Using average control chart with subgroups to monitor turnaround time

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This study proposes a control chart to monitor turnaround time (TAT) of biochemical test of laboratory for a medical center. Constants of average control chart are calculated in accordance with fixing probability of type I error ($\alpha = 0.0027$) with three distributions (Weibull, lognormal and Burr) by using Monte Carlo simulation. This study also compared probabilities of type I and type II errors ($\beta$) among control charts, including weighted variance (WV), skewed correction (SC) and traditional Shewhart control charts. Control chart with asymmetrical control limits using proposed constants based on subgroups is superior in terms of two probabilities for a skewed process.

Keywords: Average control chart, Skewed distribution. Turnaround time, Type I error, Type II error

Introduction

Turnaround time (TAT) provides the most important characteristic for laboratory testing and provided TATs for various laboratory tests. Studies reported that poor laboratory performance in terms of long TAT had a major impact on patient care. So far, all TAT studies have focused on inpatient testing (especially of an emergency nature), outpatient testing and outfits. Many processes in industry violate normality assumptions (chemical processes, cutting tool wear processes and lifetime) in an accelerated life test. Valenstein & Emancipator noted that distribution of TAT data is non-normal. Skewed nature of TAT distribution may result in specimens with excessively long TATs. Hence, if traditional control charts based on normality assumption are used to monitor a non-normal process, probabilities of a type I error ($\alpha$) in control charts increases as skewness of process increases. Most studies determined related constant values of an average control chart (ACC) based on simple sizes, and not subgroups for skewed distributions. Subgroup size could affect ability of control chart on monitoring an out-of-control signal.

This study introduces a control chart into a laboratory workflow to monitor its TAT, besides determines constants of ACC (listed in a table to be consulted by practitioners) in accordance with fixing probability of type I error ($\alpha = 0.0027$) probability limits with three distributions (Weibull, lognormal and Burr) by using Monte Carlo simulation (MCS) method.

Experimental Section

Physicians often complain that it is too long for TAT of biochemical test. A medical center from Taiwan indicated the quality improvement program, especially for statistical process control (SPC) that is not established for its laboratory. TAT defined time from computerized physician order entry to appearance in electronic medical record. Laboratory information system (LIS) automatically captured receipt and reporting times for all specimens submitted to the laboratory. Non-urgent specimens are delivered to laboratory every 2 h, but urgent specimens are immediately transported. This study monitors TAT of non-urgent specimens using an ACC based on subgroups.

Laboratory Workflow

It is a critical component of laboratory quality assurance programs for evaluating clinical laboratory specimen TAT. Accuracy of laboratory test results depends on methods of specimen collection, handling and preservation (Fig. 1).
Phase I: Preanalytical
A physician orders a test based on clinical diagnosis for a patient. Phlebotomists or nursing personnel prepares appropriate containers of specimen collection in accordance with the test orders. Specimens can be delivered to a laboratory by pneumatic tube systems, hand delivery, mechanized vehicles and dumbwaiters. Laboratory personnel checks test orders and judges accuracy of specimen collected. Specimen is numbered and logged into LIS.

Phase II and III: Analytical and Postanalytical
Specimen is tested and analyzed in accordance with the test order. After analyzing, laboratory personnel roughly interpret the analysis data. After confirming accuracy of test report, the data is logged into LIS. If test result has a panic value, laboratory personnel have to notify the physician or health care units of the phenomenon. Rare study uses a statistical process control method to manage TAT, so this study suggests that statistical quality control methods should be conducted into the workflow to control TAT. Laboratory staff can use control chart to determine whether TAT is stable and follow the out-of-control signals to find out the special causes for impacting TAT.

Fig. 1—Workflow of laboratory

**Determination of Constants of Average Control Chart (ACC)**
Through Weibull, lognormal and Burr distributions, the study will propose an ACC in accordance with various subgroups \((m)\), skewed degrees when type I risk \((\alpha_1)\) is fixed as 0.0027 [corresponding to classical 3 \(\sigma\) limits for a Shewhart control chart (SH)].

**Determination of Skewness**
Skewness coefficient \((\alpha_3)\) is most frequently used to evaluate skewness of a distribution. For practitioners with no sufficient training in statistics, \(\alpha_3\) could be confusing and requires considerable computation. This study does not utilize \(\alpha_3\) for skewness of a distribution and uses a simple method as follows: Suppose that \(m\) are available, each containing \(n\) (sample size) observations on quality characteristic. When a population is unknown, \(\hat{P}_x\) [Probability that a random variable \(X \leq \hat{x}\)] is used to estimate \(P_x(\mu)\) [Probability that a random variable \(X \leq \mu\)]. Hence, \(\hat{P}_x\) is shown as

\[
\hat{P}_x = \frac{1}{m} \sum_{i=1}^{m} \sum_{j=1}^{n} I(\bar{x} - x_{ij}),
\]

where grand average \(\bar{x} = \frac{\bar{x}_1 + \bar{x}_2 + \cdots + \bar{x}_m}{m}\) and \(\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_m\) are the average of each subgroup. If \(\bar{x} - x_{ij} \geq 0\), \(I(\bar{x} - x_{ij}) = 1\). On the contrary, \(\bar{x} - x_{ij} < 0\), \(I(\bar{x} - x_{ij}) = 0\).

**Average Control Chart (ACC)**
This study utilizes \(\bar{x}\) and range \((R)\) to estimate \(\mu\) and \(\sigma\). Hence, ACC when a population is unknown can be shown as

\[
UCL = \bar{x} + \frac{z_u}{d_2/\sqrt{n}} R = \bar{x} + A_{2u} R
\]

\[
LCL = \bar{x} - \frac{z_l}{d_2/\sqrt{n}} R = \bar{x} - A_{2l} R
\]

where \(R\) is a mean of range; \(d_2\) is mean of relative range; constant of upper control limit (UCL), \(A_{2u} = \frac{z_u}{d_2/\sqrt{n}}\); and constant lower control limit (LCL),
\[ A_{2l} = \frac{z_l}{d_2 \sqrt{n}} \]. The \( d_2 \) can be determined by using a reported function. Computed constants, \( A_{2u} \) and \( A_{2l} \), will be used to construct an ACC and then monitor a skewed process. The \( z_u \) and \( z_l \) values of Weibull, lognormal and Burr distributions can be determined by using a numerical computation of Fortran and fixing \( \alpha = 0.0027 \).

**Monte Carlo Simulation (MCS) with Golden Section Search (GSS)**

GSS method can find min. or max. of a unimodal continuous function over an interval without using derivatives\(^\text{22}\). Because Weibull, lognormal and Burr distributions are unimodal continuous functions, their distributions of mean could also be unimodal ones. Therefore, GSS is suitable to determine the values of \( z_u \) and \( z_l \). Simulation consists of two segments.

**Segment 1: Determination of Control Chart Constants**

**S1.1**—Compute golden section ratio \( \tau = \left( \frac{\sqrt{5}-1}{2} \right) \) and specify tolerance values \( \varepsilon_1 \) and \( \varepsilon_2 \) be equal to \( 10^{-3} \) and \( 10^{-6} \), respectively.

**S1.2**—Determine search interval \( [z_u^{(lo)}, z_u^{(hi)}] \), where initial \( z_u^{(lo)} = 0 \) and \( z_u^{(hi)} = 20 \) and calculate \( z_u^{(1)} = z_u^{(hi)} - \tau (z_u^{(hi)} - z_u^{(lo)}) \) and \( z_u^{(2)} = z_u^{(lo)} + \tau (z_u^{(hi)} - z_u^{(lo)}) \).

**S1.3**—Go to segment 2 to determine \( \alpha_u^{(1)} \) and \( \alpha_u^{(2)} \) by \( z_u^{(1)} \) and \( z_u^{(2)} \).

**S1.4**—Set stopping criteria and \( \alpha = 0.00135 \). If
\[
\left| \left( z_u^{(hi)} - z_u^{(lo)} \right) \right| < \varepsilon_1, \quad \left| \left| \alpha_u^{(hi)} - \alpha / 2 \right| \right| \leq \varepsilon_2 \quad \text{and}
\]
\[
\left| \alpha_u^{(lo)} - \alpha / 2 \right| < \varepsilon_2, \quad \text{where } || \text{ is an absolute value function, then search will be stopped and compute}
\]
\[
z_u = \frac{z_u^{(hi)} + z_u^{(lo)}}{2}, \quad \text{else go to step 1.5 or step 1.6.}
\]

**S1.5**—If \( \alpha_u^{(1)} < \alpha_u^{(2)} \), \( z_u^{(hi)} = z_u^{(2)} \), \( z_u^{(lo)} = z_u^{(1)} \). Recalculate new \( z_u^{(1)} = z_u^{(hi)} - \tau (z_u^{(hi)} - z_u^{(lo)}) \) and go to step 1.3.

**S1.6**—If \( \alpha_u^{(1)} > \alpha_u^{(2)} \), \( z_u^{(lo)} = z_u^{(1)} \), \( z_u^{(hi)} = z_u^{(2)} \). Recalculate new \( z_u^{(2)} = z_u^{(lo)} + \tau (z_u^{(hi)} - z_u^{(lo)}) \) and go to step 1.3.

**Segment 2: Evaluation of Average Type I Risk**

**S2.1**—Generate \( m \), each of size \( n \) i.i.d. Weibull, lognormal or Burr variates using Fortran IMSL subroutines.

**S2.2**—Compute limits of control charts for \( z_u^{(1)} \) and \( z_u^{(2)} \).

**S2.3**—Generate \( n \) i.i.d. Weibull, lognormal or Burr variates using Fortran IMSL subroutines.

**S2.4**—Compute sample statistics \( \bar{x} \).

**S2.5**—Record whether \( \bar{x} \) fall outside control limits of \( z_u^{(1)} \) and \( z_u^{(2)} \) or not.

**S2.6**—Repeat steps 2.3 to 2.5, 100 times and calculate type I risks of control limits for \( z_u^{(1)} \) and \( z_u^{(2)} \).

**S2.7**—Repeat steps 2.1 to 2.6, 50,000 times and estimate average type I risk \( (\alpha_u^{(1)}) \) of \( z_u^{(1)} \) and that \( (\alpha_u^{(2)}) \) of \( z_u^{(2)} \).

Similarly, \( z_l \) can also be determined. Table 1 presents values of \( A_{2u} \) and \( A_{2l} \) for a given \( m, n \) and \( P_i (\mu) \). Layout of \( n \) and \( P_i (\mu) \) is set by reported methods\(^\text{14,19}\). When \( n \leq 5 \), values of \( A_{2u} \) and \( A_{2l} \) are obviously difference between \( m=5 \) and \( m=1000 \), particularly for \( P_i (\mu) \geq 0.62 \). When \( n>5 \), they still have slight differences. Number of subgroup taken is often set as 20 or 30 in practice. Values of \( A_{2u} \) and \( A_{2l} \) for \( m=20 \) or 30 is slightly different with that for \( m=1000 \). Thus if \( m \) is small, tolerance of its control limits is larger than that of theoretical control limits. Type I risks of control chart using small subgroups will be larger than that using \( m=1000 \).

**Comparison with Other Control Charts**

To know type I risks and type II risks of other control charts, this study compares weighted variance (WV), skewed correction (SC) and Shewhart methods from ACCs using reported method\(^\text{19}\).

**Comparison of Type I Risks**

The study will compare three methods with proposed method \( (\alpha = 0.0027) \) for Weibull and Burr distributions when \( m = 20 \). Normally, observations in a process are
### Table 1— Constants, $A_{2n}$ and $A_{21}$ of average control chart

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LCL, lower control limit; UCL, upper control limit
collected between \( m=20 \) and \( m=30 \) and \( n=4 \) or \( n=5 \). Probabilities are presented (Tables 2 & 3) of a type I error (\( \alpha/2 \)) that an out–of–control falls outside UCL or LCL for four control charts for Weibull and Burr distributions when \( m=20 \). For \( n=2 \), \( \alpha \) s of control charts using proposed control chart (SZ), WV and SH methods are higher than the other sample sizes (Tables 2 & 3). When \( \alpha \) is increasing, \( \alpha \) s of control charts also increase. When \( n \leq 7 \), \( \alpha \) s of control charts using three methods for a Burr distribution is higher than those for a Weibull distribution. The most of \( \alpha \) s using WV, SC and SH are very high. Regardless of Weibull and Burr distributions, control chart using SZ gives good effectiveness for monitoring an out–of–control signal. The phenomenon indicates that constants (Table 1) based on three distributions are suitable for controlling the process of Weibull or Burr, because values of \( \alpha \) s are not large difference with 0.00135.

Comparison of Type II Risks

To know monitoring ability of average chart using proposed method, this study will calculate its type II risks (\( \beta \)s) and compare with \( \beta \)s of Exact, WV, SC and SH methods for \( m=20 \) and 100 when \( n=5 \). According to reported methods\(^\text{14,19}\), this study considers that the process obeys an exponential distribution (Weibull with \( \alpha=1 \) and scale parameter \( \lambda=1 \)). The study will determine \( \beta \)s of charts using SZ, WV and SC methods by referring Table 1 and reported methods\(^\text{14,19}\). When process is unstable, original mean \( (\mu_m) \) will shift to a new mean \( (\mu_m + \delta) \) but standard deviation of process remains at \( \sigma_m \). Mean \( (\mu_m) \) and standard deviation \( (\sigma_m) \) of distribution are equal to \( \lambda \). \( \beta \)s of ACC is expressed as

\[
\beta = 1 - P\{LCL \leq \bar{X} \leq UCL | \mu_m + \delta \sigma_m \}
\]

Before \( \beta \)s are determined, UCL and LCL of ACCs using Exact, SZ, WV, SC and SH methods have to be considered. When exponentially distributed data are collected by using \( m \), \( 2mn \bar{X}/\sigma_m \) has a chi-squared (\( \chi^2 \)) distribution with \( 2mn \) degrees of freedom. Let \( \chi^2_{(2mn,1-\alpha/2)} \) be \( (1-\alpha/2) \)th quantile of \( \chi^2 \) with \( 2mn \) degrees of freedom. Hence, UCL and LCL using Exact method are shown as...
KAO: USING AVERAGE CONTROL CHART WITH SUBGROUPS TO MONITOR TURNAROUND TIME

Table 3— Probabilities of type I error of control charts using three methods for selected combinations of n and (c, k) in Burr distribution when m=20

<table>
<thead>
<tr>
<th>(c, k) (αα)</th>
<th>method</th>
<th>LCL</th>
<th>UCL</th>
<th>LCL</th>
<th>UCL</th>
<th>LCL</th>
<th>UCL</th>
<th>LCL</th>
<th>UCL</th>
</tr>
</thead>
<tbody>
<tr>
<td>(7, 3) 0.00</td>
<td>SZ</td>
<td>0.0037</td>
<td>0.0036</td>
<td>0.0020</td>
<td>0.0024</td>
<td>0.0016</td>
<td>0.0020</td>
<td>0.0014</td>
<td>0.0016</td>
</tr>
<tr>
<td></td>
<td>WV</td>
<td>0.0046</td>
<td>0.0047</td>
<td>0.0017</td>
<td>0.0024</td>
<td>0.0021</td>
<td>0.0024</td>
<td>0.0018</td>
<td>0.0020</td>
</tr>
<tr>
<td></td>
<td>SC</td>
<td>0.0073</td>
<td>0.0047</td>
<td>0.0028</td>
<td>0.0026</td>
<td>0.0027</td>
<td>0.0024</td>
<td>0.0021</td>
<td>0.0020</td>
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<td>0.0048</td>
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<td>0.0025</td>
<td>0.0022</td>
<td>0.0023</td>
<td>0.0020</td>
<td>0.0021</td>
</tr>
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<td>0.0029</td>
<td>0.0034</td>
<td>0.0016</td>
<td>0.0021</td>
<td>0.0013</td>
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<tr>
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<td>0.0008</td>
<td>0.0035</td>
<td>0.0011</td>
<td>0.0034</td>
<td>0.0012</td>
<td>0.0028</td>
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<td>0.0043</td>
<td>0.0028</td>
<td>0.0023</td>
<td>0.0052</td>
<td>0.0022</td>
<td>0.0039</td>
<td>0.0020</td>
</tr>
<tr>
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<td>SH</td>
<td>0.0013</td>
<td>0.0079</td>
<td>0.0008</td>
<td>0.0043</td>
<td>0.0008</td>
<td>0.0038</td>
<td>0.0009</td>
<td>0.0034</td>
</tr>
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<td>(2, 7) 1.01</td>
<td>SZ</td>
<td>0.0018</td>
<td>0.0035</td>
<td>0.0013</td>
<td>0.0021</td>
<td>0.0012</td>
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<td>0.0013</td>
<td>0.0017</td>
</tr>
<tr>
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<td>WV</td>
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<td>0.0102</td>
<td>0.0003</td>
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<td>0.0005</td>
<td>0.0047</td>
<td>0.0006</td>
<td>0.0038</td>
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<tr>
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<td>0.0043</td>
<td>0.0023</td>
<td>0.0025</td>
<td>0.0151</td>
<td>0.0013</td>
<td>0.0133</td>
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<tr>
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<td>0.0127</td>
<td>0.0001</td>
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<td>0.0002</td>
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<tr>
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<td>SZ</td>
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<td>0.0044</td>
<td>0.0014</td>
<td>0.0030</td>
<td>0.0012</td>
<td>0.0022</td>
<td>0.0011</td>
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</tr>
<tr>
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<td>0.0001</td>
<td>0.0079</td>
<td>0.0002</td>
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<tr>
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<td>0.0028</td>
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</tr>
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<td>0.0000</td>
<td>0.0108</td>
<td>0.0001</td>
<td>0.0098</td>
<td>0.0001</td>
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<td>0.0014</td>
<td>0.0037</td>
<td>0.0012</td>
<td>0.0033</td>
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</tr>
<tr>
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<td>0.0153</td>
<td>0.0001</td>
<td>0.0094</td>
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<td>0.0080</td>
<td>0.0003</td>
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<tr>
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<td>SC</td>
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<td>0.0068</td>
<td>0.0031</td>
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<td>0.0013</td>
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</tr>
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<td>(1.2, 6) 2.63</td>
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<td>0.0010</td>
<td>0.0045</td>
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<td>0.0029</td>
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<td>0.0116</td>
</tr>
</tbody>
</table>

LCL, lower control limit; UCL, upper control limit; SZ, proposed control chart; WV, weighted variance; SC, skewed correction; and SH, shewhart control chart.

Results and Discussion

Case Study: TAT of Biochemical Test in a Medical Center, Taiwan

Data in this example are obtained from a medical center (over 2000 beds and 4000 employees) in Taiwan, where > 120 patients / h are ordered to test by physicians. To establish control chart, 30 subgroups, each of size 2 were taken. Time interval between subgroups is 2 h.
These TAT data presents a non–normal distribution and fails to satisfy normal assumption according to Kolmogorov–Smirnov test at a 5% level of significance. Computed grand average and mean range are 107.85 and 25, respectively. $P_{10} = 0.63$ and $\alpha_1 = 1.5$.

Hence, after consulting Table 1 and using a linear interpolation, $A_{2\mu} = 4.325$ and $A_{2\nu} = 1.231$. LCL and UCL of SZ are 215.86 and 77.10.

ACC using four (SZ, WV, SC and SH) methods showed (Fig. 3) following out–of–control signals: SZ, 5 (4th, 6th, 17th, 19th and 20th subgroups); SC, 5 (4th, 6th, 8th, 19th and 20th data); WV, 3 (8th, 19th and 20th data) and SH, 4 (8th, 13th, 19th and 20th subgroups). Both WV and SH control charts can not detect 4th and 6th signals since they have lower LCLs than SZ. This result is similar to the analyses shown in Tables 2 and 3. LCLs of these other control charts using WV, SC and SH are less effective in monitoring a highly skewed process since their $\alpha$s are equal or close to zero.

The laboratory director indicated that the first 8th (178.02 min) and 13th (162.18 min) points are not out-of-control signals since max. threshold of TAT for simple biochemical test for medical center is 180 min. Special causes of TAT for 19th and 20th data are produced by pneumatic tube systems malfunction (PTSM) and staff shortage. Because PTSM, specimens are waiting for delivering to the laboratory. Moreover, temporary operator workings are utilized to solve the staff shortage problem. However, temporary staffs do not have ability to deal with complicated test tasks, so they spend a lot of time understanding the test operations. For 4th, 6th and 17th out–of–control signals, they spend small time on inspecting, because their biochemical test items are very simple, such as ammonia and $CO_2$ tests. Hence, 8th and 13th signals are false alarmed data. This result shows that SZ is superior for detecting out–of–control signals than other control charts. This study suggests that the laboratory should enhance the maintenance of PTSM and staff training and scheduling.

### Conclusions
In order to improving TAT, this study introduced the concept of control chart into laboratory workflow and utilized average SZ to monitor it. Computed constants of SZ based on $\alpha = 0.0027$ and subgroups are listed in Table 1. Laboratory staff can follow the three procedures, calculate skewness ($P_{10}(\mu)$), check $A_{2\nu}$ and $A_{2\mu}$ from Table 1 for a given $m$, $n$ and $P_{10}(\mu)$, determine control limits using and to construct an ACC for a skewed process. Regardless of Weibull and Burr distributions, the control chart using SZ gives good effectiveness for monitoring an out–of–control signal. Finally, SZ is superior for detecting out–of–control signals of TAT than the other control charts for a skewed process of biochemical test.

### References


