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Performance assessment and optimization of thermal power plants by DEA BCC and multivariate analysis

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This study presents an integrated Data Envelopement Analysis (DEA) methodology using Banker Charnes Cooper (BCC) input oriented model for assessment and optimization of conventional thermal power plants (gas, steam and combined cycles). Installed capacity, fuel consumption, labor cost, internal power, forced outage hours and operating hours are used as input parameters whereas total power generation is used as output parameter. Moreover, 40 power plants in Iran were used as decision-making units and DEA-BCC model was used to assess their efficiency and rank during 1997-2000. Principal Component Analysis (PCA) and Numerical Taxonomy (NT) together with Spearman correlation technique were used to verify and validate the findings of DEA-BCC approach. In addition, all regional power plants have been ranked, assessed and optimized in comparison with all thermal power plants.

Keywords: Thermal power plants, Data envelopment analysis

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Introduction

The cost associated with construction and operation of electricity generation facilities constitute a significant portion of GNP in most developed countries[1]. Increasing demand and consumption of electricity requires higher efficiency for power generation units and lower cost for consumers. A significant number of methodologies have been developed to characterize such efficiency[2]. Parametric and non-parametric approaches are two competing paradigms widely used in the efficiency measurement. The first includes estimation of both deterministic and stochastic frontier functions (SFF), which is based on econometric regression theory and has been widely accepted in econometrics field. The latter includes Data Envelopement Analysis (DEA) and Free Disposal Hull (FDH), which are based on a mathematical programming approach. In these methodologies, frontier is defined by the most efficient decision-making unit (DMU) of the sample. Frontier methods are high-reliability analysis tool and have largely been used for studies in electrical field[3,4].

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There are studies[5-10] on the effect of property (public versus private companies) on the efficiency of generation, transmission, and distribution and also there are reports about outcomes from DEA, corrected ordinary least squares (COLS), and stochastic frontier analysis (SFA) methodologies. Golany *et al*[1] presented DEA for measuring and evaluating operating efficiency of power plants in the Israel Electric Corporation (IEC). Lam & Shiu[10] applied DEA to China's thermal power generation. For coal-fired electric generation facilities, considerable opportunities for cost reduction to electricity consumers have been observed[11].

Park & Lesord[12], using DEA and stochastic-frontier method, determined efficiencies of 64 conventional fuel power plants operating in South Korea. Meibodi[13] applied efficiency measurement techniques in a sample of electricity industries in developing countries and 15 power plants in Iran. Nemoto & Goto[14] measured productive efficiencies of a firm employing quasi-fixed inputs that cannot be instantaneously adjusted to their optimal levels. Coelli[15] suggested to select input oriented models because DMUs have particular orders to fill (electricity generation) and hence input quantities appear

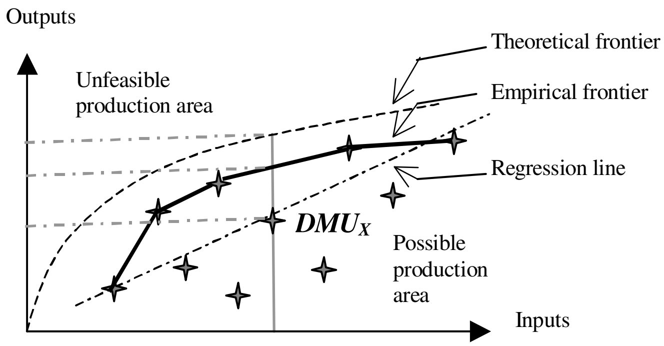

Fig. 1— Empirical and theoretical frontiers

to be primary decision variables. Knittel[16] investigated the effect of individual programs on technical efficiency of a large set of coal and natural gas generation units within a stochastic frontier framework.

This study presents an integrated DEA methodology for assessment and optimization of conventional power plants with respect to international inputs and outputs. This study deals with an investigation into technical efficiency of the Iranian generation electricity industry.

Methodology

After collecting data for input and output factors for each power plant, DEA- Banker Charnes Cooper (BCC) -input oriented model was applied for assessment and optimisation purpose. The model is input oriented because of selected applications, which DMUs (power plants) have particular orders to fill (electricity generation) and hence input quantities appear to be the primary decision variables. PCA and Numerical Taxonomy (NT) are used to verify the findings of DEA. Therefore, multivariate methods are applied to the problem of comparing and evaluating thermal power plants in Iran. DEA is based on a linear programming (LP) model for evaluating relative efficiencies of DMUs with common inputs and outputs. It is used for ranking and analysis of DMUs such as industries, universities, hospitals, cities, and facilities layouts. DEA has been compared with other methods[17].

PCA is used to reduce the number of variables under study and consequently ranking and analysis of DMUs[18-22]. NT approach is capable of identifying homogeneous DMUs from non-homogeneous DMUs. Furthermore, a group of DMUs by given input/output(s) is divided to homogeneous sub-groups[20-21]. It also ranks DMUs in a particular group. In order to conduct PCA and NT, individual output (power generation) to input (installed capacity, fuel, manpower, operating hours, forced outage hours and internal power) ratios for each DMU (power plant) are defined. Then in PCA, principal components for newly defined measures are determined. Finally, a single measure is obtained by weighting principal components in terms of information on Eigen values. PCA gives the weights among various output/input ratios defined by output and multiple inputs of DMUs. A performance ranking of DMUs according to PCA scores can be given[23]. After defining new measures, for following NT process, distance of every two DMUs for each input/output is computed. And by notice of the distance matrix, it is possible to rank DMUs. On the other hand, DEA is modified by excluding DMU under evaluation from the reference set[19]. At the end, DEA, NT and PCA rankings are compared using Spearman measures of correlation[24]. The tested hypotheses are H_o: DEA (NT) and PCA (NT) rankings are independent; and H_1: DEA (NT) and PCA (NT) rankings are directly related.

Data Envelopment Analysis (DEA)

Consistent with DEA terminology, DMU will refer to individuals in the evaluation group. DEA generates a surface called frontier that follows peak performers and envelops remainder[25]. Empirical and theoretical production frontiers (Fig. 1) are illustrated in a two-dimensional surface to generalize the case of a multi-dimensional surface. Theoretical frontier represents absolute maximum possible production that a DMU can achieve in any level of inputs; however, theoretical relationships between input and output parameters to a system are generally difficult to identify and to express mathematically. Therefore, theoretical frontier is usually unknown. Empirical frontier, based upon real DMU, is therefore used. Empirical frontier connects all "relatively best" DMUs in the observed population. If the performance of all of observed DMUs is generally poor, then empirical frontier only gives the best of a bad lot. Theoretical frontier would clearly indicate that the poor DMUs were indeed poor. Two efficiency values for DMU$_X$ (Fig. 1) are defined as absolute efficiency (A/C) and relative efficiency (A/B). DMUs that lie on the empirical frontier have a relative efficiency score of 1.0, whereas those that lie under this frontier are inefficient and have an efficiency score other than 1.0.

By providing observed efficiencies of individual DMUs, DEA may help identify possible benchmarks towards which performance can be targeted. The weighted combinations of peers and the peers themselves

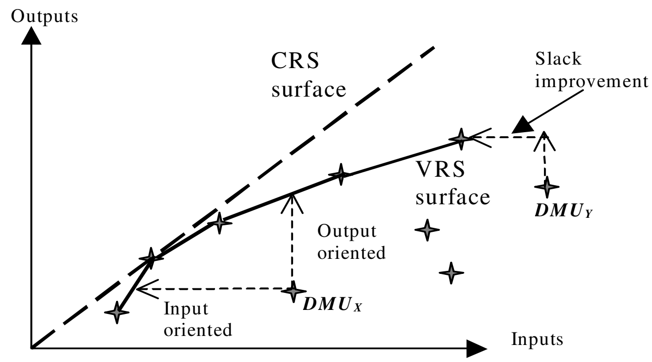

Fig. 2 — Envelopment surface and orientation

may provide benchmarks for relatively less efficient organizations. Actual levels of input use or output production of efficient organizations (or a combination of efficient organizations) can serve as specific targets for less efficient organizations, while processes of benchmark organizations can be promulgated for the information of managers of organizations aiming to improve performance. The ability of DEA to identify possible peers or role models as well as simple efficiency scores gives it an edge over other measures.

Out of four basic DEA models [Charnes Cooper Rhodes (CCR) model, Banker Charnes Cooper (BCC) model, multiplicative model, and additive model], only CCR and BCC will be discussed here. These models can be distinguished by the envelopment surface and the orientation (Fig. 2). Envelopment surface can take the form of constant-return-to-scale (CRS) or variable-return-to-scale (VRS) as evaluated in the CCR model and the BCC model, respectively.

Given the assumption of CRS, size of DMU is not considered to be relevant in assessing its relative efficiency. CRS surface is represented by a straight line that starts at the origin and passes through the first DMU that it meets as it approaches the observed population. The models with CRS envelopment surface assume that an increase in inputs will result in a proportional increase in outputs. Small DMUs can produce outputs with the same ratios of input to output, as can larger DMUs. This is because there are no economies (or diseconomies) of scale present, so doubling all inputs will generally lead to a doubling in all outputs. However, this assumption is inappropriate for services, which have economies of scale (or increasing returns to scale). In these services, doubling all inputs should lead to more than a doubling of output because providers are able to spread their overheads more effectively or take advantage of procuring supplies and other items in bulk. For other services, DMUs might become too large and diseconomies of scale (or decreasing returns to scale) could set in. In this case, a doubling of all inputs will lead to less than a doubling of outputs. It would be to a DMU's advantage to ensure that its operations are of optimal size - neither too small if there is increasing returns nor too large if there are decreasing returns to scale. If it is likely that the size of service providers will influence their ability to produce services efficiently, the assumption of constant returns to scale is inappropriate. The less restrictive variable returns to scale frontier allows the best practice level of outputs to inputs to vary with the size of DMUs in the sample.

VRS surface envelops the population by connecting outermost DMUs, including the one approached by CRS surface. VRS model allows an increase in input to result in a non-proportional increase of output levels — increasing returns to scale (IRS) occur below the point where CRS and VRS meet, and decreasing returns to scale (DRS) above. CRS surface passes through the points where DMUs have highest output to input ratios, given their relative size, and then runs parallel to respective axes beyond the extreme points. Scale efficiency of an organization can be determined by comparing technical efficiency scores of each service producer under constant returns to scale and variable returns to scale Distance from respective frontier determines technical efficiency under each assumption. Distance between constant returns and variable returns frontiers determines scale efficiency component. Distance from variable returns frontier determines technical efficiency resulting from factors other than scale. Thus, when efficiency is assessed under the assumption of variable returns, efficiency scores for each organization indicate only technical inefficiency resulting from non-scale factors. Technical efficiency scores calculated under variable returns, therefore, would be higher than or equal to those obtained under constant returns.

Other essential characteristic of DEA models is orientation, which indicates the direction an inefficient DMU approaches efficient frontier; either an increase in its output levels while maintaining the same level of input (output oriented) or a decrease in its input while keeping the same output level (input oriented). The output oriented may be more relevant for many government service providers, particularly those supplying human services, as the community often wants more of these services while budgetary pressures make it difficult to increase inputs. Input-oriented efficiency scores range between 0 and 1.0, and whereas output-oriented efficiency scores range between 1.0 and infinity; in both cases 1.0 is efficient. If a DMU is technically inefficient from an input-oriented perspective, then it will also be technically inefficient from an output-oriented perspective. However, the values of two technical efficiency scores typically will differ, as will the presence and extent of slacks. Depending on whether an input-saving or output-expanding orientation is utilized, peers for DMU_X will also differ. Output-oriented optimization of DMU_Y (Fig. 2) is slightly different in that the DMU is not fully enveloped by the surface. In this case, DMU_Y first approaches the frontier by increasing its output and then by using the "slack" variables to reach the efficient frontier. VRS surface is used in this research; therefore, only the BCC model will be discussed in detail.

The objective of DEA is to obtain the weights that maximize efficiency of DMU under evaluation, while limiting efficiency of all DMUs to less than or equal to 1.0 (for input-oriented models). Variables of this model are the efficiency score and the input–output weights; inputs and outputs of DMUs are known.

Charnes *et al*[25] recognized the difficulty in seeking a standard set of weights to calculate relative efficiency and that DMUs might value inputs and outputs differently and therefore adopt different weights. Charnes *et al*[25] therefore proposed that each DMU should adopt weights that show it in the most favorable light relative to other DMUs. This flexibility in weights is both a weakness and strength of this approach. It is a weakness because the judicious choice of weights by a DMU possibly unrelated to the value of any input or output may allow a DMU to appear efficient but it may have more to do with the choice of weights than operational efficiency. This flexibility may also be considered strength because a DMU that is inefficient with even the most favorable weights cannot argue that the weights are unfair.

Efficiency values produced by DEA are only valid within that particular group of peers. A DMU that is efficient in one group may be quite inefficient when compared with another group. In other words, if a group of very poor DMUs were evaluated using DEA, one would still have efficient DMUs. Further, if the set of DMUs is small, then there is little discrimination between them. In Fig. 2, five of the nine DMUs are on the efficient frontier. There should be at least three times the DMUs as there are variables (inputs + outputs) in the model[27]. In the case of DMU prequalification, often there are 10 or fewer DMUs to evaluate per contract. This would restrict the number of evaluation variables to three, which is simply too few for a thorough and reliable prequalification. It is, therefore, desirable to identify a standard set of best performers in construction prequalification, i.e., to identify practical frontier that would act as a fixed framework from which all DMUs could be compared.

Basic models of DEA

The original fractional CCR model (1) evaluates relative efficiencies of n DMUs (j = 1…n), each with m inputs and s outputs denoted by $x_{1j}, x_{2j},\ldots, x_{mj}$ and $y_{1j}, y_{2j},\ldots, y_{sj}$ respectively. This is done so by maximizing the ratio of weighted sum of output to the weighted sum of inputs:

$$\begin{aligned} &\mathrm{Max}\ \theta = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} \\ &\text{s.t.}\quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1\ ,\quad j = 1,\ldots, n,\quad r = 1,\ldots, s \\ &\qquad u_r, v_i \ge 0,\ i = 1,\ldots, m,\quad r = 1,\ldots, s \end{aligned} \qquad \ldots(1)$$

In model (1), efficiency of DMU_o is θ and u_r and v_i are factor weights. For computational convenience, fractional programming model (1) is re-expressed in linear program (LP) form as

$$\begin{aligned} &Max\ \theta = \sum_{r=1}^{s} u_r y_{ro} \\ &s.t.\quad \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0\ ,\quad j = 1,\ldots n, \\ &\sum_{i=1}^{m} v_i x_{io} = 1 \\ &\qquad u_r, v_i \ge \varepsilon,\ i = 1,\ldots,m, r = 1,\ldots,s \end{aligned} \qquad \ldots(2)$$

where ε is a non-Archimedean infinitesimal introduced to ensure that all the factor weights will have positive values in the solution. The model (3) evaluates the relative efficiencies of n DMUs (j = 1,…, n), respectively, by minimizing inputs when outputs are constant. The dual of linear program (LP) model for input oriented CCR is as follows[18]:

$$\begin{aligned} &Min\ \theta \\ &s.t.\quad \theta x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}\ ,\quad i = 1,\ldots m, \\ &\qquad y_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1,\ldots,s \\ &\qquad \lambda_j \geq 0 \end{aligned} \qquad \ldots(3)$$

Output oriented CCR model is as follows:

$$\begin{aligned} &Max\ \theta \\ &s.t.\quad x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}\ ,\quad i = 1,\ldots m, \\ &\qquad \theta y_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1,\ldots,s \\ &\qquad \lambda_j \geq 0 \end{aligned} \qquad \ldots(4)$$

If $\varepsilon\lambda_j = 1\ (j=1,\ \ldots,\ n)$ is added to model (3), BCC model is obtained which is input oriented and its return to scale is variable[19]. The calculations provide a maximal performance measure using piecewise linear optimization on each DMU with respect to the closest observation on the frontier. The linear programming system[24] for BCC input-oriented model is given in expression (5), and the output-oriented model in expression (6).

$$\begin{aligned} &Min\ \theta \\ &s.t.\quad \theta x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}\ ,\quad i = 1,\ldots m, \\ &\qquad y_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1,\ldots, s \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1 \\ &\qquad \lambda_j \geq 0 \qquad , j = 1,\ldots,n \end{aligned} \qquad \ldots(5)$$

$$\begin{aligned} &Max\ \theta \\ &s.t.\quad x_{io} \geq \sum_{j=1}^{n} \lambda_j x_{ij}\ ,\quad i = 1,\ldots m, \\ &\qquad \theta y_{ro} \leq \sum_{j=1}^{n} \lambda_j y_{rj} \quad r = 1,\ldots, s \\ &\qquad \sum_{j=1}^{n} \lambda_j = 1 \\ &\qquad \lambda_j \geq 0 \qquad , j = 1,\ldots,n \end{aligned} \qquad \ldots(6)$$

Original DEA model is not capable of ranking efficient units and therefore it is modified by Andersen and Petersen for DEA based ranking purposes to rank efficient units[31].

Principle Component Analysis (PCA)

PCA is used to reduce the number of variables under study and consequently ranking and analysis of DMUs, such as industries, universities, hospitals, cities, etc. These DMUs utilize a variety of sources as inputs to produce several outputs. The purpose of another paper was to describe and demonstrate the applicability of two multivariate statistical techniques, namely PCA and corresponding analysis, as analysis tools for quality professional. Using a principal component factor analysis with a rotation technique identifies seven hotels out of 33 hotel attributes and determines the level of satisfaction and service quality of Hong Kong hotels among Asian and Western travelers[29]. A multivariate analysis proposed a framework for measuring the efficiency of investment in IT that addresses some shortcomings including measurement errors, lags between investment and benefits, redistribution of profits, and mismanagement of IT resources. A paper suggests the utilization of multivariate technique as an objective optimization approach for the comparative efficiency evaluation of the maintenance department[30]. Furthermore, PCA captured measurement correlations and reconstructed each variable to define associated residuals and sensor validity index. The beverage data was analyzed using PCA and cluster analysis[31]. A multivariate analysis was used to test whether there is any relationship between airline flight delays and the financial situation of an airline[32]. PCA methodology has the advantage to be fairly simple to exploit, since it has been available in "off-the-shelf" computer packages for decades[35].

In other words, the objective of PCA is at first, to identify a new set of variables such that each new variable, called a principal component, is a linear combination of original variables. Second, the first new variable y1 accounts for the maximum variance in the sample data and so on. Third, the new variables (principal components) are uncorrelated. PCA is performed by identifying Eigen structure of the covariance or singular value decomposition of the original data.

Numerical Taxonomy (NT)

NT approach identifies homogeneous from non-homogeneous cases. Furthermore, a group of DMUs by given indexes is divided to homogeneous sub-groups[20,21]. It also ranks DMUs in a particular group. After defining new measures, for following NT process, the distance of every two DMUs for each indicator is computed and by notice of distance matrix, DMUs are ranked.

Results and Discussion

Data Collection

Electricity supply industry in Iran has been based mostly upon thermal power plants, which are based on steam turbine, gas turbine and diesel generator. In 2003, electricity produced in Iran[36] by the Ministry of Energy came from: gas turbines, 12; hydro, 2; steam power plants, 58; combined cycle power plants, 23; and diesel generators, 5%. Efficiency analysis of thermal power plants in Iran (1997-2000) has been conducted taking 40 DMUs including steam plants (S), gas plants (G), combined cycle plants (C) and the average of steam units (S-AVR), the average of gas units (G-AVR), the average of combined cycle units (C-AVR) and the average of regional electricity companies.

This study is based on standard input and output indicators recommended under international studies. Two inputs (manpower, operating hours) and one output (electric power generation) are indirectly related to economics factors. Electric power (MWh) generated from thermal power plants in each DMU is used as the output variable, while capital[35,36] (installed thermal generating capacity, MW), fuel (TJ), manpower, operating hours, forced outage hours and internal power (Ic) are five inputs (MWh) used for power generation. Various natural elements (natural gas, gasoil and Mazute) have been used as fuel in electric power generation in various thermal plants in Iran. Although, forced outage (hours) is considered as an output, it should be minimized and therefore it is considered as an input. The choice of fuel depends on availability, cost and environmental concerns and each fuel has its limitations. Internal power is the amount of energy consumed (MWh) within the site (for electrically powered equipment etc.). Other selected parameters (operating hours and forced outage hours) are applied to include various important aspects of plant's performance. Table 1 presents the actual inputs and outputs for evaluating performance of power plants in 2000[34].

Multivariate Models

A BCC-input oriented model is used to assess and optimize power plants. Throughout this analysis, the technology is modeled in terms of input-based orientation with the objective of providing electricity with a minimum resource level. An insufficient number of power plants for a DEA model would tend to rate all DMUs 100% efficient, because of an inadequate number of degrees of freedom. A proper power plant number is required for identifying a true performance frontier. A rule of thumb for maintaining an adequate number of degrees of freedom when using DEA is to obtain at least two DMUs for each input or output measure. The results from solving DEA would generate those performance frontiers that then become the final candidate designs.

BCC model evaluates relative efficiencies of 40 power plants ($j = 1\ldots40$), each with 5 inputs (installed capacity, fuel, manpower, operating hours, forced outage hours and internal power) and 1 output (power generation) denoted by x_{1j}, x_{2j}, x_{3j}, x_{4j} and x_{5j} and y_{1j} respectively. The model, which is input oriented and its return to scale is variable for the application, is as follows:

$$
\begin{aligned}
& e_o = \text{Min} \ \theta \\
\text{s.t.} \quad & \theta x_{io} \geq \sum_{j=1}^{40} \lambda_j x_{ij}, \quad i = 1,\ldots 5, \\
& y_{1o} \leq \sum_{j=1}^{40} \lambda_j y_{1j} \\
& \sum_{j=1}^{40} \lambda_j = 1 \\
& \lambda_j \geq 0
\end{aligned}
\qquad \ldots(7)
$$

Values of zero for each of the inputs (Table 2) indicate that there isn't any surplus in that production factor. The low value of technical efficiency indicates that inputs were over-used. For instance Mashhad power plant has surplus in labor and fuel. Average of inputs surplus for

Table 1 — Actual inputs and outputs during 2000

DMU	Inputs						Output
	Installed capacity	Fuel consumption	Labor	Internal power	Forced outage hours	Operating hours	Total power generation
G-rey	1243	30998	242	14264	838	2746	2130518
G-sof	96	2852	35	1168	1682	2709	169037
G-bsh	143	901	76	358	642	735	50848
G-shn	142	6015	53	1675	1035	4019	403269
G-shr	398	17592	87	4964	1571	5675	1453377
G-ghn	71	572	53	444	682	883	39095
G-shz	187	30588	124	4506	1483	7166	880809
T-tab	800	44805	463	382298	1040	7220	4936895
S-AVR	749.3	40016.8	363.7	272874.1	1286	6352.6	4452208.4
G-AVR	211.9	8166	73.9	2458.7	1062	3418	502912.8
C-AVR	805	29168.7	170.7	41275.6	1011	6065.2	3243419.3
azr-AVR	300.5	14895.5	169	97614.3	1675	5034.3	1531581.3
esf-AVR	843.7	45785.1	323	373424.3	423	4006.7	5120239.3
teh-AVR	1113	39653.2	309.6	164543	1153	5198.2	4203304.2
khr-AVR	379.8	19299.5	209.5	55670	1394	4884.3	1693543.5
far-AVR	287.6	18748.8	112.6	5131.6	1692	5606	1287765
tav-AVR	750.5	33171.8	353	179607.3	1995	5562.5	3825366.5
khz-AVR	1154	48048.5	469.5	318756.5	1656	4833	5957850.5

Table 2 — Efficiency and surplus of some of the plant by variable return to scale assumption

DMU	VRS Efficiency	Input Slacks						Output Slack
		Installed capacity	Fuel consumption	Labor	Internal power	Forced outage hours	Operating hours	Total power generation
G-rey	1	0	0	0	0	0	0	0
G-sof	0.92	0	698.59	0	0	1175.73	1349.55	0
G-shz	0.98	0	19893.48	12.95	0	288.82	1217.27	0
T-tab	0.97	0	0	186.2	0	670.16	0	0
S-AVR	0.99	0	0	110.06	0	777.12	0	0
G-AVR	0.81	0	473.79	0	0	0	309.37	0
azr-AVR	0.92	0	0	16.26	0	691.16	352.23	0
esf-AVR	0.99	0	0	37.19	3352.04	0	0	0
teh-AVR	0.81	22.98	0	34.76	9115.25	0	0	0
khr-AVR	0.85	0	0	31.07	0	323.23	0	0
far-AVR	0.99	0	4217.56	17.75	0	960.65	1613.57	0
khz-AVR	0.98	0	0	25.84	21668.97	198.75	1098.6	0
Average		7.285	684.196	39.0722	1753.883	241.0245	359.179	0

all DMUs in 2000 are: installed capacity, 7.3 MW; fuel, 684 TJ; labor, 40; internal power, 1754 MWh; forced outage hours, 242 h; and operating hours, 360 h. Inputs surplus indicate that excessive consumption of which inputs can be the reason of inefficiency in power plants. For example, labor and forced outage hours' surplus are the main reason of the inefficiency in S-AVR unit.

Calculating Input Targets

DEA suggests that inefficient units can become efficient by simply reducing their input consumption proportionately to their efficiency score level. The targets defined by efficient projections give an indication of how a DMU could improve to be efficient. Target values of some of the DMUs for all of inputs are shown in Table3. For instance, T-tab unit should reduce its install capacity from 800 to 775, fuel consumption from 44805 to 43386, number of labour from 463 to 263, internal power value from 382295 to 370193, forced outage hours from 1040 to 337 and operating hours from 7220 to 6992 to be placed on efficiency frontier and become efficient. For efficient units, target and real values of input/output(s) are equal. Target value for each input/output is computed as:

$$(X_o, Y_o) \rightarrow (\hat{X}_o = \theta_B^* X_o - s^{-*}, \hat{Y}_o = Y_o + s^{+*})$$

Benchmark for Inefficient Power Plants

By means of DEA, the reference set of inefficient power plants may be introduced. Those DMUs are the peer group for inefficient units (Table 4) known as the efficient reference set. As the inefficient units are projected onto the envelopment surface, efficient units closest to the projection and whose linear combination comprises this virtual unit form the peer group for that particular DMU.

After running BCC model for the inefficient power plants, the coefficients λ of reference set DMUs are none zero and for other DMUs equal to zero. Coefficient λ of

Table 3 — Input targets for some of the DMUs

DMU	Input Targets						Output Target
	Installed capacity	Fuel consumption	Labor	Internal power	Forced outage hours	Operating hours	Total power generation
G-rey	1243.00	30997.94	242.00	14264.00	837.80	2746.00	2130518.00
G-sof	88.44	1928.74	32.24	1076.02	373.82	1146.11	169037.00
G-bsh	143.00	900.88	76.00	358.00	642.10	735.00	50848.00
G-shn	142.00	6015.18	53.00	1675.00	1035.29	4019.00	403269.00
G-shr	398.00	17592.43	87.00	4964.00	1570.84	5675.00	1453377.00
S-zar	60.00	2151.67	190.00	14887.00	1240.59	3748.00	172142.00
C-ghm	1293.54	52799.33	352.97	217080.67	1135.30	5127.42	6551978.00
C-raj	1740.00	72172.26	524.00	424442.00	212.27	5737.00	8211320.00
C-gil	1312.00	46562.58	254.00	123614.00	1328.61	6978.00	6275607.00
C-kaz	256.00	15032.11	120.00	6952.00	210.62	7522.00	1395445.00
C-khy	214.10	8957.71	60.30	5166.09	399.11	2221.38	849910.00
C-far	740.00	41166.75	186.00	13557.00	416.80	7483.00	3774650.00
C-nsh	740.00	33390.44	144.00	16489.00	1190.14	7457.00	2851996.00
C-qom	712.00	23174.44	135.00	63983.00	1583.30	6011.00	3033272.00
S-AVR	739.26	39483.23	248.74	269235.80	491.98	6267.92	4452208.44
G-AVR	170.97	6115.80	59.61	1984.06	857.12	2448.82	502912.81
C-AVR	679.05	29056.07	152.00	41116.24	667.07	5112.68	3243419.30
azr-AVR	277.75	13768.06	139.95	90225.69	857.42	4300.97	1531581.25
esf-AVR	834.21	45272.02	282.19	365887.63	417.97	3961.77	5120239.33
teh-AVR	876.86	32058.81	215.55	123914.40	932.23	4202.64	4203304.20
far-AVR	284.17	14307.59	93.51	5070.39	711.50	3925.56	1287765.00
tav-AVR	713.20	31523.28	212.65	170681.24	917.33	5286.06	3825366.50

Table 4 — Benchmark for some of the inefficient power plants

DMU	Efficiency	Benchmarks									
G-rey	1.37038	0.550	G-bsh	0.052	T-sal	0.398	C-far				
G-shr	1.08344	0.673	G-shn	0.013	G-drd	0.313	C-far				
G-ghn	1.13307	0.188	G-bsh	0.050	G-shn	0.002	G-shr	0.760	G-drd		
G-shz	0.98139	0.606	G-orm	0.299	C-kaz	0.095	C-far				
S-AVR	0.98667	0.216	S-moh	0.098	S-fir	0.288	S-bis	0.097	C-gil	0.300	C-kaz
G-AVR	0.80696	0.082	G-shn	0.237	G-shr	0.468	G-ghn	0.185	G-drd	0.028	C-far
C-AVR	0.99614	0.334	G-drd	0.290	C-gil	0.376	C-far				
esf-AVR	0.98879	0.351	G-drd	0.353	S-moh	0.031	S-ram	0.247	S-bis	0.018	C-raj
teh-AVR	0.80848	0.405	G-drd	0.104	S-ram	0.026	C-raj	0.465	C-gil		
far-AVR	0.98807	0.234	G-orm	0.437	G-drd	0.330	C-far				
khz-AVR	0.97799	0.526	S-ram	0.386	S-fir	0.088	C-gil				

Table 5 — Actual data for the input/output(s) and reference set of G-shz

	Real Inputs						Real output	
	i_1	i_2	i_3	i_4	i_5	i_6	o_1	λ
G-orm	60	2808.31	91	1733	1757.3	4710	170483	0.6056
C-kaz	256	15032	120	6952	211	7522	1395445	0.2989
C-far	740	41167	186	13557	417	7483	3774650	0.0955
G-shz	187	30588	124	4506	1483	7166	880809	

Table 6 — Values of inputs and outputs for virtual DMU as a benchmark of G-shz

	Virtual inputs						Virtual output
	i_1	i_2	i_3	i_4	i_5	i_6	o_1
	36.33	1700.65	55.11	1049.46	1064.18	2852.27	103240.54
	76.53	4493.68	35.87	2078.24	63.08	2248.63	417155.37
	70.66	3930.73	17.76	1294.46	39.82	714.50	360413.09
Virtual DMU	183.52	10125.05	108.74	4422.16	1167.07	5815.39	880809

each of the power plants is equal to its weigh in the reference set. For instance (Tables 5 and 6), the reference set of G-shz unit include G-orm, C-kaz and C-far, with co-efficiencies values of $\lambda_{orm} = 0.606$; $\lambda_{kaz} = 0.299$ and $\lambda_{far} = 0.095$. These values indicate that there exists a virtual power plant on the efficiency frontier that its levels of inputs and outputs are equal to sum of multiplications of inputs and outputs of the reference set in their co-efficiencies equal to target values of input/output(s) for G-shz. This unit can consume fewer inputs than G-shz to produce at least the same levels of G-shz's outputs. G-shz can become efficient by simply reducing its inputs consumption to reach inputs' levels of this virtual power plant.

Ranking

DEA, PCA method and NT are applied to the data set of 40 DMUs. Scores of DEA efficiency and PCA together

Table 7 — Results of DEA, PCA and Taxonomy for the 40 power plants

DMU	Efficiency in AP	AP rank	PCA rank	Taxonomy rank	DMU	Efficiency in AP	AP rank	PCA rank	Taxonomy rank
C-far	3.18778	1	4	2	esf-AVR	0.98879	21	17	7
T-sal	3.12268	2	1	3	far-AVR	0.98807	22	24	20
C-kaz	1.81698	3	5	1	S-AVR	0.98667	23	15	13
S-moh	1.38800	4	7	5	C-khy	0.98252	24	25	25
G-rey	1.37038	5	33	33	G-shz	0.98139	25	22	18
G-bsh	1.34515	6	39	39	khz-AVR	0.97799	26	18	17
G-orm	1.27258	7	34	34	S-esf	0.96887	27	10	9
C-gil	1.22656	8	3	11	T-tab	0.96834	28	13	10
C-raj	1.22343	9	11	6	tav-AVR	0.95030	29	21	23
S-fir	1.20000	10	37	37	T-beh	0.93103	30	23	22
S-bis	1.19990	11	2	4	azr-AVR	0.92431	31	27	28
S-ram	1.14512	12	12	8	G-sof	0.92125	32	35	36
G-ghn	1.13307	13	38	38	S-abs	0.91585	33	16	19
S-zar	1.09310	14	36	35	C-ghm	0.87023	34	19	21
G-shr	1.08344	15	14	15	T-mad	0.86711	35	26	30
G-shn	1.01661	16	31	31	T-bes	0.86353	36	29	29
C-qom	1.00363	17	6	14	T-msh	0.85540	37	32	27
G-drd	1.00321	18	40	40	khr-AVR	0.85042	38	28	26
C-nsh	1.00135	19	9	16	teh-AVR	0.80848	39	20	24
C-AVR	0.99614	20	8	12	G-AVR	0.80696	40	30	32
Correlation		DEA and PCA: 0.85			DEA and Taxonomy: 0.86		PCA and Taxonomy: 0.83		

with NT rankings of power plants in 2000 are shown in Table 7.

DEA

LP model (7) does not allow for ranking of efficient units as it assigns a common index of one to all the efficient power plants in the data set. Therefore, modified model by Andersen and Petersen for DEA based ranking purposes is applied as followsr:[28]:

$$
\begin{aligned}
e_o &= \text{Min } \theta \\
\text{s.t.} \quad \theta x_{io} &\geq \sum_{j=1, j \neq j_o}^{n} \lambda_j x_{ij}, \quad i = 1, \ldots S, \\
y_{1o} &\leq \sum_{j=1, j \neq j_o}^{n} \lambda_j y_{1j} \\
\lambda_j &\geq 0, \quad j \neq o.
\end{aligned}
\qquad \ldots(8)
$$

Model (8), which excludes DMU_o, is under evaluation from the input-output constraints so that the efficient units are assigned an index (>1) and the index for inefficient units is identical with that of model (7). DEA results show that 19 out of 40 power plants are relatively efficient. However, exact ranking cannot be obtained for these DMUs.

PCA

There are 5 variables and 40 DMUs, and $d_{jir} = y_{rj}/x_{ij}$ $(i = 1 \ldots 5;\ r = l)$ for each DMU_j $(j = l \ldots 40)$. Bigger the d_{jir}, better the performance of DMU_j in terms of r^{th} output and i^{th} input. Let $d_{jk} = d_{jir}$ where $k = 1, \ldots, 5$ and $5 = 5*1$. It is needed to find some weights that combine 5 individual ratios of d_i for DMU_j. Consider the following 40*5 data matrix composed by d_{jk}: $D = (d_1, \ldots, d_5)_{40*5}$ with each row representing 5 individual ratios of d_i for each DMU and each column representing a specific output/input ratio, i.e, $d_k = (d_{k1}, \ldots, d_{k40})^T$. PCA is employed to find out principal components, which are respectively different linear combinations of $d_1 \ldots d_5$ so that principal components can be combined by their Eigen values to obtain a weighted measure of d_{jk}. PCA process of D is carried out as:

Step 1 —Calculate the sample mean vector and covariance matrix S.

Step 2 —Calculate the sample correlation matrix R.
Step 3 —Solve the following equation:

$$\left|R - \lambda I_p\right| = 0$$

It is obtained the ordered p characteristic roots (Eigen values) $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_5$ with $\Sigma\lambda_j = 5$ *(j =1, ..., 5)* and the related p characteristic vectors (Eigen vectors) $(l_{m1}, l_{m2}, ..., l_{m5})$ *(m =1 ,...,5)*. Those characteristic vectors compose principal components Y_i. Components in Eigen vectors are respectively the coefficients in each corresponding Y_i:

$$Y_m = \sum_{j=1}^{p} l_{mj}\hat{x}_{ij} \quad \text{for} \quad m = 1...5 \quad \text{and} \quad i = 1...40$$

Step 4 —calculate the weights (w_i) of principal components and PCA scores (z_i) of each DMU (*i = 1, ..., 40*). Furthermore, the *z* vector $(z_1, ..., z_5)$ where z_j shows the score of j^{th} DMUs is given by:

$$z = \sum_{j=1}^{5} w_j Y_j \qquad i = 1...40$$

Numerical Taxonomy

NT approach is as follows:
Step 1 —There are 5 variables and 40 DMUs, and $t_{jir} = y_{rj}/x_{ij}$ *(i = 1,...,5; r = 1)* for each DMU_j *(j = l, ..., 40)*. It can be shown by a *40*5* data matrix composed by t_{jk}: $T = (t_1, ... ,t_5)_{40*5}$. Each row represents p individual ratios of t_i for each DMU and each column a specific output/input ratio.

Step 2 —The 40 by 5 matrix is standardized such that all variables have mean of 0 and variance of 1.

Step 3 —Distance of every two DMUs for each variable is computed to homogenize DMUs. Therefore, distance matrix $D = [d_{ij}]_{5*5}$ and vector $d = [d_i]_{5x1}$ where d_i is the minimum of ith row of matrix D are identified. To identify homogenous power plants, upper (*L1*) and lower (*L2*) limits of vector d is computed as $L1 = \ + 2s_d$ and $L2 = \ - 2s_d$ where and s_d are mean and standard deviation of vector d respectively. If all d_{ij} are within *L1* and *L2*, homogeneity is achieved and could go to next step. Otherwise, cluster analysis is performed until all DMUs are homogenous.

Step 4 —Distance of each DMU from the ideal DMU for each variable is computed as:

$$c_i = \sqrt{\sum_{j=1}^{5}(\hat{t}_{ij} - \hat{t}_{j_{\max}})^2}$$

where, $\hat{t}_{j_{\max}}$ is the maximum of the j^{th} variable. Growth level for each DMU is:

$$f_i = \frac{c_i}{c^*}$$

where $c^* = \bar{c} + 2s_c$, and $\bar{c}$ and s_c are mean and standard deviation of c_i 's respectively. f_i 's are between 0 and 1 with 1 as the worst and 0 as the best scores.

Validation and Verification

There is a direct relationship between DEA, PCA and NT in terms of data set with respect to 40 power plants (Table 7). Particularly, the Spearman tests statistic ($r_s > 0.8$) indicate a strong direct relationship. Looking into efficiency scores and ranking for regional companies (Table 8) and average of power plants (Table 9) in 2000, esf-AVR has the best performance and teh-AVR has the worst performance because of existing surplus in installed capacity and labor and internal power. Combined cycle plants (Table 9) operate better than the other kind of plants because of their higher efficiency. Also, gas plants because of their low levels of production with respect to steam plants have the lower efficiency. By notice of the design of gas plants, gas turbines are seen as the quick way of adding new generating capacity. In general, gas turbines tend to be used for peaking purposes because they require much quality fuel and have relatively low technical efficiency.

The procedure for year 2000 is also repeated for 1997 to 1999. The results indicate that C-far, T-sal and C-gil

Table 8— Efficiency scores and ranking for regional companies

DMU	Efficiency	Rank
esf-AVR	0.98879	21
far-AVR	0.98807	22
khz-AVR	0.97799	26
tav-AVR	0.95030	29
azr-AVR	0.92431	31
khr-AVR	0.85042	38
teh-AVR	0.80848	39

Table 9 — Efficiency scores and ranking for plant's average

DMU	Efficiency	Rank
C-AVR	0.99614	20
S-AVR	0.98667	23
G-AVR	0.80696	40

in view of average ranking during the 4 years are ranked first to third, respectively. Combined cycle power plants in these 4 years have operated better than other types of power plants.

Conclusions

This study presented an integrated DEA methodology for assessment and optimization of conventional power plants (gas, steam and combined cycles) with respect to international inputs and outputs. DEA provided a comprehensive analysis of relative efficiencies of DMUs with respect to multiple inputs and output by evaluating each DMU and measuring its performance relative to an envelopment surface composed of other DMUs. Efficiency scores of 40 power plants in Iran and ranking of regional electricity companies during 1997-2000 were analyzed, evaluated and optimized. PCA and NT together with Spearman correlation technique were used to verify and validate the findings of DEA-BCC approach. By utilizing proposed model of this paper, management can obtain detailed information about the performance of each power plant. Moreover, the means of improving efficiency of power plants and optimum resource allocation to power plants could be identified. The unique features of this study are: 1) utilization of an integrated DEA-BCC model for assessment and optimization of thermal power plants and regional companies; and 2) Utilization of a robust PCA-NT approach for verification and validation of DEA-BCC approach.

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