Experimental measurement of coherent to incoherent cross-section ratio of elements in the range $6 \leq Z \leq 82$ for 59.54 keV gamma photons

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The coherent (Rayleigh) to incoherent (Compton) scattering cross-section ratio of elements in the range $6 \leq Z \leq 82$ for 59.54 keV gamma photons, have been measured. An HPGe gamma detector, placed at 90° to the incident beam, detects gamma photons scattered from the target under investigation. The intensity ratio of Rayleigh to Compton scattered peaks observed in the recorded spectra, corrected for photo-peak efficiency of gamma detector and absorption of photons in the target and air, along with the other required parameters provides differential cross-sections ratio for Rayleigh to Compton scattering. The measured values of cross-section ratio are found to agree with theory for low $Z$ elements, but deviate from theory for high $Z$ elements.

Keywords: Rayleigh scattering, Compton scattering, Scattered spectra, Cross-section ratio

1 Introduction
Rayleigh (coherent) scattering results in scattering of gamma photon without any change in energy, and is predominant at low incident photon energies, small scattering angles and high atomic number. The atomic Compton (incoherent) scattering results in degradation of gamma photon energy, and can be calculated with the incoherent scattering function modification to Klein-Nishina formula for Compton scattering. Rayleigh to Compton scattering ratio, $R$, has a power relation to $Z$ in the region of elemental interest and this power dependence is based upon the ratio $F^2/S$, with $F(q, Z)$ being the form factor and $S(q, Z)$ is the incoherent scattering function. Hubbell et al.¹ have provided theoretical model for the calculation of Rayleigh to Compton cross-section ratio from the parameters $F(q, Z)$ and $S(q, Z)$.

Various theories have been developed to calculate atomic form factor based on non-relativistic form factor¹, relativistic form factor² modified relativistic form factor³ and S-matrix theory⁴. According to these theories, the atomic form factor, $F(q, Z)$, is the Fourier transform of atomic charge distribution and can be evaluated by different wave functions. Icelli and Erzeneglu⁵ have measured the ratio of differential cross-sections for coherent and Compton scattering of 59.54 keV at scattering angles of 55° and 115° for Fe, Ni, Cu, Zn, Zr, Nb, Mo, Ag, Sn, Ta, Au and Pb targets using Ge(Li) detector. They found that the measured differential cross-section ratio increases with increasing atomic number and are in agreement with non-relativistic and relativistic form factor theories.

The knowledge of coherent to incoherent scattering cross-section ratio is useful in the calculations of radiation attenuation, reactor shielding, industrial applications and also in the field of medical sciences in a number of ways. In the present experiment, the Rayleigh to Compton scattering cross-section ratio values are measured for elements in the range, $6 \leq Z \leq 82$, for 59.54 keV incident gamma photons. The measured results are compared with various existing theories available in literature.

2 Experimental Details
The principle of present measurements is to observe the intensities of Rayleigh and Compton scattered gamma photons at a particular scattering angle. One of the major advantages of this method is that by taking the intensity ratio of Rayleigh to Compton scattered photons, a number of parameters such as absolute source strength, solid angles subtended by source and detector at the target are eliminated in the expression of ratio technique, otherwise these parameters introduce large amount of error in the final measured results.

The present measurements are performed using $^{241}$Am (59.54 keV) radioactive source of activity 0.74 GBq, which is placed at the end of a cylindrical cavity of depth 20 mm and diameter 11 mm fabricated in a lead cube having each side 160 mm (Fig. 1). An
aluminium hollow sleeve of internal diameter 8 mm, fitted in the centre of a rectangular block of lead having dimensions 80 mm × 80 mm × 15 mm, is placed coaxially adjacent to the cavity to obtain a narrow beam of photons. The distance of thin target under study from the source collimator (hole-size 4 mm in diameter) is kept as 100 mm. The source-target assembly is aligned in such a way that the incident photon beam is confined to the target element. The intense collimated beam of gamma photons from the radioactive source is made to impinge on a given target, which is placed symmetrically with respect to incident and finally scattered directions. An HPGe solid-state gamma detector of dimensions 56.4 mm diameter and 29.5 mm length placed at scattering angle of 90° detects radiations scattered from the target. The field of view of HPGe detector is confined to target only. A cylindrical collimator (hole-size 2 mm in diameter and thickness 20 mm) of aluminium is placed in front of gamma detector. The axes of source collimator, gamma detector and detector collimator pass through centre of the target. The distance between target under study and detector collimator (hole-size 2 mm in diameter) is kept as 100 mm so the angular spread about median ray in direction of gamma detector is ±1.2°. The HPGe detector is properly shielded by a cylindrical lead shielding having inner side covered with 2 mm thick iron and 1 mm thick aluminium, with iron facing lead to absorb K X-rays emitted by lead shielding. It has been checked that radiations scattered from source collimator opening do not reach directly to the active volume of HPGe detector. In the present measurements, Canberra HPGe detector and electronic modules (power supply and amplifier) are used. The experimental data are accumulated on a PC based ORTEC Mastreo-32 Multi channel analyser.

The formula used for the measurement of ratio of Rayleigh to Compton scattering cross-sections is given as:

\[
\frac{d\sigma_R(\theta)}{d\sigma_C(\theta)} = \frac{N_R}{N_C} \frac{\beta_C}{\beta_R} \frac{\gamma_C}{\gamma_R} \frac{\varepsilon_C}{\varepsilon_R}
\]

where \(N_R/N_C\) is the ratio of the number of counts under Rayleigh and Compton scattered peaks, respectively; \(N_R/N_A\) is the ratio of number of electrons and atoms per unit volume in the target; \(\beta_C/\beta_R\) is the ratio of self-absorption correction factor in the target for Compton and Rayleigh scattered energies; \(\gamma_C/\gamma_R\) is the ratio of absorption of Compton scattered and Rayleigh scattered gamma rays in air, and \(\varepsilon_C/\varepsilon_R\) is the ratio of the photo-peak efficiency of the HPGe detector for Compton and Rayleigh scattered energies.

3 Results and Discussion

In the present experiment, the intensity ratio of Rayleigh to Compton scattered peaks originating from interactions of primary gamma flux with the element, for a fixed geometrical source-sample-detector arrangement (Fig. 1) and incident gamma photon energy, have been measured. In the first step, the properly shielded HPGe gamma ray detector is placed at the desired angular position (\(\theta = 90^\circ\)) relative to the primary incident gamma beam. The spectrometer is calibrated using standard calibration gamma-sources of known energy. Measurements are then carried out by placing thin targets of known atomic number (6 ≤ \(Z\) ≤ 82) and thickness (40-500 mg-cm\(^2\)) in the primary incident gamma beam. The following procedure is adopted for the present measurements.

The target-in scattered spectra are recorded for a period of 10 ks by placing each of the targets (elements) in the primary gamma beam. The background is recorded after removing the target out of the primary beam to permit registration of events due to cosmic rays and to any other process independent of target.

The measurements for different elements are performed in the above stated sequence to minimize effect of any possible drift in the system. The subtraction of events recorded under condition (ii), from those under condition (i) results in events originating from interaction of primary gamma rays in the given target. A typical scattered spectrum, corrected for background events, originating from tantalum target is shown in Fig. 2. The Rayleigh and Compton peaks are clearly visible at 59.54 and 53.7 keV energies and distinguished from each other in the observed spectrum. The spread in observed Compton peak is caused by finite angular aperture of the source and detector collimators and Doppler
broadening of Compton peak in addition to inherent energy resolution of the HPGe detector. It has also been observed that Compton peak dominates the Rayleigh peak for low Z targets, but for higher atomic number element like Ta (Fig. 2) the Rayleigh peak is dominant than Compton peak in the observed spectra. The intensities under Rayleigh and Compton scattered peaks (Fig. 2) are deduced from the recorded scattered spectra. These observed intensities are corrected for photo-peak efficiency of the HPGe detector, absorption in air present between target (element) and the detector, and self-absorption in elements and the detector, and self-absorption in the target, as per relation 2:

\[ N_{\text{actual}} = \frac{N_{\text{obs}}}{\epsilon \beta \gamma \beta_r} \]  

where \( N_{\text{obs}} \) is the observed intensity under Rayleigh (or Compton) peak; \( \beta_p \) is correction factor for absorption of photons in the air present between target and detector; \( \beta_r \) is self-absorption correction factor for scattered photons in the target; and \( \epsilon \) is photo-peak efficiency of gamma detector for Rayleigh (or Compton) scattered photons.

The measured values of differential cross-section ratio for Rayleigh to Compton scattering, are given in column 2 of Table 1. The statistical error involved in the cross-section measurements in the range from 6.8-12%. The columns 2, 3 and 4 of Table 1 presents the cross-section ratio deduced from theory for non-relativistic form factor 1, relativistic form factor 2 and modified relativistic form factor 3, respectively.

Figure 3 shows the plot of Rayleigh to Compton cross-section ratio as a function of Z-number of the target. It is observed that as the atomic number increases, the value of Rayleigh to Compton scattering cross-section ratio increases non-linearly and the trend is similar to that predicted by theoretical models 1-4. The measured differential cross-section values for Rayleigh to Compton ratio are compared with theoretical values calculated from various theories employing non-relativistic form factor 1, relativistic form factor 2 and modified relativistic form factor 3. It has been observed that the measured values agree well with theory for low and high Z elements, but deviate from theory for intermediate Z elements. Our experimental results are found to be in agreement with theoretical values for low Z elements like carbon, Table 1—Experimental and theoretical values of Rayleigh to Compton scattering cross-section ratio

<table>
<thead>
<tr>
<th>Atomic number (Z)</th>
<th>Present results</th>
<th>Theory 1</th>
<th>Theory 2</th>
<th>Theory 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.00115 ± 0.00002</td>
<td>0.00157</td>
<td>0.00159</td>
<td>0.00151</td>
</tr>
<tr>
<td>13</td>
<td>0.0251 ± 0.003</td>
<td>0.0410</td>
<td>0.0387</td>
<td>0.0372</td>
</tr>
<tr>
<td>30</td>
<td>0.110 ± 0.011</td>
<td>0.153</td>
<td>0.140</td>
<td>0.123</td>
</tr>
<tr>
<td>40</td>
<td>0.524 ± 0.095</td>
<td>0.348</td>
<td>0.352</td>
<td>0.290</td>
</tr>
<tr>
<td>73</td>
<td>0.737 ± 0.054</td>
<td>0.959</td>
<td>1.016</td>
<td>0.869</td>
</tr>
<tr>
<td>79</td>
<td>0.958 ± 0.071</td>
<td>1.152</td>
<td>1.260</td>
<td>1.067</td>
</tr>
<tr>
<td>82</td>
<td>1.397 ± 0.095</td>
<td>1.254</td>
<td>1.398</td>
<td>1.178</td>
</tr>
</tbody>
</table>

Fig. 2—Typical observed spectrum from Ta-target when irradiated by 59.54 keV incident photons for 10 ks duration

Fig. 3—Rayleigh to Compton scattering cross-section ratio as a function of Z-number
aluminium and are very close to the MRFF theoretical model for Ta and Au, but for Pb its value is higher. For high Z elements (except lead), the measured experimental values are lower than that deduced from theory.\textsuperscript{1-3}

The photo-peak efficiency values are also measured experimentally using single energy sources of known source strength of \textsuperscript{203}Hg (279 keV) and \textsuperscript{137}Cs (662 keV). The experimental measured values of photo-peak efficiency using single energy sources of \textsuperscript{203}Hg and \textsuperscript{137}Cs of known source strengths come to be 2.5\% and 1.12\%, respectively and are nearly in agreement with Canberra Germanium detector users manual values of 2.4\% and 1.10\% respectively. The targets used in the present experiments are thin, with $\mu t \ll 1$, here $\mu$ is the linear attenuation coefficient and $t$ is the thickness of the target under study. The contribution of events resulting from Bremsstrahlung originating from slowing down of photoelectrons and Compton recoil electrons is estimated on the basis of an experimental technique suggested by Sandhu \textit{et al.}\textsuperscript{6}, and is found to be less than 1\% under the present experimental conditions.

There are no data available in literature for comparison with the present results at scattering angle of 90°. The present measurements do not require absolute source strength of radioactive source and solid angles subtended by the source and detector at the target. At low incident photon energies, the probability for Rayleigh scattering is enhanced significantly in comparison to Compton scattering, so there is need to explore this technique at low energy incident photons (59.54 keV) for samples of various interests at different possible scattering angles.

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\textbf{References}