

## An upper bound analysis and determination of the barreling profile in upsetting

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Received 25 April 2011; accepted 28 November 2011

The aim of this work is to investigate the barreling of aluminium cylinders having two different aspect ratios in terms of barrel profiles and required loads. A modified upper bound solution including a dimensionless optimizing parameter for uniaxial upsetting has been developed and the loads calculated thereby have been compared with the results of several experiments conducted. Furthermore, the barrel profile is firstly assumed as a second-order polynomial for the upsetting process and an empirical correlation between the billet dimensions and the barrel profile has been established using experimental data. AA6082 is used as test material. The experiments are carried out on a 150 metric ton capacity hydraulic press. Besides, the bulging profile and effective strain distribution of the billets are obtained with a commercially FEM program called DEFORM 3D.

**Keywords:** Barreling profile, Upper bound, Upsetting, FEM

All types of metal forming processes consist of metal flow. The main factors affecting the flow are friction, flow stress of the metal, the geometry of the tools and the workpieces. The change in geometry of the free surface of the deformed part is an indication of the metal flow. So, valuable information about metal flow is provided by the change in geometry of the free surface. In a metal forming process, the final shape of the product, the mechanical properties related to deformation, and the deformation of defects such as cracks or folds at the surface or at the center of the product are determined considering the flow of the metal. The axial upsetting of cylinders is widely used in forging processes. It is well known that when a solid cylinder is compressed axially between two flat-faced parallel platens, the friction between the cylinder and the platens at their surfaces of contact causes heterogeneous deformation, which in turn produces barreling of the cylinder as shown in Fig.1. Friction has an important role in determining the life of the tool, the formability of the work material and the quality of the finished product. Friction causes to decrease in the homogeneity of deformation, leading to defects in the finished products, e.g., surface finish, internal structure. So, a number of studies have already been made in an attempt to obtain quantitative data on friction in metal processing by using the actual metal forming operation or simulative laboratory tests<sup>1-3</sup>.

Axial compression of solid cylinders has been previously studied by many researchers regarding its industrial importance and applications in various metal forming processes, such as forging and upsetting. Lubrication is an important way to decrease the friction; however, friction could not be entirely expelled from the contact surface during the upsetting. The barreling shape of a cylinder under compression testing has been quantitatively investigated by many researchers. Kulkarni and Kalpakjian<sup>4</sup> offers a review of the previous theoretical and experimental work in upsetting. That review also

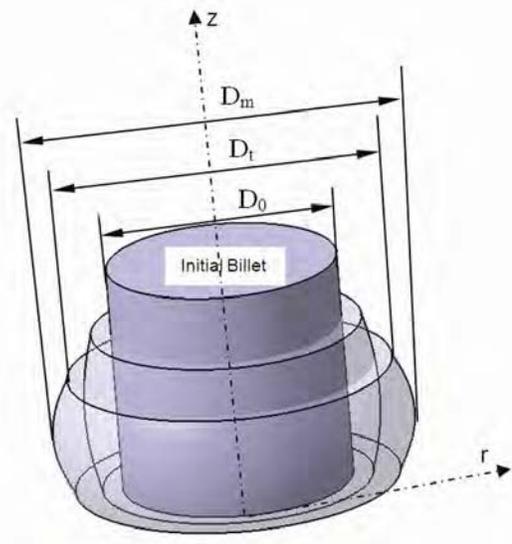


Fig. 1—The initial billet and barreling profiles at different stages

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reports on some experiments on barreling in the upsetting of aluminum billets with and without lubrication. Based on their measurements, Kulkarni and Kalpakjian<sup>4</sup> concluded that the profile of the barreled billets can be assumed as an arc of a circle and the shape of the barrel is affected by the initial  $h_0/d_0$  ratio and by the friction conditions. Resting on the work by Latham and Cockcroft<sup>5</sup> who suggest the profile of the barreled surface as an approximation of an arc of a circle, the researchers use the ratio of the current height to the current radius of curvature of specimen as a parameter ( $H_{ur}$ ) to characterize the extent of barreling.  $H_{ur}$  was found out to be not sensitive to aspect ratio and to deformation speed. It increased linearly with percentage reduction for unlubricated specimen, but this correlation was not observed for lubricated workpieces. However, another disturbing feature is that the variation of  $H_{ur}$  versus strain depends on the material being deformed. Schey *et al.*<sup>6</sup> announce that the shape of the barrel is affected by the geometrical factors such as, the  $h_0/d_0$  ratio, the reduction ratio and the diameter ratios. They state that a power law can represent the barreling profiles of steel and aluminum specimens well for both low and high friction conditions.

Some investigators<sup>7</sup>, who studied the barrel formation in cylindrical upsetting, suggested velocity fields to predict the forming loads. Banerjee<sup>8</sup> theoretically proved that the barrel radius could be given as a function of height strain by using the extrapolation technique and verified the same through experimental data. A numerical technique was used by Hashmi<sup>9</sup> to analyze the upsetting of cylindrical specimens between the free falling ram and the stationary anvil which have unequal friction properties. Gupta *et al.*<sup>10</sup>, Tseng *et al.*<sup>11</sup> and Narayanasamy *et al.*<sup>12</sup> found out that the barreled shape can be reasonably characterized as the arc of a circle or a circular curvature. Kobayashi<sup>13</sup> also assumed the barreling profile to be parabolic. Narayanasamy *et al.*<sup>14</sup> theoretically showed that the barrel radius could be expressed as a function of axial strain and confirmed the same through experimental verification. Yang *et al.*<sup>15</sup>, considering the dissimilar frictional conditions on flat die surfaces, used an upper bound method to determine the forging load during the upset forging of cylindrical billets. Narayanasamy and Pandey<sup>16</sup> studied the prediction of the barreling profile during the hot upsetting of the sintered iron billets which have a variety of initial

aspect ratios. He compared the theoretical radius value which he found using the empirical expressions with the experimental results, taking the barrel profile as an arc of a circle. Malayappan and Narayanasamy<sup>17</sup> aimed to find a relationship between the measured radius of the curvature of the barrel and a new geometrical shape factor based on the contact diameters, barrel diameters, initial height and hydrostatic stress while using a die with constraints on one end. Lin and Lin<sup>18,19</sup> investigated both the effects of die geometry on the barrel profile and fold defects occurring during the upsetting by using FEM. Baskaran and Narayanasamy<sup>20</sup>, in their theoretical and experimental studies, prepared billets having three different aspect ratios, namely 0.5, 0.75, 1.0 and cold upset forged them. The calculations were made under the assumption that the curvature of the barrel followed the form of a circular arc. He used the elliptical shaped billets and expressed that the barrel radius follows a power law relationship with the geometrical shape factor. Chen *et al.*<sup>21</sup> used the Hill's general method to calculate the flow stress of hot aluminum cylindrical billets under uniaxial simple compression. They also investigated the correlation between the barreling and the friction coefficient. Hence they simulated the upsetting process using a FEM software called DEFORM 2D. The barreling behaviors of truncated billets of aluminium, zinc and copper were studied by Abu Thaheer and Narayanasamy<sup>22</sup> and the barrel radius was defined as a power law equation changing with stress ratio.

A study similar to the one presented here was done by Ebrahimi and Najafizadeh<sup>23</sup>. They conducted several cylindrical compression tests to determine the constant friction using the upper bound method. They developed an upper bound equation by using the velocity fields which were first given by Avitzur<sup>24</sup> and could calculate the constant friction factor ( $m$ ), only by measuring the degree of barreling (i.e. the maximum radius and height of cylinder after deformation).

As seen in all studies related to barreling, the barreling profile is generally assumed to be the arc of a circle and formulated in this way. In a study carried out with the injection upsetting method by Altinbalik<sup>25</sup>, the barreling profile was suggested as a polynomial equation resting on the measured results. The basic parameters of the equations were taken into consideration as instantaneous heights and strokes. Looking at the sufficiently satisfactory results, the equation was accepted as the surface equation of the

specimens. This surface equation was substituted with the barreling parameter in the equation given by Ebrahimi<sup>23</sup>. Moreover, the load calculation was done using the upper bound method. A modified velocity field is proposed in the present study. Thus, the friction coefficient of the process was determined precisely by comparing the measurement results with the calculated load. The results were also compared with a commercially FEM program called DEFORM 3D.

### Experimental Procedure

AA6082 has been chosen as experimental material. AA6082 has excellent strength and cold formability, thus extensively used by the automotive industry. The cylindrical specimens of 20 mm diameter and 20 and 30 mm lengths were prepared from the bar, so the aspect ratios of specimens were obtained as 1.0 and 1.5. As it is expressed in the literature, specimens with aspect ratios higher than 2 suffer a column-like buckling, resulting in a concave-convex surface where no single radius can be defined or measured. The specimens were annealed for 2 h at 425°C and allowed to cool in furnace. The dies of the experiment were made of 1.2344 hot worked steel and hardened in 53HRC and their surfaces were ground. The experiments were carried out on a 150 metric ton capacity hydraulic press with 5 mm/s ram speed. The upset forging tests were conducted at room temperature under unlubricated conditions and the specimens were carefully cleaned with acetone so as to provide a similar friction condition before deformation. Extreme care was taken to place the axis of the cylindrical specimen concentric with the axis of the ram. Compressive upsetting was carried out up to a true height strain ( $\ln h_0/h_f$ ) of "0 to 1.0 with the increment of 0.1. Loads were recorded by using a pressure-current transducer and experiments were automatically stopped. The upper plate of the press was adjusted to a previously determined position by means of true height strains. Two specimens were used for each strain value. After the experiments, the following parameters were measured: (i) the height of the deformed specimen ( $h_f$ ); (ii) the top and the bottom diameters of the specimen ( $D_t$ ); (iii) the maximum diameter of the part ( $D_m$ ) and (iv) the distance between the outer surface and the center was measured for each specimen using the horizontal lines which were drawn along the parts by the serigraphy technique which was previously used by Çetinarslan and Sahin<sup>26</sup> before the experiments as shown in Figs 2a and 2b.

True stress-strain curves under uniaxial compression were obtained by an extrapolation technique<sup>15</sup> assuming that the volume was constant during the deformation and barreling processes. The stresses were calculated in each case using the simple expression

$$\sigma = 8F / \pi \left( \frac{3h_0 d_0^2}{h_f} - D_m^2 \right) \quad \dots (1)$$

where  $\sigma$  is the compressive stress,  $F$  is the load,  $h_0$  and  $d_0$  are the initial height and the initial diameter of the cylinder, and  $D_m$  and  $h_f$  are the maximum diameter and the height of the barrel after each stage of loading. From the test, the material was characterized by the following true stress-strain relationship where dry friction was employed:

$$\sigma = 341 \varepsilon^{0.17} \quad \dots (2)$$

### Determination of the Barreled Surface

In the present study, it is aimed to describe the outer surface profile during the deformation process through an equation and comparing the results with the measured values. Two basic parameters of barreling form are stroke and instantaneous height. The outer surface of the sample was assumed as a secondary polynomial by taking into consideration the previous study<sup>25</sup> and the barreling geometry of the current specimens concerning the vertical movement of the punch. In order to obtain the equation form, firstly the extent of barreling was obtained by measuring the diameters at various heights as mentioned above. The empirical formulation of the barrel profile was put forward using the measured values. Then, the profile of the barreled surface and height is given as:

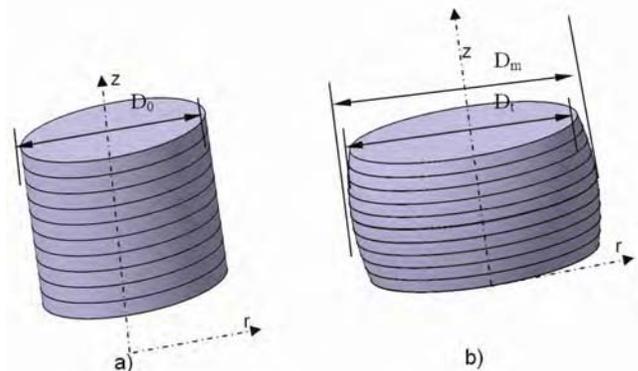


Fig. 2—Schematic representation of horizontal lines drawn by serigraphy technique (a) initial billet and (b) deformed billet

$$f(h, s) = a_1 h^2 + a_2 s^2 + a_3 h + a_4 s + a_5 \quad \dots (3)$$

where  $s$  and  $h$  are the stroke and height of the billet, respectively. The coefficients  $a_1, a_2, a_3, a_4,$  and  $a_5$  are determined statistically by using the least squares of the experimental data. Then the sum of squares of the residuals equation was differentiated with respect to each of the coefficients. This resulted in five equations.

$$S_r = \sum_{i=1}^n (x_i - a_1 h^2 - a_2 s^2 - a_3 h - a_4 s - a_5)^2 \quad \dots (4)$$

The coefficients of  $a_1$ - $a_5$  were obtained by means of the Gauss elimination technique. The obtained coefficients were given in Table 1. The values of the correlation coefficients were given in the last column in this table. As seen from Table 1, the quadratic equation represents a reasonable fit and a good agreement was found between the experimental data and predicted surface.

**Upper Bound Analysis**

The upper bound theorem was first formulated for rigid perfectly plastic materials, and states that among all the kinematically admissible strain rate fields, the actual one minimizes the power. The upper bound formulation for total rate of energy dissipation during the deformation is given as:

$$\dot{W} = \frac{2\sigma_o}{\sqrt{3}} \int_V \sqrt{\frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}} dV + \frac{\sigma}{\sqrt{3}} \int_{\Gamma_s} |\Delta V| dS + m \frac{\sigma}{\sqrt{3}} \int_{\Gamma_f} |\Delta V| dS \quad \dots (5)$$

The first right hand term of Eq. (5) represents internal power dissipated over the volume  $V$ , the second term represents shear losses and third term represents frictional losses on the tool-workpiece interface.  $\sigma_o$  is the flow stress of material,  $\epsilon_{ij}$  the derived strain rate tensor,  $|\Delta V|$  is the velocity discontinuity over the shear surfaces,  $m$  is the constant friction factor.

In this study, the upper bound method was used for load prediction. The axial velocity in the plastic deformation of the circular billet is a nonlinear

function of the vertical coordinate. Thus, the  $C$  parameter was introduced for non-uniform axial velocity distribution along any radial cross-section as done by Hsu<sup>27</sup> through a new upper bound analysis. Therefore, a modified velocity field has been defined for upper bound analysis (without accepting  $D_t = D_m$  as in the study of Ebrahimi<sup>23</sup>). Thus, the friction coefficient of the process was determined by comparing the measurement results with the calculated load.

**Proposed Upper Bound Solution**

The velocity fields in the cylindrical coordinates are assumed as follows:

$$u_r = \left( \frac{U \cdot r}{2H} \right) \left[ 1 + C \left( \frac{2z}{H} - 1 \right) \right],$$

$$u_z = - \left[ \left( \frac{U \cdot z}{H} \right) \left[ 1 + C \left( \frac{z}{H} - 1 \right) \right] \right], u_t = 0 \quad \dots (6)$$

The strain rates are obtained from the velocity fields given above as:

$$\dot{\epsilon}_{zz} = - \frac{U}{H} \left[ 1 + C \left( \frac{z}{H} - 1 \right) \right] - U \frac{z}{H^2} C$$

$$\dot{\epsilon}_{rr} = \frac{1}{2} \frac{U}{H} \left[ 1 + C \left( \frac{2z}{H} - 1 \right) \right] \quad \dots (7)$$

$$\dot{\epsilon}_{\theta\theta} = \frac{1}{2} \frac{U}{H} \left[ 1 + C \left( \frac{2z}{H} - 1 \right) \right]$$

The volume constancy is obtained from the following equation.

$$\dot{\epsilon}_{rr} + \dot{\epsilon}_{\theta\theta} + \dot{\epsilon}_{zz} = \frac{U}{H} \left[ 1 + C \left( \frac{2z}{H} - 1 \right) \right] - \frac{U}{H} \left[ 1 + C \left( \frac{z}{H} - 1 \right) \right] - U \frac{z}{H^2} C = 0 \quad \dots (8)$$

$$\epsilon_{rz} = \frac{1}{2} U \frac{r}{H^2} C, \epsilon_{r0} = \epsilon_{0z} = 0 \quad \dots (9)$$

The effective strain rate is obtained as:

$$\bar{\dot{\epsilon}}_{eff} = \frac{1}{2} \left[ \frac{U^2}{H^2} \left[ 1 + C \left( \frac{2z}{H} - 1 \right) \right]^2 + 2 \left[ - \frac{U}{H} \left[ 1 + C \left( \frac{z}{H} - 1 \right) \right] - \frac{Uz}{H^2} C \right]^2 + U^2 \frac{r^2}{H^4} C^2 \right]^{0.5} \quad \dots (10)$$

Table 1—Coefficients of Eq. (1) according to aspect ratio

	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$(s_r - s_t)/s_t$
$h_0/d_0=1.0$	-0.01797	0.06543	0.257	0.03598	20.36	0.9942
$h_0/d_0=1.5$	-0.01053	-0.02827	0.2255	0.008432	20.32	0.9859

The internal power dissipation can be calculated by using the strain rates derived from the velocity field as:

$$W_i = \frac{2\pi\sigma \int_0^{H/2} \int_0^R \dot{\epsilon}_{eff} r dr dz}{\sqrt{3}} \dots (11)$$

$$W_i = \frac{2\pi\sigma}{\sqrt{3}} \int_0^{H/2} (3H^2 12CzH - 6CH^2 + 12C^2 z^2 - 12C^2 zH + 3C^2 H^2 + R^2 C^2) \frac{U^2}{H^4} dz \dots (12)$$

$$W_i = 0.577 U^2 \frac{(3H^2 - 3CH^2 + C^2 H^2 + R^2 C^2)}{H^3} \pi\sigma \dots (13)$$

The friction loss is calculated as:

$$W_f = -\frac{4\pi m\sigma}{\sqrt{3}} \int_0^R \frac{1}{2} U \frac{r}{H} (1+C) r dr = -\frac{2}{3\sqrt{3}} \pi m\sigma R^3 \frac{U}{H} (1+C) \dots (14)$$

The total internal energy dissipation is obtained as:

$$J = W_i + W_f \dots (15)$$

The required force is obtained by equating the total internal energy dissipation to the external work as:

$$F = \frac{J}{V_0} \dots (16)$$

As explained above, the constant *C* is a parameter used for a non-uniform axial velocity distribution along any radial cross-section including the central line of the billet, i.e., the axial velocity in the plastic deformation region is a nonlinear function of the *z*-coordinate. A flow chart of theoretical analysis in Fig. 3 shows that all the steps are followed in the analysis. In consequence of the calculations regarding the deduced coefficient, the upper bound equation is optimized with respect to *C* parameter for a certain friction for the experiment  $h_0/d_0=1.0$  and  $h_0/d_0=1.5$ .

**Results and Discussion**

In the present study, the barreling profiles of upset billets having different aspect ratios were measured and the admissible barreling surface was defined according to these profiles. Then, the process was modeled by an upper bound analysis. The theoretical loads which are known from the literature as a general situation were

applied to the specific situation concerning the process under investigation. The theoretical load results were compared with the measured ones and the friction factor was determined sensitively.

The theoretical and measured barreling curves have to do with the samples having the aspect ratios of 1.0 and 1.5 for four different true strain steps as shown in Figs 4a and 4b. The values of all strains are not represented in order to avoid barreling curves which are too close to each other in the diagram. Bold lines

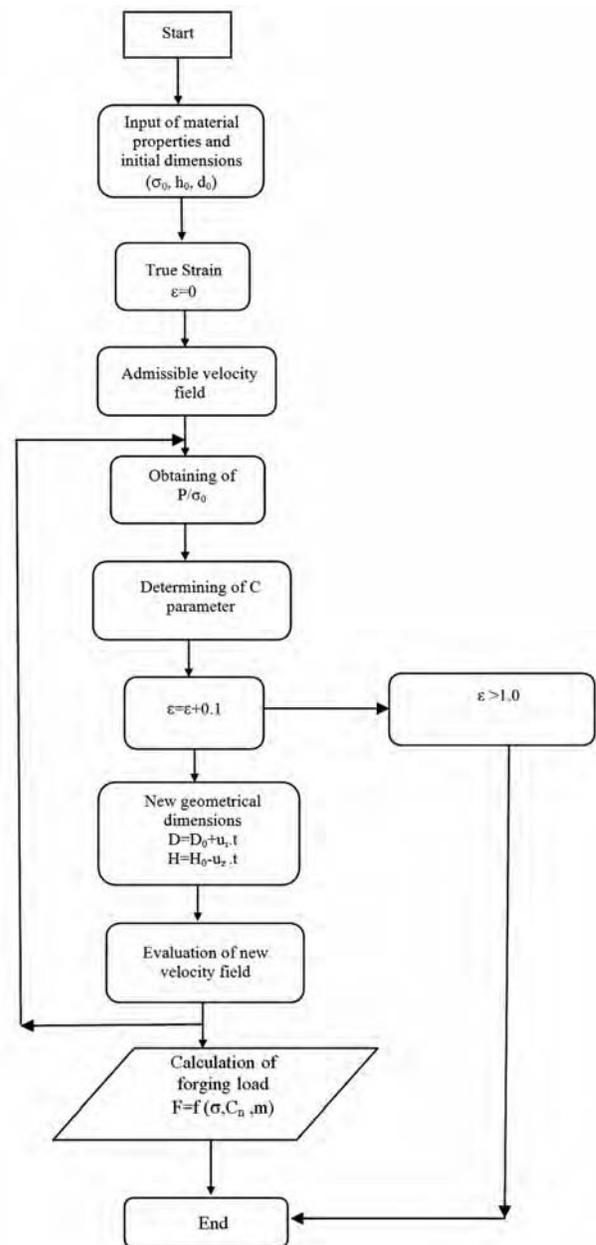


Fig. 3—Flow chart

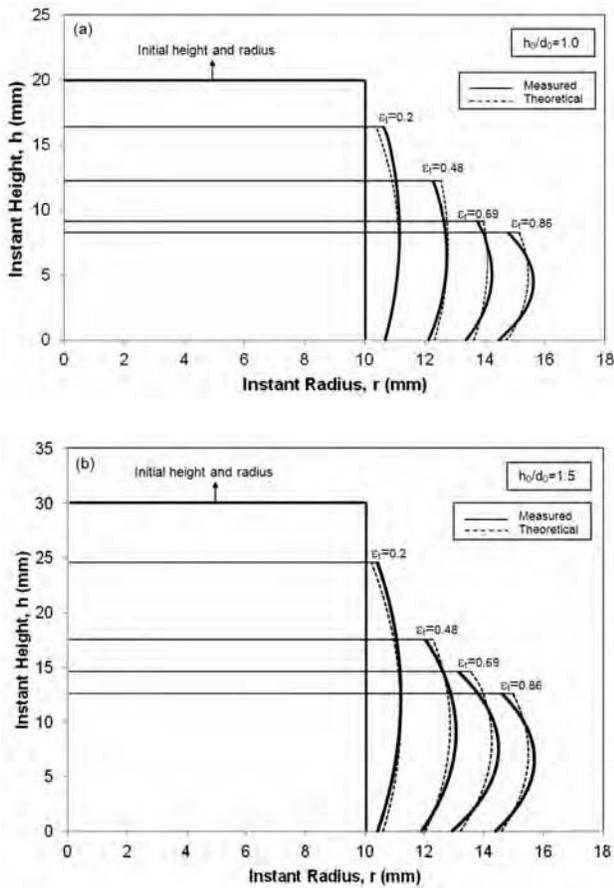


Fig. 4—Theoretical and measured barreling curves for some strain values (a)  $h_0/d_0=1.0$  and (b)  $h_0/d_0=1.5$

represent measured results whereas weak lines show theoretical curves attained from Eq. (3). For both aspect ratios, the curves in Figs 4a and 4b stand respectively for the same true strain values. It can be seen that the theoretical curves yielded more proper results especially for smaller strain values for each aspect ratio. The maximum radius of the theoretical curve is lower than its value measured by increasing the true strain but it is higher than  $D_t$ . The maximum deviation between the determined diameter and the measured diameter value is 1.1% for  $h_0/d_0=1$  and 1.5% for  $h_0/d_0=1.5$ . Thus, it is possible to calculate the diameter for any height value and the true strain by using only one formula instead of using different radius values for each step of the process. On the other hand, as seen in Figs 4a and 4b the curvature of the parts having bigger aspect ratios is much more excessive. The original end diameter always expands but the expansion decreases as the  $h_0/d_0$  ratio increases. This is consistent with the previous studies

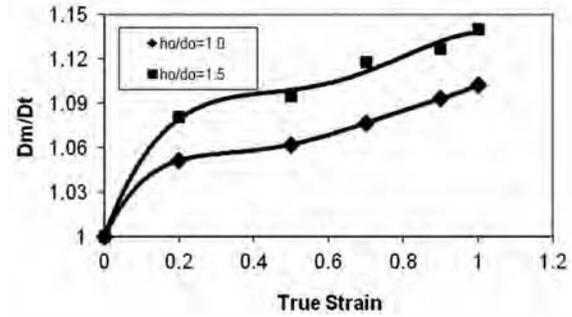


Fig. 5— $D_m/D_t$  ratios versus true strain during deformation

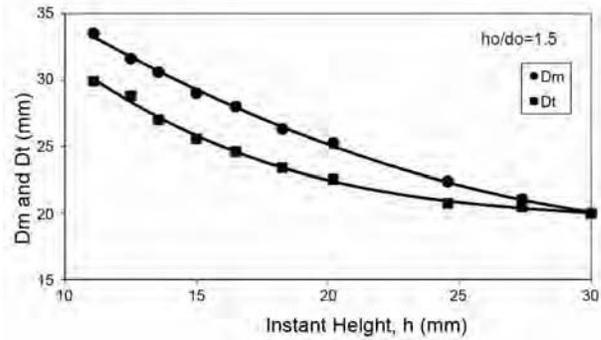


Fig. 6—Change of maximum and top diameter versus height of cylinder during deformation

where the barreling curve is theorized as an arc of circle<sup>10</sup>. Another diagram supporting this event is shown in Fig. 5. The variation of the true strain ( $\epsilon_t$ ) versus the ratio of the maximum diameter to the top diameter of the billet is presented in Fig. 5. The ratio of  $D_m/D_t$  allows better comparison between results and also represents the degree of barreling. According to Fig. 5, the difference between the maximum and top diameters increases as the height of the billet decreases. As it is seen, the ratio of  $D_m/D_t$  increases as both the true strain and aspect ratio get higher and it reaches its maximum value, 13%, for  $h_0/d_0=1.5$  at  $\epsilon_t = 0.86$ .

Another diagram related with the  $D_m$  and  $D_t$  is given in Fig. 6 which shows the shape geometry of the billet during the upsetting for  $h_0/d_0=1.5$  in order to compare with the values given in Ref.<sup>23</sup>. As seen in the figures, the difference between the maximum and the top diameter is remarkable and measurable. This result is consistent with the study of Ebrahimi<sup>23</sup>. The diameter and height of the billet were chosen bigger than that used by Ebrahimi by keeping  $h_0/d_0=1.5$  because the sensitivity of this method will increase

with a rise in dimensions of the cylinder as stated in Ref.<sup>23</sup>. For the specimens the aspect ratio of which is  $h_0/d_0=1.5$ , this difference gets clearer if it is evaluated together with Fig. 5. In the present study, the dimensions of the billets with aspect ratio of 1.5 are four times greater than the billets used Ref.<sup>23</sup>. It is observed that the  $D_m/D_t$  ratio does not vary for the same aspect ratio of 1.5 and the friction factor  $m = 0.23$ .

The theoretical load curves obtained from the upper bound analysis, the measured load values and the values obtained in Ref.<sup>23</sup> are shown in Figs 7a and 7b. In order to make a comparison between present study and Ebrahimi's work<sup>23</sup> in terms of load the surface equation was substituted with the barreling parameter in the equation given by Ebrahimi<sup>23</sup>. In Fig. 7a, the analysis results are given with the measured ones for  $h_0/d_0=1.0$  and  $m=0.23$  values. The calculated results are lower than the measured ones until  $\epsilon_t = 0.3$ . This difference comes from the assumed velocity field. Furthermore, the most important point in load analysis is the load value emerging from the final stroke for finding out the press capacity required for accomplishing a process. In this respect, the upper bound analysis satisfies 70% of the process in accordance with the literature. On the other hand, the predicted load value is only 10% higher than the measured value at

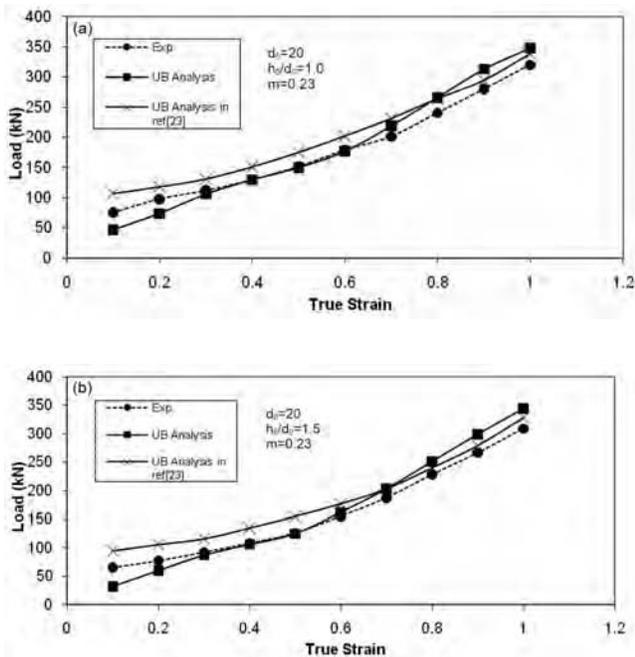


Fig. 7—Comparison between the experimental and theoretical load values (a)  $h_0/d_0=1.0$  and (b)  $h_0/d_0=1.5$

the final stage of the process. On the same diagram, the results of ref.<sup>23</sup> are also presented and these values are found satisfactorily close to the measured values. This verifies that the suggested second-order surface model and the coefficients as given in Table 1 are quite closer to the measured values. Similar diagrams are given in Fig. 7b for  $h_0/d_0=1.5$ . A similar situation is also available in this diagram and the upper bound analysis result is 12% higher than the measured values in the experiments.

The results were also compared with those of a commercially FEM program called DEFORM 3D. The required data has been given to the software and results were obtained after automatic meshing. A good agreement has been obtained with measured results for both  $D_t$  and  $D_m$ . The results for the maximum diameter are very close to the measured values for  $h_0/d_0=1.5$  and the difference is 0.22 mm. The load values obtained from DEFORM 3D and the upper bound analyses and the measured load values are shown in Figs 8a and 8b. It is seen that DEFORM3D gives satisfactory results for all strain values when compared to experimental results. Figure 9 gives effective strain distributions for the parts deformed at different true strains for two different aspect ratios.

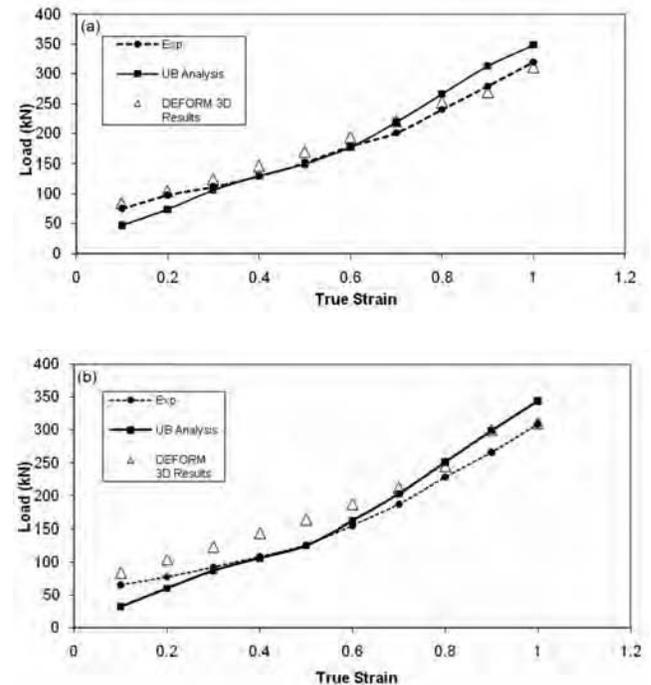


Fig. 8—Comparison of load values obtained from upper bound, DEFORM3D and measured from the experiments (a)  $h_0/d_0=1.0$  and (b)  $h_0/d_0=1.5$

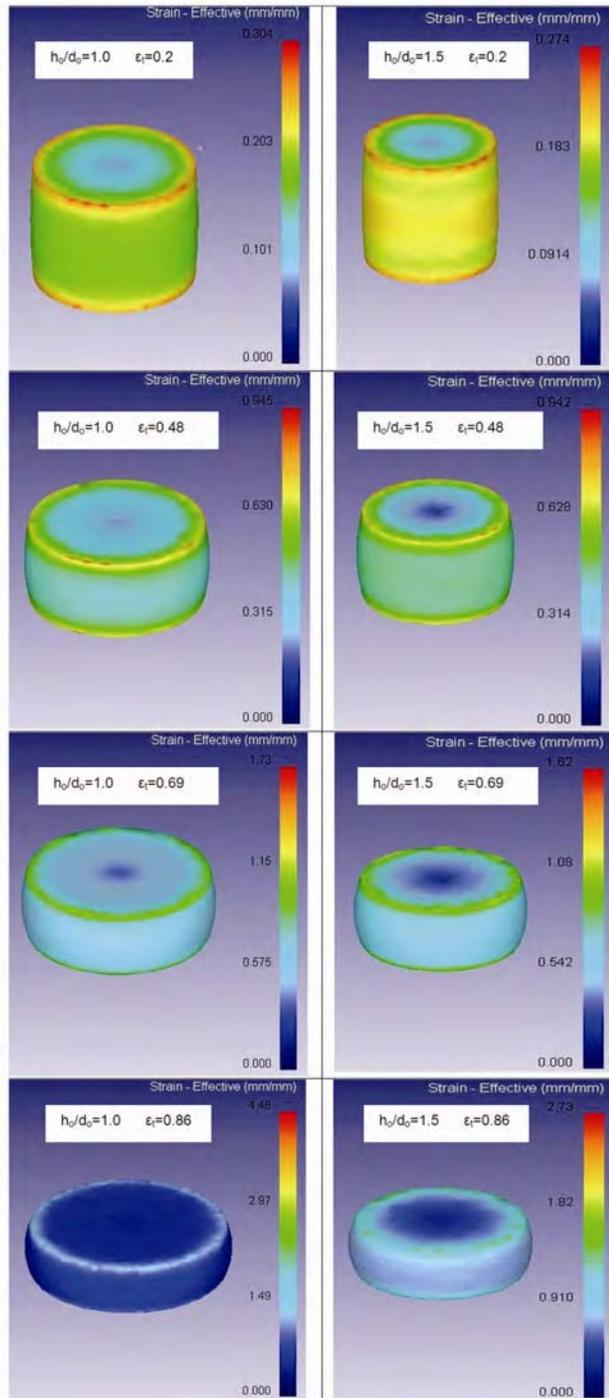


Fig. 9—Strain distribution of the billet with different deformation degrees

**Conclusions**

A series of upsetting experiments were carried out for different aspect ratios of 1.0 and 1.5 and the barreling profiles were measured. The measured results indicate that the profile of the barrel fits

closely to a second-degree polynomial during each stage in the axial upsetting of the billet and so the barreling surface was assumed as a second-order polynomial function. The prediction of the barreling profile of deformed billets showed a good agreement with the experimental measurements.

On the other hand, there is a good agreement between the theoretical forging loads and the experimentally measured ones. In general, the theoretical velocity field does not express very accurately the metal flow conditions. However, the velocity field proposed in the present work can be used conveniently for the prediction of forging load and deformation in upset forging of cylindrical billets due to the *C* coefficient which represents the non-uniform axial velocity. So, the friction factor can be determined sensitively by using the suggested formulas.

It is observed that the  $D_m/D_t$  ratio of the upset billet varies depending on the aspect ratio but does not vary in billets having the same aspect ratio but in different dimensions. It can be suggested that the experiments may be carried out in the ranges of 2.0-2.5 for the aspect ratios which are given the maximum limits in literature.

Also a good harmony was found between the experimental results and the results obtained from DEFORM3D by means of load and strain.

**Nomenclature**

- $V_0$  = punch velocity
- C* = optimization parameter
- h* = instantaneous height of billet
- $h_0$  = initial height of billet
- $U_r, U_\theta, U_z$  = radial, circumferential and axial velocity component
- $\dot{\epsilon}_{ij}$  = strain rate tensor
- $\bar{\epsilon}_{eff}$  = effective strain rate
- $\dot{W}_i, \dot{W}_f$  = rate of energy dissipation of plastic deformation, and rate of frictional energy loss
- $|\Delta V|$  = velocity discontinuity
- F* = forming load
- $\sigma, \sigma_0$  = flow stress
- $p_{av}$  = punch pressure
- A* = cross section area of billet
- $d_0$  = initial diameter of the billet
- $D_t$  = top diameter of the billet after deformation
- $D_m$  = maximum diameter of the billet after deformation

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