PLACING Ramanujan along side John Keats may seem a bit paradoxical – the former belonging to mathematics and the latter to English literature. Moreover Ramanujan has never been marked having an interest in literature nor in English language. But, the juxtaposition is apt and appropriate in the sense that both the heroes of respective faculties are the best examples of the proverb “Men live in deeds and not in years”.

John Keats lived only for twenty-six years. In this short span of life granted to him, he dived deep into the ocean of literature and gave many precious verses that are peculiar and unique. He left an everlasting imprint on the body of English literature by devoting his life to the service of English literature. He lived through many odds in life. Miseries dominated his life. He died of Tuberculosis. He stands in no way less than the other (poets), some of whom were fortunate enough to spend even a century.

Likewise, Srinivasan Ramanujan lived only for thirty-three years. For him also, life was not a bed of roses – trials and tribulations became a part and parcel of his life. But he remained engrossed in his mathematical passion notching up several milestones in his short life span.

John Keats was in love with Fanny Browne who could not become his. Throughout his life the longing for his lady love did not proceed to fruition. Something of the same kind happened with Srinivasa who could not enjoy the company of his wife Janaki when he needed it so much and this perhaps is the sad reason why his health went on deteriorating both internally and externally leading to his final departure. Incidentally, both the geniuses fell victim to the same disease – tuberculosis.

The researches done and the formulas given by Srinivasa are of much significance without which mathematicians could not do in modern times. He was one of the greatest mathematicians of modern times. Ramanujan’s life was a lesson in triumph and tragedy. By his profound and amazingly original work he has carved for himself a permanent place in the history of mathematics. Although he had little formal education, Ramanujan has left a memorable imprint on mathematical thought. He had no formal training in mathematics to speak of and was truly an untutored genius, an uncut diamond but of uncommon brilliance.

Early Life of a Prodigy

Srinivasa Ayyangar Ramanujan was born at Erode in the Madras Presidency on 22 December 1887 to Komalathmmal and K. Srinivasa Iyengar. He was a mathematical prodigy. During his school days, he impressed his teachers, senior students and classmates with his intuition and astounding proficiency in several branches of mathematics – algebra, trigonometry, arithmetic and number theory.

In later years, a friend of his recounted the following incident. In an arithmetic class on division, the teacher was explaining that if three bananas were given to three boys, each boy would get a banana. The teacher generalised this idea. Ramanujan is said to have asked: “Sir, if no banana is distributed to no student, will every one still get a banana?”

Another friend who took private tuitions from Ramanujan also recalled that Ramanujan used to ask about the value of zero divided by zero and then answer that it can be anything since the zero of the denominator may be several times the zero of the numerator and vice-versa and that the value cannot be determined. To this intriguing question Bhaskara proved the answer to be infinity.

In fact, the uncommon brilliance of Srinivasan in the subject may be marked by putting only one example of his adolescent stage when he was asked by a senior school student to find out the unknown quantities of the equation $x + y = 7$ and $x + y = 11$ and Ramanujan solved it quickly and said that $x = 9$ and $y = 4$. At that time he was in the fourth year at school and such a question was expected to be tackled only by a sixth year student. This won for him a friend who in later years took him to the Collector of Nellore.
\[ x + y = 7 \]
Squaring both the sides, we get

\[ x = (7-y)^2 \]
\[ x + y = 11 \quad y = 11 - x \]
Again squaring both the sides, we have

\[ Y = (11-x)^2 \]

Using the value of \( x \) from (i) in (ii), we get

\[ y = 11 - (7 - y)^2 \]
\[ y = 121 + (7-y)^2 - 211 \cdot (7-y)^2 \]
\[ y = 121 + 2401 - 1372y + 294y^2 - 28y^3 + y^4 - 2 - 22y^2 + 308y \]
\[ y^4 - 28y^3 + 272y^2 - 1065y + 1444 = 0 \]
\[ y^4 - 28y^3 + 272y^2 - 1065y + 1444 = 0 \]
\[ (y - 4) (y^3 - 24y^2 + 176y - 361) = 0 \]
\[ y - 4 = 0 \quad y = 4 \]
\[ y + 4 = 11 \quad x = 9 \]
\[ x = 9 \quad y = 4 \]

Since \( x + y = 11 \)
\[ x = 9 \quad y = 4 \]

Thus \( x = 9 \) and \( y = 4 \) are the solutions of the given equations.

CASE–II
When the R.H.S. of (iii) is \( 4 \cdot x \cdot 1 \), then we have

\[ x - y = 4 \quad x + y - 1 = 1 \]
\[ i.e. \quad x - y = 4 \quad x + y = 2 \]
On solving, we get
\[ 2x = 6 \quad x = 3 \quad y = 9 \]
And \( 2y = -2 \quad y = -1 \quad y = 1 \)
Again these values of \( x \) and \( y \) do not satisfy the equation (i) and (ii).

CASE–IV
When the R.H.S of (iii) is \( -4 \cdot x \cdot -1 \), then we have

\[ x - y = -4 \quad x + y - 1 = -1 \]
\[ i.e. \quad x - y = -4 \quad x + y = 0 \]
On solving these pairs of equations, we get
\[ 2x = -4 \quad x = -2 \quad x = 4 \]
And \( 2y = 4 \quad y = 2 \quad y = 4 \)
Again these pairs of values do not satisfy the equation (i) and (ii).

CASE–V
When the R.H.S of (iii) is \( 1 \cdot x \cdot 4 \), we get

\[ x - y = 1 \quad x + y - 1 = 4 \]
\[ i.e. \quad x - y = 1 \quad x + y = 5 \]
On solving these two equations, we get
\[ 2x = 6 \quad x = 3 \quad x = 9 \]
And \( 2y = 4 \quad y = 2 \quad y = 4 \)
We find that these values of \( x \) and \( y \) satisfy the equation (i) and (ii).

CASE–VI
When the R.H.S of (iii) is \( -1 \cdot x \cdot -4 \), we have

\[ x - y = -1 \quad x + y - 1 = -4 \]
\[ i.e. \quad x - y = -1 \quad x + y = -3 \]
On solving these two equations, we get
\[ 2x = -4 \quad x = -2 \quad x = 4 \]
And \( 2y = -2 \quad y = -1 \quad y = 1 \)
Again these values of \( x \) and \( y \) do not satisfy the equation (i) and (ii).

In this way we observe that the only solution of equation (i) and (ii) can be obtained by CASE–V as \( x = 3 \) i.e. \( x = 9 \) and \( y = 2 \) i.e. \( y = 4 \).

The senior mathematics teacher of the school Ganapathy Subbier had such confidence in Ramanujan’s ability that he even entrusted him the task of preparing for the school a conflict-free timetable.

In his fourth year at school, Ramanujan mastered Loney’s Trigonometry, part II. In 1903, through his friends from the Kumbakonam Government College Ramanujan obtained G.S. Carr’s Synopsis, a book on pure mathematics, containing propositions, formulae and methods of analysis with abridged demonstrations. It contained about 6000 formulae without proofs.
Ramanujan used to ask about the value of zero divided by zero and then answer that it can be anything since the zero of the denominator may be several times the zero of the numerator and vice-versa in geometry, algebra, trigonometry and Calculus. Through the new world thus opened to him, Ramanujan went bounding with delight. It was this book that awakened his genius. He set himself to establish the formulae given therein.

In 1904, from Town High School, Madras, Ramanujan passed his Matriculation Examination in the first class. This gained him the Subramanian Scholarship. He thus entered the Government College in Kumbakonam. He had to study English, Sanskrit, Mathematics, Physiology and History.

The urge to pursue mathematics became irresistible in him. Topics such as magic squares, continued fractions, prime numbers, elliptic integrals and hyper-geometric series engaged his attention. His excessive devotion to mathematics resulted in his neglect of other subjects, and he failed in the F.A. examination in 1905 and was advised to continue his studies in some other college.

Subsequently, Ramanujan joined the F.A. class at Pachaliappa's College, Madras in 1906. J.A. Yates, the Principal of the college at that time, recognized his abilities and gave him a scholarship. Ramanujan continued his mathematical activities with vigour. He could hardly study a few months at the college as his health was getting affected. When he fell ill he discontinued his studies and returned to Kumbakonam. Urged by his father he appeared as a private candidate for the F.A. examination in 1907 but failed. This marked the end of his formal education.

Years of Adversity

The years between 18 and 25 are the most critical years in a mathematician's career. During his five unfortunate years (1907-1912) Ramanujan's genius was misdirected, sidetracked and to a certain extent distorted, says Prof. G.H. Hardy. Despite the great financial difficulties he had, inspired by Carl’s Synopsis, Ramanujan started noting down his own results in Notebooks.

A college mate of Ramanujan has stated that during the college years, Ramanujan taught him the method of constructing Magic Squares, the subject of the first chapter of his first and second notebooks, which dates from his school days.

During his five unfortunate years (1907-1912) Ramanujan’s genius was misdirected, sidetracked and to a certain extent distorted, says Prof. G.H. Hardy.

Ramanujan’s investigation in continued fraction and divergent series started during this period. He had to do all this by discovering them de novo.

He recorded his results in his notebooks. Proofs were often absent. The profundity of contents of these notebooks as they are being analysed today reveal more and more staggering complexities. Intuition played a large part in these researches. There are three such notebooks in all containing 212, 352, and 33 pages respectively. Exact facsimiles of these notebooks have now, since 1957, been published in two volumes by the cooperative effort of the University of Madras, The Tata Institute of Fundamental Research and Sir Dorabji Trust.

It was during this period at the age of 22 that Ramanujan was married (betrothal) to Janaki, then 9 years old in 1909. The marriage took place near Karur. With the new responsibilities he was constantly in search of a benefactor and a job to earn enough to sustain his needs. He tutored a few students in mathematics first in Kumbakonam and later he sought employment as a tutor in mathematics to eke out a livelihood in Madras.

He had heard about Prof. V. Ramaswamy Iyer, the founder of the Indian Mathematical Society. Ramanujan approached him for help. Ramaswamy Iyer looked at some of the notebooks of Ramanujan. He was wonderstruck by the intricate theorems and the formulas contained in them. Assessing his plight, he gave a letter of recommendation to Prof. P.V. Seshu Iyer who got Ramanujan a temporary job of a clerk at the Accountant General’s Office at Madras. He worked there for a few days but was not satisfied with the job. He came to know about a vacancy of a clerk in Madras Port Trust. He made a formal application and wrote to Prof. Seshu Iyer as well as to Ramachandra Rao, Collector of Nellore for their help. Ultimately he got the job as a clerk in the Madras Port Trust on a salary of Rs. 30/month.

Turning Point

Ramanujan’s entry into the Port Trust in 1912 may well be considered as the turning point in his career. It was the beginning of the appreciation of his scholarship and researches in Mathematics. Ramanujan, disappointed at the lack of recognition, had bemoaned to a friend that he was probably destined to die in poverty like Galileo. This was not to be.

The life of Ramanujan, in the words of C.P. Snow, “is an admirable story, and one which showers credit on nearly everyone.” Sir Francis Spring, the chairman of the Port Trust, encouraged Ramanujan in his mathematical pursuits. Sir Francis Spring drew the attention of Dr. Gilbert T. Walker, F.R.S. to Ramanujan’s Notebooks.

Dr. Walker, a good mathematician and a fellow of the Trinity College, Cambridge immediately recognized the quality of Ramanujan’s work. He wrote to Mr. Francis Dewsbur, the Registrar of Madras University, commending the work of Ramanujan. On the recommendation of Dr. Walker, Ramanujan was granted a special research scholarship of Rs. 75/- per month for two years with the express consent of the Governor of Madras at the University of Madras. As the first research scholar of Madras University he began his research career as a professional mathematician from May 1913.

Dr. Walker also wanted the University to correspond with Prof. G.H. Hardy of Cambridge.
By this time Ramanujan had started publishing his work. His first article on Bernoulli numbers was published in the Journal of Indian Mathematical Society in 1911 followed by two more articles in 1912. These had a good impact. Prof. C.L.T. Griffith of the Engineering College, Madras was impressed by these articles. He told Sir Francis Spring that Ramanujan was an extraordinary mathematician and that he should send some of his works to leading mathematicians in London.

By now Ramanujan had access to foreign journals from the library of the Indian Mathematical Society at Madras. In one of these journals Prof. Hardy had raised some queries that came to the attention of Ramanujan. Ramanujan had already made some contributions in the area related to the queries raised by Prof. Hardy. It was therefore easy for him to answer these queries. He wrote his answers in the form of letters to Prof. Hardy.

In 1913, Ramanujan wrote to Hardy as follows (S. Ramanujan, Collected Papers, 2nd Ed, Chelsea, New York, 1962, p. xiii):

Dear Sir,

I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust office at Madras on a salary of only 820 per annum. I am now about 23 years of age. I have had no university education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as startling....

I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced, I would very highly value any advice you give me. Requesting to be excused for the troubles I give you.

I remain, Dear Sir, Yours truly,

S. Ramanujan

With his letter Ramanujan included about 120 theorems/formulas.

One morning in January 1913, Prof. Hardy found among his letters on the breakfast table, a large untidy envelope decorated with Indian stamps. When he opened it he found that the letter was a little out of the ordinary. It consisted of some theorems very strangely looking without any proofs. Hardy thought for a moment that the writer of the letter might be a fraud. He duly went about the day according to his daily routine. But there was something nagging him at the back of his mind. Anyone who could fake such theorems, right or wrong, must be a fraud of genius. He went that evening after dinner to argue it with his friend Prof. J.E. Littlewood. They worked together and it did not take them long to come to the conclusion that the writer of the letter was a man of genius. Hardy says, “A single look at them is enough to show that they could only be written down by a mathematician of the highest class. They must be true, if they were not true, no one would have had the imagination to invent them. ... The writer must be completely honest, because great mathematicians are commoner than thieves or humbugs of such incredible skill.”

Hardy immediately decided that Ramanujan must be brought to England. He entrusted Prof. Neville with the responsibility of bringing Ramanujan to London. With the help of Prof. Neville, Ramanujan finally sailed to England from Madras on 17 March 1914. On 14 April 1914, he reached London through the channel. Ramanujan spent four very fruitful years at Cambridge, fruitful certainly to him, but more so to the world of mathematics. Hardy records that the time he spent with Ramanujan from 1914 to 1918 was one of the “most decisive events” of his life – Hardy’s life.

Ramanujan was awarded the B.A. degree by research in 1916. Ramanujan published 27 papers, seven of them jointly with Hardy. In 1918 he was elected Fellow of the Royal Society and in the same year was also elected Fellow of Trinity College, both honours coming as the first to any Indian. The University of Madras rose to the occasion and made a permanent provision for Ramanujan by granting him an unconditional allowance of $250 a year for five years from April 1919, the date of expiry of the overseas scholarship that he was then drawing. At the same time a post of Professor was created by Madras University for Ramanujan, but alas, fate decided otherwise.

**Sharp Memory**

According to Littlewood, Ramanujan treated numbers (every positive integer) as his personal friends. There is an interesting anecdote in this connection. Around the middle of 1917, Ramanujan’s health declined and he was admitted to a nursing home in Cambridge. Once Prof. Hardy visited him in the hospital. Finding Ramanujan depressed, he thought of cheering him up and remarked that the number of the taxi by which he came was 1729 and that it seemed to him to be a dull number, and that he was afraid it was an unfortunate omen. Ramanujan replied immediately: “No, it is a very interesting number as it is the smallest number that can be expressed as the sum of two cubes in two different ways as $1^3 + 12^3 = 1729 = 9^3 + 10^3$.

Then, Hardy asked him, whether he could tell the solution of the corresponding problem for four powers and he replied, after a moment’s thought, that he knew no obvious example but felt that the first such number must be very large. In fact this observation of Ramanujan was true, since the simplest known solution of $x^4 + y^4 = z^4 + t^4$ is Euler’s $(59^4 + 158^4 = (133^4 + 134^4) = 635318657$.

The anecdote reveals Ramanujan’s remarkable feeling for numbers and his sharp memory which made him recall one entry out of several thousands he had made in his notebooks and the fact that he had not recorded in his notebook the observation he made about 1729 which came only when Prof. Hardy made an innocuous statement.

Hardy refers to a formula that Ramanujan was fond of: “There is one particularly interesting formula, viz.

$$x^{k+1}\phi(0) - x\phi(1) + ax\phi(2) - \ldots = \frac{x\phi(0)}{\sin\alpha}s$$

of which he was especially fond and made continual use. This is really an ‘interpolation formula’ that enables us to say, for example, that under certain conditions, a function that vanishes for all integral values of its arguments must vanish identically. I have never seen this formula stated explicitly by anyone else, though it is closely connected with the work of Mellin and others.”
The Rogers–Ramanujan Identities

In additive number theory, mathematicians study ways of writing integers as sums. The sums are called partitions. A partition of an integer \( N \) is a finite sequence \( a, b, c, d, e, \ldots, r \) of positive integers, called ‘parts’ of the partition such that

\[
a + b + c + d + \ldots + r = N.
\]

For example, the seven partitions of 5 are

\[
5 = 1 + 1 + 1 + 1 + 1
= 2 + 1 + 1 + 1
= 2 + 2 + 1
= 3 + 1 + 1
= 3 + 2
= 4 + 1
= 5
\]

Of these partitions, three have distinct parts: 5, 4+1, 3+2, and three have only odd parts 5, 3+1+1, 1+1+1+1+1. This phenomenon is considered in a two hundred year old theorem of Euler. Another example, 4, 3, 3, 2 is a partition of 12. We write the partition as 4332 without even the commas separating the integers. 522111 is another partition of 12. We note that we always write a partition in such a way that as we read it, the parts do not increase. How many partitions are there of a given integer \( n \)? The answer is \( p(n) \) in standard terminology.

- \( P(1) = 1 \)
- \( P(2) = 2, \text{ for } 2 \text{ and } 11 \text{ are the partitions of } 2. \)
- \( P(3) = 3, \text{ for } 3, 21 \text{ and } 111 \text{ are the partitions of } 3. \)
- \( P(4) = 5, \text{ for } 4, 31, 22, 211 \text{ and } 1111 \text{ are the partitions of } 4. \)
- And so on, \( P(200) \approx 397299029388. \) Thus \( p(n) \) becomes very large very rapidly.

Very little is known about the arithmetical properties of \( p(n) \). Even questions like whether \( p(n) \) is odd or even, for a given \( n \), is difficult to answer. Ramanujan was the earliest mathematician to enquire into such properties. Ramanujan observed properties like the following. Whatever integer \( n \) might be, \( p(5n+4) \) is divisible by 5, \( p(7n+5) \) is divisible by 7 and similar ones. In connection with these properties, Ramanujan proved a number of identities, one of which is

\[
p(4)+p(9)x+p(14)x^2+ \cdots = \frac{5((1-x^5)(1-x^{10})\cdots)}{(1-x)(1-x^2)(1-x^3)\cdots 3^a}
\]

This result has been considered to be representative of the best of Ramanujan’s work by Hardy. Hardy says: “If I had to select one formula for all Ramanujan’s work, I would agree with Major MacMahon in selecting the above.”

In 1918, Hardy and Ramanujan published a joint paper in the Proceedings of the London Mathematical Society on an exact formula for \( p(n) \), Euler’s Theorem. For any positive integer \( n \), the number of partitions of \( n \) with distinct parts equals the number of partitions of \( n \) with odd parts.

Basic properties of arithmetic can be used to prove Euler’s theorem. The theorem is proved by showing explicitly how to transform all the partitions with odd parts into the partitions with distinct parts. The procedure is as follows:

1. Collect like terms
2. Write the coefficients or sums of distinct powers of 2 (The powers of 2 are the numbers 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, \ldots.). Thus \( 4x7 + 3x3 + 7x1 \) becomes \( (4)x7 + (2+1)x3 + (4+2+1)x1 \)
3. Carry out the indicated multiplications.

Thus \( (4)x7 + (2+1)x3 + (4+2+1)x1 = 28 + 6 + 3 + 4 + 2 + 1 \),
Which is a partition of 44 with distinct parts.

Let us now apply our three rules to the partition of 5 with odd parts:

\[
5 \quad 1 \times 5 \quad = 5
3 + 1 + 1 \quad 1 \times 3 + 2 \times 1 \quad = 1 \times 3 + (2)x1 \quad = 3 + 2
1 + 1 + 1 + 1 + 1 \quad 5 \times 1 \quad = (4+1)x1 \quad = 4 + 1
\]

Lo and behold, our rules have, as was suggested, yielded all the partitions of 5 with distinct parts. One may use the fundamental properties of arithmetic to establish that indeed our rules always provide a proof of Euler’s theorem.

For almost 150 years no other theorem of this type was found. Ramanujan and L.J. Rogers independently found a theorem that is much deeper than Euler’s though it looks almost the same. Odd numbers may be characterized as integers whose last digit is 1, 3, 5, 7 or 9. Let us call integers strange numbers if their last digit is 1, 4, 6, or 9.

Janaki– Ramanujan’s Wife
The Last Days

Ramanujan's achievements were all about elegance, depth and surprise beautifully intertwined.

Unfortunately, Ramanujan contracted a fatal illness in England in 1918. He convalesced there for more than a year and returned to India in 1919. His condition then worsened, and he died on 26 April 1920.

One might expect that a dying man would stop working and await his fate. However, Ramanujan spent his last year producing some of his most profound mathematics. A moving description of this time is given by Ramanujan’s wife:

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He returned from England only to die, as the saying goes. He lived for less than a year. Throughout this period, I lived with him without break. He was only skin and bones. He often complained of severe pain. In spite of it he was always busy doing his mathematics. That evidently helped him to forget the pain. I used to gather the sheets of paper that he filled up. I would also give the slate whenever he asked for it. He was uniformly kind to me. In his conversation he was full of wit and humor.

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Even while mortally ill, he used to crack jokes. One day he confided in me that he might not live beyond thirty-five and asked me to meet the event with courage and fortitude.

He was well looked after by his friends. He often used to repeat his gratitude to all those who had helped him in his life.

Ramanujan was born 124 years ago. His life was tragically short. However, his mathematical discoveries are still alive and flourishing. "Ramanujan is important not just as a mathematician but because of what he tells us that the human mind can do," said Prof. Askey. "Some one with his ability is so rare and so precious that we can't afford to lose them. A genius can arise anywhere in the world. It is our good fortune that he was one of us. It is unfortunate that too little of Ramanajan's life and work, esoteric though the latter is, seem to be known to most of us."

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Ramanujan's notebooks

Srinivasan Ramanujan lived only for thirty-three years. For him also, life was not a bed of roses — but he remained engrossed in his mathematical passion noting up several milestones in his short life span.

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