Maths Play

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Question 1. Given that X, Y and Z denote single digit integers in the range of 1 to 9, what are the values of X, Y and Z in the following equation:

XYZ + ZYX = YYZY.

Solution: [Note: The above question falls into a class known as Cryptorithms. There is a sub-class of such questions known as Alphametics. Alphametics are essentially Cryptorithms that make sense when read, for example, SEND + MORE = MONEY, because its letters form words which are used in day-to-day life.]

In questions like these it is assumed that (i) there is a one-to-one mapping i.e. the same letter always denotes the same digit and the same digit is denoted by the same letter, (ii) different letters denote different numerals and (iii) there are no leading zeroes.

The Left Hand Side of the equation suggests that both XYZ and ZYX are three-digit numbers and the Right Hand Side indicates that YYZY is a four digit number which implies that the first Y from the left, in YYZY, is a carry-over digit.

Now, if we add only two digits in the range of 0 to 9 the minimum carry can be zero and the maximum carry can be one. For instance 4 + 5 = 09 (the carry digit is 0) and 9 + 9 = 18 (the carry digit is 1).

Therefore the value of Y is 1.

The equation (XYZ + ZYX = YYZY) also suggests that Z + X = YY (looking at the first and last letters from the left); which implies that the following form is valid:

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   Y '! carry
  XYZ
+ ZYX
 YYZY
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Therefore, Z = Y + Y + Y = 3. This implies that 3 + X = 11, thus the value of X is 8.

Question 2. What are the values of A, B, and C in the following equation:

AB x 4 = CA.

Solution. What is evident from the question is: (i) AB and CA are necessarily two digit numbers, and

(ii) AB should be less than or equal to 24. Why? Because 25 x 4 = 100.

Which implies that the value of A is either 1 or 2 and the value of B can be any value from the set {0,1,2,3,4}.

Also, if A is 1, then B cannot be 1; and, if A is 2 then B cannot be 2 as each letter corresponds to a different digit.

(iii) CA is an even number, because CA is the product of AB and 4. Therefore A is necessarily an even digit, which implies that the value of A is 2.

(iv) In CA, the value of A is 2. This implies that B should be 3 or 8. Why? Because 3 x 4 = 12 and 8 x 4 = 32. To get 2 in the unit's place it is necessary to multiply 4 by a number which contains 3 or 8 in the unit's place.

(v) From the discussion above, clearly the value of B cannot be 8, hence the value of B is 3.

(vi) 23 x 4 = 92

(viii) Therefore, A = 2, B = 3 and C = 9.

Question 3. If ALFA + BETA + GAMA = DELTA, then what are the values of ALFA, BETA, GAMA and DELTA?

Solution:

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  ALFA
  BETA
 +GAMA
 DELTA
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From the question it is evident that A+A+A = CA where C is for the carry digit. If we add the same digit thrice and the unit's digit of the result is the same digit, then obviously the digit has to be 0 or 5; because 0+0+0=0 and 5+5+5=15.

Obviously, the value of A should be 5, because 5 + 5 + 5 = 15 (the last digit from the left in ALFA, BETA, GAMA and DELTA is A). One more possibility is that the value of A might be 0, because 0 + 0 + 0 = 0. But since there are no leading zeroes herein, obviously the value of A should be 5.

Substituting A = 5, the equation takes the following form:

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 5LFA
 BET5
+G5M5
 DELT5
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D can be either 1 or 2 because D indicates the carry digit. Assuming D = 1, we get the following form:

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  5LF5
  BET5
 +G5M5
 1ELT5
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Now, 1 + F + T + M = CT ...(1) where C represents the carry-digit obtained on adding 5+5+5.

C1 + L + E + 5 = L ...(2) C1 represents the carry-digit obtained on adding 1 + F + T + M

and C2 + 5 + B + G = 1E ...(3) C2 represents the carry digit obtained on adding C1 + L + E + 5.

From (1), it follows that 1 + F + M = 10, because on adding 10 + T, the unit's digit of the answer will be T.

Therefore F + M = 9

So F and M can have any one of the following values:

(9,0), (8,1), (7,2), (6,3), (5,4), (4,5), (3,6), (2,7), (1,8), (0,9).

But, neither F nor M can be 5 or 1 (as we know A=5 and D=1),

If we add only two digits in the range of 0 to 9 the minimum carry can be zero and the maximum carry can be one. For instance 4 + 5 = 09 (the carry digit is 0) and 9 + 9 = 18 (the carry digit is....

therefore the following combinations are ruled out: (8,1), (5,4), (4,5), (1,8).

That means F and M can take values from the following set:

(9,0), (7,2), (6,3), (3,6), (2,7), (0,9). [Let us call this Inference 11, for convenience]

From (2), it follows that C1 + E + 5 = 10. Assuming that the carry-digit, C1 is 1, then E = 4

Substituting A = 5, D=1, E=4 the equation takes the following form:

5LFA
B4T5
+G5M5
14LT5

Assuming the value of C2 as 1, it follows from (3)

1 + 5 + B + G = 14.

Or B + G = 8.

Therefore, B and G can have any one of the following values:

(8,0), (7,1), (6,2), (5,3), (4,4), (3,5), (2,6), (1,7), (0,8).

From the discussion above, it is clear that the possibilities (7,1), (5,3), (4,4), (3,5) and (1,7) can be safely ruled out because A=5, D=1, and E=4.

Which implies that the values of B and G can be drawn from the set (8,0), (6,2), (2,6), (0,8). [Let us call this Inference 12, for convenience]

From **Inference 11** and **Inference 12,** the values of F and M can be 9 and 0 (or 0 and 9); the values of B and G can be 6 and 2 (or 2 and 6), since each letter corresponds to a unique digit.

The following values are known:

A	B	D	E	F	G	L	M	T
5	6	1	4	9	2	?	0	?

Therefore, the equation takes the following form:

5L95
64T5
+2505
14LT5

Only two variables L and T and three values 3,7,8, are left.

Suppose L = 3 and T = 7, then,

5395
6475
+2505
14375

Alternatively, if L = 7 and T = 8, then,

5795
6485
+2505
14785

Thus, for this question multiple solutions are possible.

Question 4. Two engineers (say A and B) from a premier institution meet each other after eleven years. The following conversation takes place between them:

A: How are you?
B: Great! I am married and I have three daughters.
A: Wow! How old are they?
B: The product of their ages is 72 and the sum of their ages is the number written over there (Points to a number which only A and B can see)
A: Difficult to figure out the ages.
B: My eldest daughter celebrated her birthday yesterday
A: Well. My eldest daughter is also of the same age.

What are the ages of B's daughters?

Solution: From the question it is evident that the product of the ages is 72 and the sum is a number which we do not know. Let us determine the ages that yield a product of 72.

Product	Sum
1, 1, 72	74
1, 2, 36	39
1, 3, 24	28
1, 4, 18	23
1, 6, 12	19
1, 8, 9	18
2, 2, 18	22
2, 3, 12	17
2, 4, 9	15
2, 6, 6	14
3, 3, 8	14
3, 4, 6	13

We get a **sum** of 14 if the factors are 2,6,6 or 3,3,8. In all the other cases we get a distinct and different sum. Therefore, A gets confused. But when he is told that the eldest daughter celebrated her birthday, then he understands that the elder ones are not twins and hence rules out 2,6,6 and can make out that their ages are 3, 3, 8.

Question 5. A 5-digit number has the following feature:

If we put the numeral 1 at the beginning, we get a number three times smaller than if we put the numeral 1 at the end of the number. Which is this 5-digit number?

Solution: Let Y denote the five-digit number.

3 x (100000 + Y) = 10Y + 1. Adding 100000 puts a 1 at the beginning of the number. Multiplying by 10 and then adding 1 puts a 1 at the end of the number.

Therefore, 300000 + 3Y = 10Y + 1

implies that 7Y = 299999

which means Y = 42857.

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