Application of the multisource thermal model in pseudorapidity distributions of charged particles produced in $p\bar{p}$ or $pp$ collisions over an energy range from 0.053 to 7 TeV

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The pseudorapidity distributions of charged particles produced in $p\bar{p}$ or $pp$ collisions over a center-of-mass energy range from 0.053 to 7 TeV have been studied by using the multisource thermal model. The difference between the rapidity and pseudorapidity for massive particles (pions) has been considered. The calculated results are found to be in agreement with the experimental data of the UA5, UA1, P238, CDF, CMS and ALICE Collaborations.

Keywords: Pseudorapidity distribution, Charged particle, $p\bar{p}$ or $pp$ collisions

1 Introduction

Pseudorapidity ($\eta$) or rapidity ($y$) distributions are one of the basic measurement quantities in high energy collisions. According to these distributions, one can study some important information such as the particle statistical behaviour, dynamic fluctuation, longitudinal scaling, interaction mechanism, and others. In the past decades, the UA5$^1$, UA1$^2$, P238$^3$, CDF$^4$, and other Collaborations measured the pseudorapidity distributions in inelastic (or non-single diffractive) $p\bar{p}$ or $pp$ collisions over a center-of-mass energy range from 0.053 to 1.8 TeV. Recently, the CMS$^5$, ALICE$^6$ and other Collaborations have measured successfully the pseudorapidity distributions of charged particles produced in non-single diffractive $pp$ collisions at the CERN Large Hadron Collider (LHC) with energy of 0.9, 2.36, and 7 TeV. According to Ref. (3), the single diffractive interaction corrections are very low (about 0.5%). It is not necessary to distinguish the inelastic and non-single diffractive processes in this paper.

As basic processes, $p\bar{p}$ and $pp$ collisions are very useful in understanding of proton-nucleus and nucleus-nucleus collisions. In a recent work$^7$, we obtained experimentally a group of linear dependence of parameters on $\ln \sqrt{s}$ in the description of pseudorapidity distributions in $p\bar{p}$ or $pp$ collisions at the non-LHC energies, where $\sqrt{s}$ denotes the center-of-mass energy and is given in units of GeV. These linear dependences were based on the multisource thermal model$^8$ and the condition of $\eta=y$ for massless particles. According to the linear dependences, the parameter values can be obtained at different $\ln \sqrt{s}$. Then, the pseudorapidity distributions at the LHC energies can be easily described$^9$.

In the case of neglecting the difference between $\eta$ and $y$, some misleading conclusions may be obtained. If we introduce a temperature parameter $T$ and consider the difference between $\eta$ and $y$ for massive particles (pions), an accurate result can be calculated from the multisource thermal model. It is necessary if we extend the work of Ref. (7) which has already done fits of experimental data with this model assuming massless particles to the situation assuming massive particles. In this paper, the description is extended and the fits to data are repeated assuming the pion mass. We analyze the pseudorapidity distributions of charged particles produced in $p\bar{p}$ or $pp$ collisions over an energy range from 0.053 to 7 TeV. This energy range covers the main energies available at the past and present colliders.

2 The Model

In the framework of the multisource thermal model$^{7,9}$, many sources are assumed to form in high energy collisions. In the rapidity space, in the laboratory reference frame, these sources distribute...
homogeneously in a target cylinder and a projectile cylinder, with rapidity \( y_c \) being in the ranges \([y^\min_c, y^\max_c]\) and \([y^\min, y^\max]\), respectively. Similar to Maxwell’s ideal gas model in thermodynamics, each source is assumed to emit particles isotropically in the source rest frame. Due to the normalization, the contributions \( (K_T \text{ and } K_P) \) of target and projectile cylinders to (pseudo) rapidity distribution satisfy \( K_T + K_P = 1 \).

The transverse momentum \( p'_T \) of a particle with mass \( m \) emitted from a rest source with temperature \( T \) is assumed to obey Boltzmann distribution:

\[
f(p'_T) = c p'_T \exp \left( -\frac{\sqrt{p'^2 + m^2}}{T} \right) \quad (1)
\]

where \( c \) is a normalization constant and \( T = (13.7 \pm 0.7) \ln^\sqrt{s} + (115 \pm 2.8) \) (MeV) as fitted in Ref. (11). The longitudinal momentum \( p'_\perp = p'_T / \tan \theta' \), energy \( E' = \sqrt{p'^2 + p'^2 + m^2} \), and rapidity \( y' = \frac{1}{2} \ln \frac{E' + p'_T}{E' - p'_T} \) of the considered particle can be given by the relation among momentum components, the relation between momentum and energy, and the definition of rapidity, respectively, where \( \theta' \) is the emission angle of the considered particle in the rest source and obeys the distribution of \( \frac{1}{2} \sin \theta' \). In the laboratory frame, the considered source has a rapidity of \( y_c \) and the considered particle has a rapidity of \( y = y_c + y' \). The transverse and longitudinal components of the considered particle momentum in the final state can be given by

\[
p_T = p'_T \quad \text{and} \quad p'_\perp = \frac{p_T}{\cos \theta' + E' \sin \theta'} \quad (2)
\]

respectively. Then, in the final state, the emission angle \( \theta = \arctan \frac{p_T}{p'_\perp} \) and pseudorapidity \( \eta = -\ln \tan \left( \frac{\theta}{2} \right) \) of the considered particle can be given by their definitions. Meanwhile, the rapidity \( y \) is obtained due to \( y = y_c + y' \) or by its definition \( y = \frac{1}{2} \ln \frac{E + p_T}{E - p_T} \), where \( E = \sqrt{p'^2 + p'^2 + m^2} \).

In \( p\bar{p} \) and \( pp \) collisions, the symmetry of target and projectile gives:

\[
y_{T\max} - y_{T\min} = y_{P\max} - y_{P\min} \quad \ldots (4)
\]

\[
y_{P\max} = -y_{T\min} \quad \ldots (5)
\]

\[
y_{P\min} = -y_{T\max} \quad \ldots (6)
\]

\[
K_P = K_T \quad \ldots (7)
\]

In the above discussion, a numerical calculation can be given by the Monte Carlo method. Because we have considered the massive particles and introduced the temperature, the leading target and projectile contributions are not necessary to be included. This indicates that our previous work is superfluous in consideration of leading nucleons due to the trouble of massless particles. On the other hand, the number of free parameters in the present work is 2, and that in the previous work is 4. The improvement of the model is obvious.

3 Comparison with Experimental Data

Figure 1 shows the pseudorapidity distributions of charged particles produced in \( p\bar{p} \) collisions over an energy range from 0.053 to 1.8 TeV. The solid circles represent the experimental data measured by the UA5\(^1\), UA1\(^2\), P238\(^1\), and CDF\(^2\) Collaborations, respectively, and the open circles are symmetrical reflection at the mid-pseudorapidity \((\eta=0)\). The solid and dotted curves are our results calculated in the present work and previous work\(^7\), respectively. The parameter values in the present work are obtained by fitting the experimental data and all of the charged particles are regarded as pions. The obtained values of free parameters \( y_{P\max}(=-y_{T\min}) \) and \( y_{P\min}(=-y_{T\max}) \), normalization constant \( N_c \), and corresponding \( \chi^2 \) per degree of freedom \( \chi^2/\text{dof} \) for the solid curves are presented in Table 1. One can see that the modified model describes the experimental data over an energy range from 0.053 to 1.8 TeV. The values of \( \chi^2/\text{dof} \) are rather low due to the fact that large experimental systematic and statistical uncertainties are included in the fit. These are usually correlated and we may treat as such in the fitting procedure. Although the solid
and dotted curves are similar in the available data range, the number of free parameters for the solid curves is 2, and that for the dotted curves is 4. In the fragmentation ranges, the difference between the two curves is obvious. We would rather believe that the modified model is better than the previous one due to inclusion of the mass term and less number of the free parameters.

In Fig. 1(e), there is a large difference in the central rapidity region between the two curves. Because the data in the central region is not available, the parameter values have space to be adjusted and to give a small difference.

To see a clear relation between the parameters and \( \sqrt{s} \), the correlations between \( y_{\text{max}} \) \( (=-y_{\text{min}}) \) and \( \ln \sqrt{s} \), \( y_{\text{min}} \) \( (=-y_{\text{max}}) \) and \( \ln \sqrt{s} \) as well as \( N_c \) and \( \ln \sqrt{s} \) are shown in Fig. 2. The symbols represent the parameter values obtained from Fig. 1 and the lines are our fitted results. One can see the linear dependence of the parameters on \( \ln \sqrt{s} \). These linear relations can be described by:

\[
\begin{align*}
y_{\text{max}} &= -y_{\text{min}} = (0.649 \pm 0.026) \ln \sqrt{s} - (0.366 \pm 0.159) \\
y_{\text{min}} &= -y_{\text{max}} = (0.212 \pm 0.022) \ln \sqrt{s} - (0.626 \pm 0.135) \\
N_c &= (8.498 \pm 0.409) \ln \sqrt{s} - (25.292 \pm 2.542)
\end{align*}
\]

with \( \chi^2/\text{dof} \) being 0.024, 0.202, and 0.055, respectively.

We notice that a linear relation of the parameter \( N_c \) with \( \ln \sqrt{s} \) is obtained. However, the CMS Collaboration\(^5\) shows that the charged particle pseudorapidity density in the central rapidity region is better described by a second-order polynomial of \( \ln \sqrt{s} \) which is significantly dominant at the LHC.
energies. Also the ALICE Collaboration reports that the similar density shows a power-law of \( \ln \sqrt{s} \). It seems that our results are inconsistent with those of the CMS and ALICE Collaborations. In fact, the present work concerns the normalization parameters, and Refs 5 and 6 report the pseudorapidity density in the central rapidity region. Although there is a relation between the normalization parameter and the pseudorapidity density in the central rapidity region, the two kinds of parameters are different. Our results in fact are not inconsistent with those of the CMS and ALICE Collaborations.

By using Eqs (8-10) and \( T = (13.7 \pm 0.7) \ln \sqrt{s} + (115 \pm 2.8) \) (MeV)\(^{11} \), we can get the parameter values at the LHC energies. Then, the experimental data at the LHC energies can be described. Figure 3 shows the pseudorapidity distributions of charged particles produced in \( pp \) collisions at 0.9, 2.36, and 7 TeV. The circles represent the experimental data of the CMS\(^5 \) and ALICE\(^6 \) Collaborations and the solid curves are our calculated results. Corresponding to the solid curves in Figs 3(a-d), the values of \( \chi^2/\text{dof} \) are 0.024, 0.042, 0.014, and 0.113, respectively. One can see that the model describes the experimental data at the LHC energies. Because the data at 0.9 and 2.36 TeV are below or close to the CDF results at 1.8 TeV which are already included in the fit, it is not so surprising that the model describes these data too. Instead, it proves that the LHC results are consistent with previous experiments. In Fig. 3, the dotted curves are another calculation which will be discussed later.

![Figure 3](image)

Fig. 3 — Pseudorapidity distributions of charged particles produced in NSD \( pp \) collisions at 0.9, 2.36 and 7 TeV.

Similar to the heavy ion experimental data quoted or reported in Refs (12-15), the most pseudorapidity distributions quoted in Fig. 1 show in almost full \( \eta \) range. Recently, by using a similar model and under the condition of \( \eta = y \), the dependence of charged particle pseudorapidity distributions on centrality and energy in proton and deuteron induced nuclear collisions at high energies, have been studied by us\(^{16} \).

It is indeed convenient for us to use this kind of data in almost full \( \eta \) range. However, the data used in Fig. 3 show in a narrow \( \eta \) range and are difficult to obtain the parameter values. In this case, we hope to use the experiential equations Eqs (8-10) to obtain the parameter values.

The analyzed spectra at the LHC energies are measured in a narrow range, in comparison to the full interval of pseudorapidity around the mid-pseudorapidity. We assume that the “target” and “projectile” sources are distributed equally and uniformly along rapidity within the corresponding rapidity intervals. Clearly this is only one possibility out of many which would also describe the data. Especially, for the asymmetric collisions, the equally and uniformly distributions of two kinds of sources may not be the true situation\(^{16} \). For symmetric collisions, the uniformly distribution is not the sole choice. As an example, by using a four-source model, two target sources with rapidities \( y_{T1} \) and \( y_{T2} \) and contributions \( K_{T1} \) and \( K_{T2} \) \( = 0.5 - K_{T1} \) to the pseudorapidity distribution and two projectile sources with symmetrical parameters \( y_{P1} \), \( y_{P2} \), \( K_{P1} \), and \( K_{P2} \) \( = 0.5 - K_{P1} \). the obtained pseudorapidity distributions are shown in Fig. 3 by the dotted curves. The corresponding values of parameters and \( \chi^2/\text{dof} \) are presented in Table 2. One can see that the four-source model describes the experimental data around the mid-pseudorapidity range. Because the data in Fig. 3 are in a narrow range, the values of parameters \( y_{P1} \) and \( y_{P2} \) are rather unbending. These renders that the fits with the four-source model in Table 2 show no linear dependence in \( y_{P1} \) and \( y_{P2} \).

We notice that the four-source model predicts consistently lower values of multiplicity at higher pseudorapidity compared to the uniform-source case. This difference is caused by the source distribution range in the rapidity space. In fact, the source distribution range \( \{ y_{T1}, y_{T2} \} \) in the four-source model is narrower than that \( \{ y_{\text{min}}, y_{\text{max}} \} \) in the uniform-
source model. If we use a wider \([y_{T1}, y_{p1}]\), the fitting quality in the central region is not acceptable. Because the availability of data is in a narrow kinematic range, the four-source model seems successful. However, we do not expect that the four-source model is successful for a wider kinematic range. Instead, it is expected that the uniform-source model is successful for a wider kinematic range and at higher energies.

Experimentally, rapidity spectra of pions have approximately Gaussian form at not too high energies, whereas the corresponding pseudorapidity spectra have a dip\(^{13-15}\) at \(\eta=0\). Generally, the mid pseudorapidity dip is due to the conversion from rapidity to pseudorapidity and a pure kinematical effect. At very high energy, it is expected that a mid rapidity dip to appear. Within the used model the mid rapidity dip seems to result in a mid rapidity gap between "target" and "projectile" cylinders. In fact, by using the two cylinders having a not too large mid rapidity gap, the obtained rapidity spectra also have approximately Gaussian form. As an extension of two fireballs\(^{17}\), the two cylinders can give the mid rapidity dip which is expected at very high energy.

In our calculation, the length of target or projectile cylinder is:

\[
 L_T = L_P = y_{\text{max}} - y_{\text{min}} = y_{\text{max}} - y_{\text{min}} \\
 = (0.437 \pm 0.034)\ln \sqrt{s} + (0.260 \pm 0.209) \quad \text{(11)}
\]

the length of the two cylinders is:

\[
 L_{PP} = y_{\text{max}} - y_{\text{min}} = (1.298 \pm 0.037)\ln \sqrt{s} - (0.732 \pm 0.225) \quad \text{(12)}
\]

and the gap width between the two cylinders is:

\[
 W = y_{\text{min}} - y_{\text{max}} = (0.424 \pm 0.031)\ln \sqrt{s} + (1.252 \pm 0.191) \quad \text{(13)}
\]

We see that the linear relations between the lengths and \(\ln \sqrt{s}\), as well as between the gap width and \(\ln \sqrt{s}\), are observed.

### 4 Discussion and Conclusions

According to the model in a hadron-hadron collision two tubes of sources are formed, which are associated with either the projectile, or the target. Each of the tubes covers a certain rapidity interval (\(L_P\) or \(L_T\)), and there is a rapidity gap (\(W\)) between them. Making a Lorentz boost to the rest frame of the target, one should also arrive at two tubes covering the same rapidity interval (\(L_P\) or \(L_T\)) with them and having the same rapidity gap (\(W\)) between them.

Comparing with the case of assuming massless particles, in the case of considering the difference between \(\eta\) and \(y\) for massive particles, we have to introduce the temperature parameter. The contributions of leading target and projectile particles to the (pseudo)rapidity distribution for massless particles are not any longer needed to consider. In addition, the number of parameters for massive particles is less than that for massless particles. Although we cannot obtain an analytical expression for \(\eta\) and \(y\) distributions in the present work, the Monte Carlo method is used to give a number of simulated data and the final state distribution is obtained by the statistical method. Because we give the parameter values in both the cases of massless and massive particles by fitting the data, both the comparisons with the data are good.

In the considered energy range, the temperature parameter\(^{11}\) stays about 170-230 MeV due to

\[
 T = (13.7 \pm 0.7)\ln \sqrt{s} + (115 \pm 2.8) \quad \text{(MeV)}
\]

The dependence of \(T\) on \(\ln \sqrt{s}\) is slight. This reflects the transparence of interacting system in the concerned energy range. It is not a good idea to obtain a high temperature in \(p\bar{p}\) or \(pp\) collisions at high \(\sqrt{s}\). Considering the stopping effect of target nuclei, we may study proton-nucleus collisions or nucleus-nucleus collisions at higher energy to obtain a higher temperature.

To conclude, the multisource thermal model can describe the pseudorapidity distributions of charged particles produced in \(p\bar{p}\) or \(pp\) collisions over an energy range from 0.053 to 7 TeV. In the

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**Table 2**—Parameter values corresponding to the dotted curves in Fig. 3

<table>
<thead>
<tr>
<th>Figure</th>
<th>Energy (TeV)</th>
<th>(y_{T1} - y_{T2})</th>
<th>(y_{p1} - y_{p2})</th>
<th>(K_P = -K_T)</th>
<th>(N_e)</th>
<th>(\chi^2/\text{dof})</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(a)</td>
<td>0.9</td>
<td>2.00\pm0.05</td>
<td>0.50\pm0.02</td>
<td>0.33\pm0.01</td>
<td>30.25\pm0.95</td>
<td>0.012</td>
</tr>
<tr>
<td>3(b)</td>
<td>2.36</td>
<td>2.30\pm0.05</td>
<td>0.50\pm0.02</td>
<td>0.32\pm0.01</td>
<td>42.20\pm1.00</td>
<td>0.019</td>
</tr>
<tr>
<td>3(c)</td>
<td>2.36</td>
<td>2.30\pm0.05</td>
<td>0.50\pm0.02</td>
<td>0.35\pm0.01</td>
<td>43.50\pm1.00</td>
<td>0.003</td>
</tr>
<tr>
<td>3(d)</td>
<td>7</td>
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<td>0.50\pm0.02</td>
<td>0.33\pm0.01</td>
<td>54.00\pm1.10</td>
<td>0.038</td>
</tr>
</tbody>
</table>
calculation, the initially fits are free fits per energy. In a subsequent step, the linear dependence of the parameters is concluded. The difference between the pseudorapidity and rapidity for massive particles (pions) is considered. A set of experimental equations of parameters is obtained from the distributions with almost full $\eta$ range. Then, the parameter values at the LHC energies can be obtained from the set of equations. High energy collisions are an important topic in modern physics. More experimental and theoretical works are needed in the future.

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