Production run time problem with machine breakdowns under AR control policy and rework

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Received 13 November 2006; revised 07 September 2007; accepted 12 September 2007

This paper examines the production run time problem with random machine breakdowns under abort/resume (AR) policy and reworking of defective items produced. Under AR policy, production of interrupted lot will be immediately resumed when breakdown is fixed and machine is restored. Mathematical modeling and derivation of the production-inventory cost functions for both systems with/without breakdowns are presented. These cost functions are integrated and long-run average cost per unit time is obtained. Theorems on convexity and on bounds of run time are proposed and proved. A recursive searching algorithm is developed for locating optimal run time within the bounds. Numerical example with sensitivity analysis is provided to assist in optimal operational control of such an unreliable system.

\textbf{Keywords}: Abort/resume policies, Defective items, Machine breakdowns, Manufacturing, Production run time, Rework

\textbf{Introduction}

In manufacturing sector, economic production quantity (EPQ) model is often used to deal with optimization in non-instantaneous replenishment problem\textsuperscript{1,2}. EPQ model can be considered as an extension to the well-known economic order quantity (EOQ) model\textsuperscript{3} with the difference on that in EPQ model the products are produced in-house instead of being acquired from outside suppliers as assumed by EOQ model. EOQ/EPQ models are simple and still applied industry-wide today\textsuperscript{1,4}. In classic EPQ model, all items produced are assumed to be of perfect quality. However, due to process deterioration or other factors, in real-life systems, generation of imperfect quality items is inevitable. Therefore, a considerable amount of research has been carried out to enhance classic EPQ model by addressing the issue of defective items produced\textsuperscript{5-11}. Imperfect quality items can be reworked and repaired; so overall production-inventory costs can be reduced\textsuperscript{12-17}. Hayek & Salameh\textsuperscript{12} examined an EPQ model with assumption that all defective items produced are repairable. Chiu\textsuperscript{13} considered a finite production model with random defective rate, scrap, rework of repairable defective items, and backlogging. Jamal \textsl{et al}\textsuperscript{14} studied optimal manufacturing batch size with rework process at a single-stage production system.

Groenevelt \textsl{et al}\textsuperscript{18} first presented two production control policies; first one assumes that production of interrupted lot is not resumed (called no resumption or NR policy) after a breakdown; and second policy considers that production of interrupted lot will be immediately resumed (called abort/resume or AR policy) after breakdown is fixed and if the current on-hand inventory is below a certain threshold level. Studies have since been carried out to address the issue of production systems with breakdowns\textsuperscript{19-30}.

Moinzadeh & Aggarwal\textsuperscript{20} proposed an (s, S) policy and investigated policy parameters that minimize expected total cost per unit time. Chung\textsuperscript{22} examined upper and lower bounds of optimal lot sizes for two extensions (NR and AR policies) to EPQ model\textsuperscript{18}. Kim \textsl{et al}\textsuperscript{23} studied an extended optimal lot sizing model with an unreliable machine. Makis & Fung\textsuperscript{25} investigated effects of machine failures on optimal lot size as well as on optimal number of inspections. Boone \textsl{et al}\textsuperscript{27} studied impact of imperfect processes on production run time. Giri & Dohi\textsuperscript{30} formulated EMQ model based on net present value (NPV) approach.

This paper examines an imperfect production system with random breakdowns under the abort/resume (AR)
control policy and the reworking of defective items produced.

Proposed Model

This study considers an imperfect production process that may randomly produce $x$ portion of defective items at a rate $d$. During production uptime, machine is subject to a random breakdown that follows Poisson distribution. When random breakdown occurs, AR policy is adopted, wherein machine is put under repair immediately and interrupted lot will be resumed when machine is fixed and back to operating condition. Machine repair time is assumed to be constant.

Defective rate $x$ is assumed to be a random variable with a known probability density function. All items produced are screened and unit inspection cost is included in the unit production cost $c$. All defective items are assumed to be repairable through a rework process at a rate $P_1$ in each production run, when regular process ends. Production rate $P$ is larger than demand rate $\dot{e}$. Hence, production rate of defective items $d$ can be expressed as $d=Px$. All related cost parameters considered in the proposed model include setup cost $K$, holding cost $h$, repairing cost $C_R$ and holding cost $h_1$ for each reworked item, and the repairing cost for machine breakdown $M$, besides additional notations listed under nomenclature.

For EPQ model with random defective rate and shortages not permitted, basic assumption should be that $P \geq \text{sum of demand rate and production rate of defective items}$. Hence, $(P-d-\lambda) \geq 0$ or $(1-x-\lambda/P) \geq 0$ must be satisfied. Let $t$ denote time before random breakdowns during production uptime $t_1$, then following two cases have to be investigated:

Case 1: $t < t_1$

In this case, time before a machine breakdown taking place $t$ is smaller than production uptime $t_1$. Under AR policy, production of interrupted lot will be immediately resumed when the breakdown is fixed. On-hand inventory level of perfect quality items, when a random breakdown occurs during $t_1$ (Fig. 1), provides: level of on-hand inventory when machine breakdown occurs $H_2$, level of inventory when machine is repaired $H_3$, maximum level of on-hand inventory when machine is restored and remaining production uptime is accomplished $H_4$, level of on-hand inventory when reworking of defective items are completed $H_5$, production uptime $t_1$, and cycle length $T'$. Using procedure from earlier studies\(^{12,13}\)

\[
\begin{align*}
H_4 &= H_3 - t_1 \lambda = (P - d - \lambda) t - g \lambda \\
H_5 &= H_4 + (P - d - \lambda)(t_1 - t) \\
H_3 &= H_5 + t'_1 (P - \lambda) \\
t_1 &= \frac{Q}{P}; \quad Q = t_1 P \\
T' &= t + t'_1 + (t_1 - t) + t'_2 + t'_3 \quad \text{or} \\
\end{align*}
\]

where $t_1 = g$ and $d=Px$.

Defective items produced during time $t$ (before a breakdown takes place) is $dt$ and total defective items produced during uptime $t_1$ (Fig. 2), time needed for reworking of defective items $t'_2$ and the time needed for depleting all on-hand perfect quality items $t'_3$ can be obtained as

\[
\begin{align*}
d \cdot t_1 &= x \cdot Q = x \cdot t_1 \cdot P \\
t'_2 &= \frac{dt_1}{P_1} = \frac{P t_1}{P_1} \\
t'_3 &= \frac{H_3}{\lambda} = T' - t_1 - t'_2 - t'_1 \\
\end{align*}
\]

Total production-inventory cost per cycle in the case of breakdown takes place (under AR policy) during production uptime $t_1$ is

\[
TC_1(t_1) = C(t_1 \cdot P) + K + M + C_R (t_1 \cdot x) + h_1 \left[ \frac{P t'_1}{2} (t'_1) \right] + \]

\[
\begin{align*}
&\text{...}(1) \\
&\text{...}(2) \quad H_4 = H_3 - t_1 \lambda = (P - d - \lambda) t - g \lambda \\
&\text{...}(3) \quad H_5 = H_4 + (P - d - \lambda)(t_1 - t) \\
&\text{...}(4) \quad H_3 = H_5 + t'_1 (P - \lambda) \\
&\text{...}(5) \quad t_1 = Q \cdot \frac{1}{P}; \quad Q = t_1 P \\
&\text{...}(6) \quad T' = t + t'_1 + (t_1 - t) + t'_2 + t'_3 \quad \text{or} \\
&\text{...}(7) \quad d \cdot t_1 = x \cdot Q = x \cdot t_1 \cdot P \\
&\text{...}(8) \quad t'_2 = \frac{dt_1}{P_1} = \frac{P t_1}{P_1} \\
&\text{...}(9) \quad t'_3 = \frac{H_3}{\lambda} = T' - t_1 - t'_2 - t'_1 \\
&\text{...}(10) \quad Total \ production-inventory \ cost \ per \ cycle \ in \ the \ case \ of \ breakdown \ takes \ place \ (under \ AR \ policy) \ during \ production \ uptime \ t_1 \ is \ }
\end{align*}
\]