>
<tr>
<td>[12]</td>
<td>1 Broken bar</td>
<td>FFT, wavelet transform, peak measurement, and multivariate control charts</td>
<td>Qualitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>[14]</td>
<td>Voltage supply, broken bars, and locked rotor</td>
<td>Motor model, wavelet multi resolution, artificial neural networks</td>
<td>Qualitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>[15]</td>
<td>Faulty bearings</td>
<td>Wavelet transform, support vector machine, artificial neural networks, mean square error</td>
<td>Qualitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>[16]</td>
<td>Short circuit, 5 broken bars, and faulty bearings</td>
<td>Wavelet packet, mean, variance, standard deviation, skewness, kurtosis</td>
<td>Qualitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>[19]</td>
<td>1 broken bar</td>
<td>Hilbert transform, symbol tree sliding window, information entropy</td>
<td>Quantitative analysis in software</td>
<td>Off-line</td>
</tr>
<tr>
<td>Proposed</td>
<td>1 broken bar, faulty bearings</td>
<td>SVD, information entropy</td>
<td>Quantitative analysis in Hardware (FPGA)</td>
<td>On-line</td>
</tr>
</tbody>
</table>

methodology are shown as density functions. From results (Table 1) from distinct statistical analyses on obtained startup-transient current data and singular values of generated 32×128 matrix for each treated condition, it is
clear that proposed methodology ensures 99.7% of effectiveness on BRB or FBR detection in a quantitative way, since corresponding detectability zones are clearly and completely separated. From comparison chart (Table 2), it is clear that proposed methodology offers a simple approach optimized for hardware implementation through SVD and information entropy estimation, different from earlier studies that usually require combination of standard, and non-standard techniques executed off-line for single or multiple fault detection. FPGA-based hardware implementation of proposed technique is used for online analysis in real-time applications. It employs 24.3152 ms to identify induction motor condition, whereas a 2.4GHz Intel Core Duo elapses 680.6819 ms to carry out this task.

Conclusions
This study introduced a novel methodology for early detection of BRB and FBR in induction motors. Proposed approach fuses singular value decomposition with information entropy. Results show high effectiveness of proposed technique utilizing a $3\sigma$ rule, which ensures 99.7% of certainty on estimating induction motor condition in a quantitative way. FPGA-based hardware implementation allows online analysis utilizing proposed methodology in real-time applications.

References