Characteristics of pion production in ring-like events using entropy index

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The possible chaotic behavior in production of ring like structured events and jet like structured events in $^{16}\text{O}$-AgBr interactions at 60 AGeV with the help of a newly defined parameter, entropy index $\mu'$, has been studied. The data reveal less chaoticity in the pionisation process of ring like events with respect to that of jet like events.

Keywords: Nucleus-nucleus interactions, Fluctuation phenomena, Erraticity, Entropy index

1 Introduction

In every high energy collisions, many particles are produced and distributed in the available phase space volume in various ways. Some well known physical phenomena like correlation, intermittency, etc. may be considered as the manifestation of the fact that the production of particles are dominated by large fluctuations arising out of dynamical reasons\textsuperscript{1,2}. The genuine multiparticle correlations originating from some collective effects may lead to some high multiplicity events with some special features e.g. ring like events, jets and other regular patterns\textsuperscript{3-5}. Ring-like structures are occurrences where many pions are produced in narrow regions along the rapidity axis, which are at the same time diluted over whole azimuth. On the other hand, the jet-like structures consist of cases where particles are focused in both dimensions\textsuperscript{6}. The origin of ring-like events is still unknown. Several theories have come up in this regard\textsuperscript{4,7}.

In the present paper, the data on pions obtained from $^{16}\text{O}$-AgBr interactions at 60 AGeV for ring like event in order to shed light on the possible chaotic behavior in production of ring like structured events, have been analyzed. Similar analysis on jet like structured events has also been performed.

2 Experimental Details

The data used in the present investigation have been obtained from sets of Ilford G5 emulsion plates exposed to $^{16}\text{O}$ beam with energy\textsuperscript{b} 60 AGeV at CERN. Here we are dealing with nuclear emulsion detector which is itself the target for any high energy projectile beam.

The details of scanning and measurement were reported in our earlier publications\textsuperscript{8,9}.

Nuclear emulsion covers $4\pi$ geometry and provides very good accuracy in the measurements of angles of produced particles and fragments due to high spatial resolution and thus, is suitable as a detector to study the fluctuations in the fine resolution of the phase space considered. For the present investigation, 250 events of $^{16}\text{O}$-AgBr interactions at 60 AGeV were chosen.

3 Methodology for Separation of Ring like and Jet like Events

The ring like and jet like events can be separated using a method adopted by Adamovich\textsuperscript{3} where we start with a fixed number $n_d$ of particles (shower tracks). Each $n_d$-tuple of particles put consecutively along the $\eta$-axis is then characterized by:

(a) size $\Delta \eta_e = \eta_{\text{max}} - \eta_{\text{min}}$

where $\eta_{\text{min}}$ and $\eta_{\text{max}}$ are the pseudo-rapidity values of the first and last particles in the subgroup:

(b) A density $\rho_e = n_e / \Delta \eta_e$

Thus, each subgroup of particles, dense or dilute, has the same multiplicity and hence can be easily compared with each other. The azimuthal structure of a particular subgroup can now be parametrized in terms of the following quantities:

$$S_1 = -\sum \ln(\Delta \Phi_i) \quad \ldots (1)$$
and \( S_2 = \sum (\Delta \Phi_i)^2 \) \hspace{1cm} \ldots (2)

where \( \Delta \Phi_i \) is the azimuthal difference between two neighbouring particles in the group. For the sake of simplicity, we can count \( \Delta \Phi \) in units of full revolutions and thus we have:

\[ \sum (\Delta \Phi_i) = 1 \]

Both these parameters will be large (\( S_1 \to \infty, S_2 \to 1 \)) for jet like structures and small (\( S_1 \to n_d \ln n_d, S_2 \to 1/n_d \)) for ring like structures.

\( S_2 \) distribution is used to separate ring-like and jet-like events. From the \( S_2/\langle S_2 \rangle \) distribution of the data separation of ring-like and jet-like events is done taking into account that for ring like events \( S_2/\langle S_2 \rangle \) is less than 1. Whereas, for jet like events \( S_2/\langle S_2 \rangle \) is greater than 1.

In our earlier work we separated ring like and jet like events with the help of the parameter \( S_2 \), the details of which is given in our earlier publications\(^9\).

### 4 Method of Erraticity Analysis

The single particle density distribution in pseudo rapidity space is non flat. As the shape of this distribution influences the scaling behaviour of the factorial moments, we have used the ‘cumulative’ variable\(^{10}\) \( X(\eta) \) instead of \( \eta \). The cumulative variable \( X(\eta) \) is given by:

\[ X(\eta) = \int_{\eta_1}^{\eta} \rho(\eta') d\eta' / \int_{\eta_1}^{\eta_2} \rho(\eta') d\eta' \]

where \( \eta_1 \) and \( \eta_2 \) are two extreme points in the distribution \( \rho(\eta) \). The corresponding region of investigation for \( X(\eta) \) then becomes \((0, 1)\).

If we consider a two dimensional space such as pseudo rapidity \( \eta \) space as the horizontal axis, which is divided into \( M \) bins and the vertical axis has \( N \) sites corresponding to the \( N \) events in the event space. For each event, the factorial moments are calculated according to the following formula:

\[ f_q^e (M) = \frac{1}{M} \sum_{i=1}^{N} n_i (n_i - 1) \ldots \ldots (n_i - q + 1) \]

where \( n_i \) is the number of particles in the \( i^{th} \) bin for \( e^{th} \) event. The normalized factorial moments for the \( e^{th} \) event are then defined as:

\[ F_q^e (M) = f_q^e (M) / [f_1^e (M)]^q \] \hspace{1cm} \ldots (3)

Since \( F_q^e (M) \) fluctuates from event to event, one obtains a distribution \( P(F_q^e) \) for the whole event sample. Let the average of \( F_q^e (M) \) determined from \( P(F_q^e) \) be denoted by \( <F_q^e (M) > \) and \( \Phi_q (M) = F_q^e (M) / <F_q^e (M) > \)

In order to quantify the degree of that fluctuation, a new normalized moment\(^{11}\) is defined as:

\[ C_{p,q}(M) = <\Phi_q^p (M) > \] \hspace{1cm} \ldots (4)

where \( p \) is a positive real number. If \( C_{p,q}(M) \) has a power law behaviour as the division number \( M \) goes to infinity:

\[ C_{p,q}(M) \propto M^{\Psi_q(p)}, M \to \infty \] \hspace{1cm} \ldots (5)

where \( \Psi_q(p) \) is referred to as the erraticity exponent.

Since \( C_{p,q}(M) \) are the moments of \( P(F_q^e) \), they describe the deviation of \( F_q^e \) from its average value, and thus the erratic fluctuations of \( F_q^e \) from event to event are captured by \( C_{p,q}(M) \). Again these fluctuations depend on the bin size because \( F_q^e \) itself is a description of the spatial pattern that varies according to resolution. Thus, if these fluctuations scale with bin size, then the erraticity exponent \( \Psi_q(p) \) is an economical way of characterizing an aspect of the self similar dynamics that has some order of its erratic fluctuations\(^{12}\):

If the spatial pattern never changes from event to event, \( P(F_q^e) \) would be a \( \delta \) function and hence \( \Phi_q^p (M) \) and \( C_{p,q}(M) \) would be 1 at all \( M, p \) and \( q \) resulting \( \Psi_q(p) = 0 \). The larger \( \Psi_q(p) \) is, the more erratic is the fluctuation of the spatial patterns.

One can extract the entropy index\(^{11}\) \( \mu_q \) as:

\[ \mu_q = \left. \frac{d}{dp} \Psi_q(p) \right|_{p=1} \] \hspace{1cm} \ldots (6)

It is referred to as entropy index and it describes the width of the fluctuation. The \( \mu_q \) is related to the entropy\(^{11}\) in event space as \( S_q = \ln (NM^{-\mu_q}) \) where one can think of the event space as a one-dimensional space with \( N \) sites. To emphasize that \( S_q \) is defined in event space; one may call\(^{11}\) it “eventropy”. At each site, we can register a number \( F_q^e \). If \( F_q^e \) is the same at
each site, then \( P(F_q^\epsilon) = \frac{1}{N} \) and \( S_q = \ln N \). One should think of this as being highly dispersed in the event space, since \( F_q^\epsilon \) is spread out uniformly over all space. The larger the number of events, the larger is the “eventropy”. However, a branching dynamic that results in the same \( F_q^\epsilon \) for every event does not fluctuate in the branching processes. It corresponds to nearby trajectories staying nearby throughout. In short, the dynamics is not chaotic. In other words, \( S_q = \ln N \) implies that \( \mu_q \) must vanish. Thus, small \( \mu_q \) corresponds to large “eventropy”, which in turn implies no chaotic behaviour. On the other hand, if all \( F_q^\epsilon = 0 \) except only one event \( \epsilon' \), then \( P(F_q^\epsilon) = \delta_{\epsilon\epsilon'} \) and \( S_q = 0 \). This is highly ordered in the event space, but the fluctuation of \( F_q^\epsilon \) from zero to a non-zero value is large. Thus, if the distribution \( P(F_q^\epsilon) \) is broad, the fluctuation is large, initially nearby trajectories become widely separated in the final states of different events, and the dynamics is chaotic. In order for the “eventropy” to be small \( \mu_q \) must be large. Thus large entropy index implies chaotic behaviour.

If the power law behaviour of Eq. (5) is not satisfied by the experimental data and since the general behaviour of \( C_{p,q}(M) \) is rather similar in shape, one can regard \( C_{2,2}(M) \) as the reference\(^{13} \) that carries the typical dependence on \( M \) and examine \( C_{p,q}(M) \) versus \( C_{2,2}(M) \) when \( M \) is varied as an implicit variable. If \( C_{p,q}(M) \) follows the scaling behaviour with \( C_{2,2}(M) \) as:

\[
C_{p,q}(M) \propto g(M)^{\Psi'_{\epsilon}(p)} \quad \text{... (9)}
\]

where \( \ln g(M) = (\ln M)^{c} \) and \( \Psi'_{\epsilon}(p) \) is the newly defined erraticity exponent. If Eq. (9) is approximately valid for a common \( g(M) \) for all \( p \) and \( q \), it follows from Eq. (7) that:

\[
\chi(p,q) = \Psi'_{\epsilon}(p) / \Psi'_{\epsilon}(2) \quad \text{... (10)}
\]

and the newly defined entropy index:

\[
\mu'_q = \left. \frac{d}{dp} \Psi'_{\epsilon}(p) \right|_{p=1} \quad \text{... (11)}
\]

Using Eqs. (8) and (10), one can have:

\[
\mu'_q = \Psi'_{\epsilon}(2) \chi'_q \quad \text{... (12)}
\]

These values of \( \mu'_q \) are distinctly different from \( \mu_q \) and should not be compared to one another unless \( g(M) = M \).

5 Results and Discussion

The method of erraticity analysis described above is applied to nuclear emulsion data of \(^{16}\text{O}-\text{AgBr}\) interactions for ring like and jet like events at 60 AGeV. We divide \( X(\eta) \) region into \( M \) bins where \( M \) is varied from 1 to 10 in steps of 1.

We have calculated the moment of factorial moment \( C_{p,q}(M) \) for both ring like and jet like events to probe the event to event fluctuation of \( F_q^\epsilon(M) \) using Eq. (4). Here \( p \) is the order for event to event fluctuation. For each \( q \) values (2 and 3), we have calculated the values of \( C_{p,q}(M) \) for \( p = 2, 3 \) and 4. To check whether \( C_{p,q}(M) \) follows scaling behaviour with \( M \), \( \ln C_{p,q}(M) \) is plotted against \( \ln M \) for both ring like [Fig. 1(a)] and jet like [Fig. 1(b)] events.

These plots are not linear, so the power law behaviour in Eq. (5) is not well satisfied. As all \( C_{p,q}(M) \) follow similar trend, we consider \( C_{2,2}(M) \) as the reference and plot \( \ln C_{p,q}(M) \) versus \( \ln C_{2,2}(M) \) for ring like [Fig. 2(a)] and jet like [Fig. 2(b)] events for \( p = 2, 3, 4 \) for each \( q \). Now it is evident that the power law behaviour of Eq. (7) is well satisfied. The linear best fits of \( \ln C_{p,q}(M) \) versus \( \ln C_{2,2}(M) \) are performed. The slopes \( \chi(p,q) \) of the linear fits against \( p \) values (2, 3, and 4) for different \( q \) values (2, 3) are plotted in Fig. 3(a) for ring like events and in Fig. 3(b) for jet
like events. It can be seen from Fig. 3 (a and b) that \( \chi(p,q) \) is an increasing function of \( p \) with varying slope for both ring like and jet like events. We have to determine the slopes of these plots at \( p=1 \). To do it we first calculate \( \ln C_{p,q}(M) \) for \( p=0.9 \) and \( p=1.1 \) for each \( q \) value (2 and 3) for both ring like and jet like events. We then plot \( \ln C_{p,q}(M) \) versus \( \ln C_{2,2}(M) \) for both ring like and jet like events in Fig. 4(a and b), respectively. These plots show good linear behaviour. The values of \( \chi(p,q) \) for \( p=0.9 \) and \( p=1.1 \) for both the \( q \) values are listed in Tables 2 and 1, respectively for ring like and jet like events. Using Eq. (8) we have calculated \( \chi'_{q} \), which is listed in Table 1 for jet like events and in Table 2 for ring like events.

Since \( C_{p,q}(M) \) does not show power law behaviour with \( M \), as it is also evident from Fig. 1(a and b) we will search for a more general form given in Eq. (9). To obtain a good linear behaviour of \( \ln C_{2,2}(M) \)
versus $\ln \, g(M)$ plot we vary $a$. The values of $a$ corresponding to the best fitted plots of $\ln C_{2,2}(M)$ versus $\ln \, g(M)$ is presented in Table 3 and shown in Fig. 5 for both ring-like and jet-like events. The slopes of the plots give $\Psi'_{2,2}(2)$ values, which are also included in Table 3. Using Eqs (10-12), we calculate the event-to-event fluctuation of factorial moments by calculating the newly defined entropy index $\mu'_q$ and
Table 1 — Values of $\chi'_q$ and $\mu'_q$ for jet like events for $p=0.9$ and $1.1$ and $q=2, 3$ of $^{16}$O-AgBr interactions at 60AGev

<table>
<thead>
<tr>
<th>Data</th>
<th>q</th>
<th>p</th>
<th>$\chi(p,q)$</th>
<th>$\chi'_q$</th>
<th>$\mu'_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jet like</td>
<td>2</td>
<td>0.9</td>
<td>-0.0039±0.0004</td>
<td>0.440±0.004</td>
<td>0.057±0.004</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td></td>
<td>0.0490±0.0004</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.9</td>
<td>-0.18±0.001</td>
<td>2.005±0.015</td>
<td>0.259±0.018</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td></td>
<td>0.220±0.002</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 5 — Plots of $\ln C_{p,q}(M)$ versus $\ln(M)$ for ring like and jet like events

Fig. 6 — Dependence of $\mu'_q$ on $q$ for ring like and jet like events

present in Table 2 for ring-like events and in Table 1 for jet-like events. The variation of $\mu'_q$ against $q$ for both ring-like and jet-like events is shown in Fig. 6.

Table 2 — Values of $\chi'_q$ and $\mu'_q$ for ring like events for $p=0.9$ and $1.1$ and $q=2, 3$ of $^{16}$O-AgBr interactions at 60AGev

<table>
<thead>
<tr>
<th>Data</th>
<th>q</th>
<th>p</th>
<th>$\chi(p,q)$</th>
<th>$\chi'_q$</th>
<th>$\mu'_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring-like Events</td>
<td>2</td>
<td>0.9</td>
<td>-0.0320±0.0009</td>
<td>0.370±0.009</td>
<td>0.032±0.002</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td></td>
<td>0.0420±0.0009</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.9</td>
<td>-0.207±0.002</td>
<td>2.370±0.025</td>
<td>0.204±0.012</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td></td>
<td>0.267±0.003</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 — Fit parameters corresponding to the plots of fig 5 for ring like and jet like events

<table>
<thead>
<tr>
<th>Data</th>
<th>$\alpha$</th>
<th>$\Psi'_d(2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring like events</td>
<td>1.87 ± 0.06</td>
<td>0.086 ± 0.004</td>
</tr>
<tr>
<td>Jet like events</td>
<td>1.54 ± 0.08</td>
<td>0.129 ± 0.008</td>
</tr>
</tbody>
</table>

6 Conclusions

It is evident from Fig. 6 that the values of newly defined entropy indices, $\mu'_q$, increase with $q$ for both ring like and jet like events which gives the indication that chaoticity increases with the order of spatial fluctuation, $q$. This implies that event to event fluctuations become more erratic with the increase of $q$ for both ring like and jet like events. The values of $\mu'_q$ is more in case of jet like events than that of ring like events. Thus, this suggests a very interesting observation that the pionisation process is less chaotic in ring like events than in jet like events.

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References

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