FEATURE ARTICLE

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Nature loves symmetry. Whenever deviation from symmetry occurs there happens to be a certain imbalance. The appearance of this imbalance is what allows us to study nature, making symmetry an indispensable part of every field of study.

Beauty of Symmetry

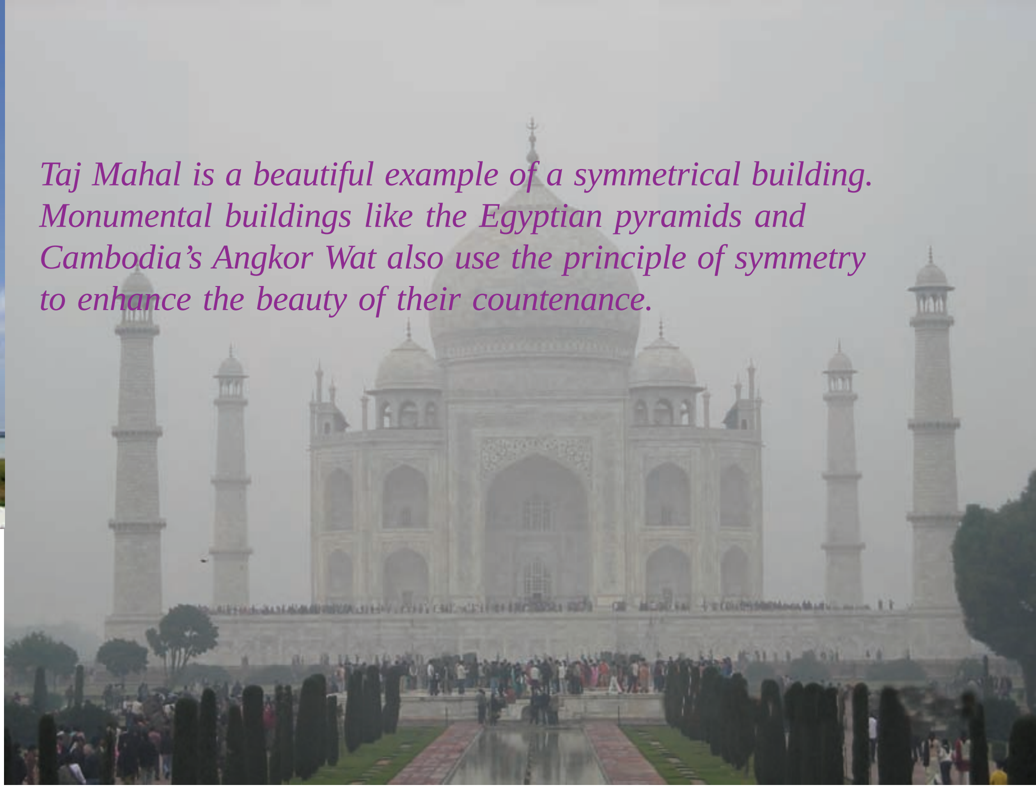
*Everything has beauty, but not everyone sees it. – **Confucius***

SYMMETRY is omnipresent in nature. When humans began to observe nature they began to see various symmetries that were being followed for a long time without deviation. Day and night follow each other without fail. It would certainly be awkward if one of our ears functioned differently from the other or is placed somewhere else on the face. Plants and animals have symmetrical bodies and shapes. When we remove the shell of a walnut, we see the wonderful symmetry of walnut halves. Nature loves symmetry and our notions of beauty are closely tied to symmetry.

The term symmetry derives from the Greek word 'SUM' (meaning 'with' or 'together') and 'METRON' (meaning 'measure'). When we look at nature, consciously or unconsciously our attention is attracted to objects that exhibit symmetry like a symmetric or a beautiful flower, a symmetrical tree or any beautiful shape. And so, humans have tried to apply the principle of symmetry to design and build balanced and harmonious structures, and in science, to study unknown systems from the known ones.

The salience of symmetry in design can be easily seen in its marvelous use in a structure, a symbol or a building. Some religious symbols have certain symmetry like the Judaic Star of David (which has six-fold rotational symmetry), Christianity's cross (which has bilateral symmetry) and the Hindu Swastika (which has four-fold point symmetry). Taj Mahal is a beautiful example of a symmetrical building. Monumental buildings like the Egyptian pyramids and Cambodia's Angkor Wat also use the principle of symmetry to enhance the beauty of their countenance.

Symmetry is also a guiding principle in scientific research. Scientists in different fields understand symmetry differently but ultimate the salient goal in many kinds of research is to arrange things with a particular similarity in a group. In the language of mathematics, a group is a set of elements that satisfy certain properties like associativity, identity and



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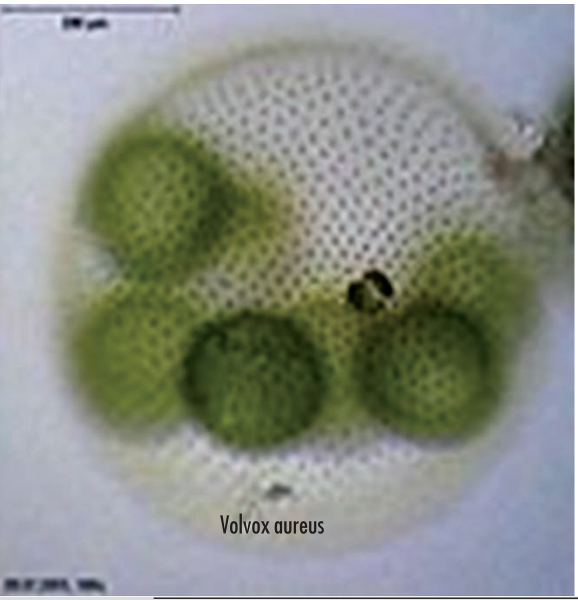
invertibility – properties that are closely related to symmetry. Specific mathematical operations such as reflections, rotations and translations are used to describe the symmetric interchange made within a system. One finds a relationship between symmetry and equivalence, between the elements of the group – symmetry leads to certain

Leaf with bilateral symmetry



Volvox aureus

relations that are satisfied by various elements that form a class.

Further, group theory is related to representation theory. Representation makes an abstract object concrete by describing its elements by matrices and the algebraic operations in terms of matrix addition and matrix multiplication making it easier for us to understand the problem leading to its solution. It is a prevalent strategy in science to simplify a problem by writing dynamical equations keeping in mind specific symmetries in the system. A representation of group helps reduce many theoretical problems in the physical sciences to problems in linear algebra, which are reasonably well understood.

We consider examples of symmetry in different scientific disciplines.

Symmetry in Biology

In biology, bilateral symmetry, spherical symmetry and radial symmetry is often talked about. In bilateral symmetry or plane symmetry, the organism is symmetric with respect to the plane passing through the centre of the body, called the sagittal plane, which divides it into right and left parts. The right part seems to be the mirror image of the left part (reflection symmetry). Most animals including humans are bilaterally symmetric, and have an identical shape on either side, as if cut by a mirror.

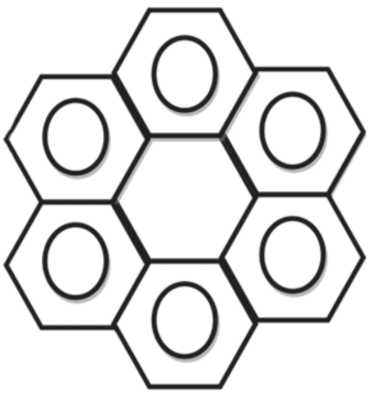
In spherical symmetry, the organism or body is symmetric with respect to any plane drawn through the body dividing it into exactly two equal halves. In nature, colonial algae called volvox exhibits this symmetry. In radial symmetry the body is cut into two similar halves by any vertical plane passing through the centre of the body. Some special forms of radially symmetric animals are sea anemone, jelly fish and sea stars.

Symmetry in Mathematics

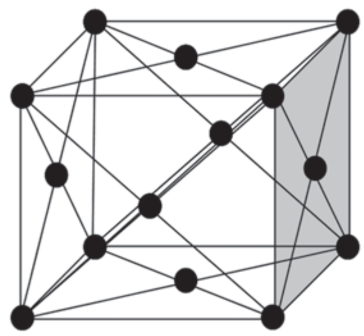
In mathematics, symmetry principles or invariance under certain operations form the basis of matrix representation theories and also help us to find a solution to some partial differential equations. Applied mathematicians address complex natural phenomena with mathematical tools; so there is often a blending of symmetrical principles from mathematical operations and the natural phenomena it is trying to address.

Symmetric and numerical solutions for systems of nonlinear reaction-diffusion equations find applications in biology and engineering in the study of transport of oxygen from blood plasma to living tissue of the skeletal muscle or brain across capillary walls. In these cases we need to solve certain diffusion reaction equations.

Sea anemone



Coronene molecule of D_{6h} Point group.



A unit face-centered cubic crystal

Symmetry principles have been applied at the micro scale also. The symmetry properties of carbon nanotubes (CNTs) are essential for interpretation of their Raman and IR spectra. Character tables for the symmetry groups satisfied by the CNTs are available in literature.

Such linear and nonlinear equations need mathematical models that are being developed using Lie-symmetries of partial differential equations and classical symmetries of systems of nonlinear diffusion equations.

Symmetry in Chemistry

In chemistry, molecular symmetry describes symmetry in molecules. Group theory is an important and useful framework for the study of symmetry in molecular orbitals. The s, p and d orbitals exist in certain symmetric shapes. Elements like helium, neon and xenon and beryllium, magnesium and calcium are very beautifully decorated in the periodic table, which is a result of grouping them with respect to common properties. The molecules are studied by placing them in various point groups on the basis of their structure. Molecules of water and pyridine are kept in C_{2v} point group, CO_3^{2-} anion is placed in D_{3h} point group whereas benzene molecule refers to D_{6h} point group. The characters of point group operations are used in the study of ligand field theory.

Many theories using various kinds of symmetries have helped to explain different chemical processes. Substitution reactions shift the molecule from a higher to a lower symmetry; this is what is called symmetry-breaking. In case of a complex molecule, where it becomes tedious to know about the number of isomers it possesses, the use of symmetry comes to our rescue. Therefore, the chemistry of symmetry lies in the molecular shape, which decides its behavior in chemical reactions.

We now have a complete set of 32 crystallographic point groups published in 1936 by Rosenthal and Murphy. Normal modes of vibration are decided on the basis of the symmetry it satisfies. E. Bright Wilson has used them to predict the symmetry of vibrational normal modes. Symmetry restrictions help chemists in understanding chemical bonding and molecular dynamics like analyzing vibrations of molecules, studying molecular orbitals and their wave functions.

For a perfectly ordered three dimensional crystallographic structure, the unit cell contains atoms arranged in characteristic configuration such that its continuous repetition throughout the crystal space results in generation of the entire crystal structure. Common unit cells are simple cubic, face-centered cubic and body-centered cubic.

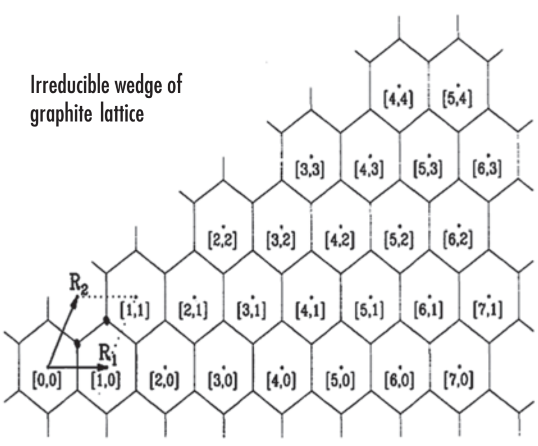
Molecules of considerable interest are fullerenes, which are a class of large molecular carbon chain substances. Fullerene is considered to be the third allotrope of carbon apart from diamond and graphite. The fullerene molecule (C_{60}) has a truncated icosahedral 'soccer-ball' structure which is spherically symmetric with a total of 12 pentagonal rings and 20 hexagonal rings.

Symmetry in Physics

Symmetry in physics has been generalized to mean invariance – lack of any visible change under any kind of transformation. The concept of symmetry then can be applied to abstract things or objects or dynamical equations of physical importance. Then one could predict that a particular system is symmetric with respect to laws rather than to objects or phenomena, or one could say that the system has internal symmetry.

In classical mechanics, the first study applying this notion of symmetry was made by Hamilton and Jacobi who used the method of applying transformations that leave Hamiltonian or Lagrangian unchanged leading to the solution of equations of motion. Using such transformations, it was possible to reduce difficult problems into simpler ones that could be solved using simple mathematics. Poincare brackets and canonical transformations were a result of the above procedure.

In relativistic dynamics, the assumption of symmetry between rest frame and moving frame of motion led to the prediction of principle of relativity which was given by Galileo (Galilean transformations). It made possible to derive laws of nature and test their validity using symmetry properties. Another turning point was Einstein's special theory of relativity that was based on two fundamental postulates: the light postulate and principle of relativity. Einstein for the first time used transformations that depended on space



Interestingly, this implies that the word 'symmetric' should not be taken as an absolute truth – there is hardly anything that possesses absolute symmetry.

and time referred to as local symmetries. Noether in 1918 derived a connection between global symmetries and conservation laws. Since then, many problems in physics derive solutions to local symmetries by taking global symmetry as a subgroup of some local symmetry.

In quantum theory, symmetry principles were found to be most effective. Wigner and Weyl were the first to apply symmetry of groups to quantum physics. Quantum states are related to each other through different transformations that implicitly conserve the symmetry of the system. The transformations are operations relating the states corresponding to operators, which in turn, are related to physical observables or quantities. Eigenvalue spectra can represent an irreducible representation of the symmetry group and the symmetry of Hamiltonian then would correspond to particular conserved quantity.

Permutation symmetry is a discrete symmetry that says that a state is symmetric if we swap indistinguishable quantum particles, which led to quantum statistics of bosons and fermions. It is related to the indistinguishable nature of particles that could be applied to Bose-Einstein statistics and Fermi-Dirac statistics. The famous CPT theorem postulated by Luders in 1952 states that C (charge conjugation), P (parity) and T (time reversal) in combined manner are a general symmetry of physical laws. In other words, the laws governing gravitational forces, electromagnetic forces and strong forces are invariant with respect to charge conjugation, parity and time reversal independently. Weak interactions violate parity (symmetry of reflection) as seen in the case of beta decay. However, beta decay respects a combination of C and P symmetry.

A new concept of 'symmetry breaking' is popular in particle physics nowadays. Passion for symmetry led to the Nobel Prize in Physics for 2008 being awarded to Yoichiro Nambu of Enrico Fermi Institute at University of Chicago for discovery of the mechanism of spontaneous broken symmetry in subatomic physics. Symmetry breaking does not imply that all of the symmetry of the system is lost. Rather that the system shifts to a lower symmetry subgroup related to original symmetry group through some transformation relation. The symmetry-breaking concept has helped us to theoretically explain the existence of mass-less bosons. According to Higgs, when internal symmetry is promoted to a local symmetry, the mass-less bosons disappear and the gauge bosons acquire mass. This phenomenon was said to be responsible for giving masses to gauge vector bosons like W and Z.

It is also believed that an unexplained broken symmetry of some kind lies behind the very origin of the cosmos in the Big Bang some 14 billion years ago. If equal amounts of matter and antimatter were created, they ought to have annihilated each other. But this did not happen; there was a tiny deviation of one extra particle of matter for every 10 billion antimatter particles. It is this broken symmetry that seems to have caused our cosmos to survive. However, what exactly happened still remains unanswered. May be some new experiments would explore an answer to this in the future.

Symmetry principles have been applied at the micro scale also. The symmetry properties of carbon nanotubes (CNTs) are essential for interpretation of their Raman and IR spectra. Character tables for the symmetry groups satisfied by the CNTs are available in literature. Helical and rotational symmetries are used to define different types of graphite tubules constructed by rolling up a single graphite sheet. Using such symmetries graphite tubules have been predicted to be moderate bandgap semiconductors.

Symmetry: The Essence of Nature or a Useful Lie for Science?

Nature appears to love symmetry! Whenever deviation from symmetry occurs, there also occurs a certain imbalance. As long as the system is symmetric or invariant, no change is observed in its behaviour or properties. When the system tries to interact with the surrounding environment or some forces

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are applied to it or its environment is changed due to some reason, a loss of symmetry occurs. This loss of symmetry changes the system's properties, which enables the system to be studied.

Interestingly, this implies that the word 'symmetric' should not be taken as an absolute truth – there is hardly anything that possesses absolute symmetry. A wide range of biological systems possess approximate symmetries. We assume the organism to be symmetric in its ideal form and then exploit the system to study the mechanics and dynamics by making slight changes in the ideal form. In this way, we are able to draw conclusions of what might happen if the symmetry is close to the ideal case and not exact. We have already seen from the CTP theorem and symmetry breaking examples how new insights have been derived even if the initial assumption of symmetry is not entirely correct.

Symmetry truly reveals the beauty of the natural world or scientific representation. The famous Victorian designer Owen Jones said, *"That which is beautiful is true; that which is true is beautiful."* We can add *"and that which is true and beautiful is symmetric!"*

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